International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
Maths 2020; 5(2): 92-99

www.mathsjournal.com
Received: 04-05-2020
Accepted: 06-06-2020

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Analysis of manpower system using multi-absorbing states Markov chain

VO Ezugwu and LI Igbinosun

Abstract
Previous studies have used single-state absorbing Markov chain in prediction of manpower systems where retired staff and dropout are lumped together. In this study, we examine the necessity of separating the two. This is achieved using multi--absorbing states Markov chain. This method gives a clearer picture on how wastages occur in academic manpower systems.

Keywords: Markov chain, multi-absorbing states, transition probability, manpower planning, wastages

Introduction

Effective manpower planning is an enormous task in managing large organizations such as federal and state administrations, large companies or academic institutions. These systems consist of employees at different grades with specific roles and job descriptions. Manpower planning is the process by which the management determines how the organization should move from its current manpower position to its desired manpower position. Through planning, management strives to have the right number and right kinds of people, at the right places, at the right time, doing things which result in both the organization and individual receiving maximum long-run benefit. Collings and Wood (2009) [2] defined manpower planning as the range of philosophies, tools and techniques that any organization should deploy to monitor and manage the movement of staff both in terms of numbers and profiles. The prediction of manpower is subject to how the current supply of employees will change internally, Kwon, *et al* (2015). These changes are observed by analyzing what happened in the past, in terms of staff retention and movement, and extrapolating into the future to see what happens with the same trend of the past. Manpower planning consists of series of activities viz:

1. Forecasting future manpower requirements, either in terms of mathematical projection of trend in the economic environment and development in industry, or in terms of judgment estimates based upon the specific plans of a company.
2. Making an inventory of present manpower resources and assessing the extent to which these resources are employed optimally.
3. Anticipating manpower problems by projecting present resources into the future and comparing them with the forecast of requirement to determine their adequacy both quantitatively and qualitatively.
4. Planning the necessary programs of requirement, selection, training, development, utilization, transfer, promotion, motivation and compensation to ensure that future manpower requirements are properly met.

Before now, some studies like Igboanugo and Edokpia (2014) [7], Ekhosuehi *et al* (2015) [5], Ogbogo *et al* (2013) [11], Rahim (2015), Ezugwu and Ologun (2017) [6] modeled a hierarchal system with single absorbing state for academic and organizational systems, where dropouts and retired staff were lumped together. In other words, little or no work has been done in separating the wastages (retired and the dropouts). In this paper, an attempt was made to group the retired staff and the dropouts into two absorbing states. The retired staff encapsulates those who successfully retired from the system by years of service or by age, while the dropouts encapsulates those who dropped out of the system by resignation, sack, death or ill- health. In this study, the interest is in the probability of staff in each grade to retire and the expected length of time to retire. A particular interest is in the probability of staff to retire as a professor and the proportion of staff who are serving professors, as the desire of every academic staff is to reach the climax (i.e become a professor). Unfortunately, not every academic staff becomes a professor before retirement. Also, professors are the pillar on which university education leans.

In this study, the academic manpower system of University of Uyo, Nigeria, is modeled using Markovian models where the retired staff are separated from the dropouts. In the university, vacancies are filled by promotion from the serving staff or by appointment from outside the system after an advert and a thorough interview. Promotion is done every year but every academic staff member is entitled to promotion from current grade to the next higher grade every 3 years after the member has completed or fulfilled the necessary requirements/ conditions for promotion peculiar to the particular origin grade. Otherwise, there is no promotion for that particular staff, hence, the reason for repetition. In the case of Graduate Assistant, he is immediately upgraded to Assistant Lecturer once he presents his M.Sc certificate. In this work, we assume that no staff member is given accelerated promotion and that there is no demotion. In the university, we have categories of academic staff: Graduate Assistants (GA), Assistant Lecturer (AL), Lecturer II (LII), Lecturer I (LI), Senior Lecture (SL), Associate Professor (AP), and Professor (P). These grades form the transient states of the Markov chain. A state in Markov chain is transient if and $p_{ij} \succ 0$ and $p_{ji} = 0, \ \forall i, j$. All the states are transient since all the movements are gearing towards absorption and away from the grades. The absorbing states of the Markov chain are the group of retired staff and group of dropouts. So, in all, we have 7 transient states and 2 absorbing states. A state in a Markov chain is absorbing if $p_{ii} = 1$ and $p_{ij} = 0, \ \forall i, j$. A Markov chain is called an absorbing Markov chain if there exists at least one absorbing state and it is possible to transition from a transient state to absorbing state perhaps in multiple steps. It is unlike a regular Markov chain for which transition matrix is primitive, primitive in the sense that it requires that $p_{ij} \prec 1, \forall i$. Regular chains never get stuck in a particular state.

Literature Review

Markov chain has been a useful tool in prediction and it has been used extensively in manpower planning. Belhaj and Tkiouat (2013) [1] presented a predictive model of numbers of employees in a hierarchal time-dependent system of human resources incorporating sub-systems, that each contains grades of the same family. The proposed model was motivated by the reality of staff development which confirms that the path evolution of each employee is usually in its family of grades. Ekhosuehi and Osagiede (2012) [4] derived a transition matrix for multi-echelon educational system using logistic and Markov chain theoretical methodologies. They used the school differential variables as the explanatory variables of the logistic model and the transition matrix of the Markov chain is the non-homogeneous empirical transition matrix (NHETM). Ekhosuehi (2013) [3] examined the passage of staff in a faculty using one state absorbing Markov chain. He considered two cases involving regardless of staff leaving intention and staff unwillingness to leave. Little effort has been made to characterize manpower flow in relation to manpower policies deriving such system in order to ascertain if they are in congruent with the rai-son d'etre of the institution. Igboanugo and Onifade (2011) [8] used Markov chain statistical tool to unravel the dynamics of staff stock and flow in a typical first generation Nigerian university with the ultimate intent of lucidly describing the existing manpower policy and pointing to its future direction. Human resources are considered as the most crucial, volatile and potentially unpredicted resource which an organization utilizes. However, manpower planning seeks to make a link between strategy, structure, and people more explicit. Parma *et al* (2013) [13] strived to get a better matching between manpower requirements and manpower availability. Ossai and Uche (2009) [12] considered the condition of maintainability with an aspect of manpower planning and control that gives the management a desirable and acceptable free hand to what looks like wastage control. Ogbogo *et al.* (2013) [11] studied the behavior of academic staff grade transition of a polytechnic institution in Nigeria using a single state absorbing Markov chain. Their objective was to determine the proportion of staff recruited, promoted and withdrawn from various grades in the institution over the years and also forecast the expected manpower structure of the institution. Touama (2015) [15] applied Markovian models and transition probability matrix to analyze the work force movement in Jordan productivity companies. His objective was to find out the product of the company that has the highest retention and lowest loss. Wan-yi and Shou (2015) [17] focused on the improved gray Markov model in human resource internal supply forecasting, so that the enterprises can reasonably predict their internal human resource supply through this method and provide an important guarantee for the enterprises to develop human resources strategic planning.

Materials and Methods

The method applied is first order time-independent Markov chain with the property that

$$p\left[X_n = j / X_0, = i_0 X_1 = i_1, \ldots, X_{n-1} = i\right] = p\left[X_n = j / X_{n-1} = i\right], \ \forall i, j \ \ldots\ldots\ldots (1)$$

This means that if the process is in state at time t, the probability that it transitions to state j at time $t+1$ depends only on its current state and not on the preceding states. The basic manpower model is given by

$$n_j(t+1) = \sum_{i=1}^{m} p_{ij}(t) n_i(t) + r_j(t), \ \forall i, j = 1,2,\ldots,m \ \ldots\ldots\ldots (2)$$

Equation (2) can be expressed as

$$n_j(t+1) = p_{ii}(t) n_i(t) + p_{ij}(t) n_i(t) + r_j(t), \ \forall i, j = 1,2,\ldots,m \ \ldots\ldots\ldots (3)$$

This is the number of staff in grade j at time $t+1$ and

$n(t)=\sum_{j=1}^{m} n_j(t+1)=\sum_{j=1}^{m}\sum_{i=1}^{m} p_{ij}(t)n_i(t)+\sum_{j=1}^{m} r_j(t)$,………………………………………. (4) is the total number of staff in the system at time $t+1$. The structure of the system at any given time t is

$$n(t)=[n_1(t), n_2(t),..., n_m(t)]. \quad ....(5)$$

Where $n_j(t)$ is the number of staff in grade j at the beginning of the period t. $p_{ii}(t)$ is the probability that a member of staff in grade i will still be in grade within the time period $(t, t+1)$. $p_{ij}(t)$ is the probability that a member of staff will be promoted from grade i to grade j within the time period $(t, t+1)$ and $r_j(t)$ is the number of recruits into grade j at time t.

We consider a system with two groups of states, S and U where $S=\{1,2,...,m\}$ is a set of m transient states corresponding to the grades of the system. The states are mutually exclusive and collectively exhaustive. $U=\{1,2,...,r\}$ is a set of absorbing states corresponding to the various forms of wastages. Then, $N=m+r$, the total number of states in the system.

Transition Probability Matrix

The transition probability matrix (TPM) of a Markov chain with multi-absorbing states is represented in the form

as
$$P=\begin{bmatrix} 1 & 0 & 0 & . & . & . & 0 & . & 0 & 0 & 0 & . & . & . & 0 \\ 0 & 1 & 0 & . & . & . & 0 & . & 0 & 0 & 0 & . & . & . & 0 \\ 0 & 0 & 1 & . & . & . & 0 & . & 0 & 0 & 0 & , & , & , & 0 \\ . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\ r_{11} & r_{12} & r_{13} & \cdot & , & , & r_{1r} & , & q_{11} & q_{12} & q_{13} & \cdot & \cdot & \cdot & q_{1m} \\ r_{21} & r_{22} & r_{23} & \cdot & \cdot & \cdot & r_{2r} & \cdot & q_{21} & q_{22} & q_{23} & \cdot & \cdot & \cdot & q_{2m} \\ . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\ r_{m1} & r_{m2} & r_{m3} & \cdot & \cdot & \cdot & r_{mr} & \cdot & q_{m1} & q_{m2} & q_{m3} & \cdot & \cdot & \cdot & q_{mm} \end{bmatrix} \quad ....(6)$$

Expressing equation (6) in a canonical form, we have

$$P=\begin{bmatrix} I & 0 \\ R & Q \end{bmatrix} \quad ....(7)$$

Where, I is an $r \ x \ r$ identity matrix. This gives the transition probabilities among the absorbing states. 0 is a $r \ x \ m$ matrix of zeros which gives the transition probabilities from absorbing states to transient states. Hence, transition from absorbing states to transient states is not possible R is an $m \ x \ r$ matrix which gives the absorption probabilities Q is an m matrix which gives the transition probabilities among the transient states, $S=\{1,2,...,m\}$ such that $\sum_{i=1}^{m} p_{ij} \prec 1$.

Iterative multiplication of equation (7) yields

$$p^2=\begin{bmatrix} I & 0 \\ R & Q \end{bmatrix}\begin{bmatrix} I & 0 \\ R & Q \end{bmatrix}=\begin{bmatrix} I & 0 \\ R+QR & Q^2 \end{bmatrix} \quad ....(8)$$

$$P^3=\begin{bmatrix} I & 0 \\ R+QR & Q^2 \end{bmatrix}\begin{bmatrix} I & 0 \\ R & Q \end{bmatrix}=\begin{bmatrix} I & 0 \\ R+QR+Q^2R & Q^3 \end{bmatrix} \quad ....(9)$$

In general,

$$p^n = \begin{bmatrix} I & 0 \\ \left(I + Q + Q^2 + \ldots + Q^{n-1}\right)R & Q^n \end{bmatrix} \ldots\ldots(10)$$

We note here that R and 0 are not necessarily square. Taking the limit of equation (10) as $n \to \infty$, we have

$$\lim_{n\to\infty} p^n = \lim_{n\to\infty} \begin{bmatrix} I & 0 \\ \left(1+Q+Q^2+\ldots+Q^{n-1}\right)R & Q^n \end{bmatrix} = \begin{bmatrix} I & 0 \\ HR & 0 \end{bmatrix} \ldots\ldots(11)$$

Since $\lim_{n\to\infty} I = 1$, $\lim_{n\to\infty} 0 = 0$ and $\lim_{n\to\infty} Q^n = 0$, because the sum of every row of Q is strictly less than 1 and the largest eigenvalue of is less than 1. In other words, Q is a sub-matrix, Where $H = I + Q + Q^2 + Q^3 + \ldots = (1-Q)^{-1}$. H is called the fundamental matrix (FM) for the absorbing chain.

$$H(i, j) = Q^0(i, j) + Q\ (i, j) + Q^2(i, j) + Q^3(i, j) + \ldots\ldots \ldots\ldots(12)$$

Where $Q^n(i, j)$ is the probability that a member of staff who starts in the grade will occupy the grade in time period.

Prediction of Withdrawal and Retirement Rates

In the canonical form, the dropout and retirement rates for grade i n years later is obtained by

$$r_{ik}^{(n)} = \sum_{j=1}^{n} p_{ij}^{(n-1)} r_{ik}, i, j = 1,2,\ldots,k \ldots\ldots(13)$$

Where $p_{ij}^{(n-1)}$ is the probability that an academic staff in grade i at time j $(n-1)$ will be in grade years later r_{ik} is the probability that an academic staff in grade at time withdraws or retires from service at k. r_{ik} is the $(i, k)th$ element of the product $Q^{n-1}R$.

Therefore the cumulative withdrawal or retirement rates is obtained by

$$r_{ik}^{(x)} = \sum_{j=1}^{x} r_{ik}^{n}, i, j = 1,2,\ldots,t, k = 1,2,\ldots,r \ldots\ldots(14).$$

$r_{ik}^{(x)}$ is the $(i, k)th$ element of $\left(I + Q + Q^2 + Q^3 + \ldots + Q^{N-1}\right)R$

Expected Length of Stay before Absorption

The expected number of years before dropout or retirement can be obtained using

$$H = 1 + Q + Q^2 + Q^3 + \ldots = (1-Q)^{-1} \ldots\ldots(15)$$

The total number of years a member of staff will stay in the system before dropout or retirement is

$$(1-Q)^{-1} e \ldots\ldots(16)$$

Where e is a k x 1 column vector of ones

Absorption Rates

Let b_{ij} be the probability that an absorbing chain will be absorbed in the absorbing state j if it starts in the transient state i. Let B be the matrix with entries b_{ij}.

Then B is the $m \times r$ matrix and $B = HR$ ....(15)

Where, H is the fundamental matrix and R is as defined in the canonical form.

That is $B = (1-Q)^{-1} R$ ....(16)

Estimation of Transition Probability

Let $n_{ij}(t)$ be the number of staff flow from grade i to grade j in time t and $n_{i0}(t)$ be the number of staff leaving grade i in period t. These flows of staff are governed by transition probabilities and the grades are assumed independent with respect to these probabilities. Flows from grade i to grade j follow a multinomial distribution with probabilities $p_{ij}(t)$ $\forall i, j = 1,2,...,m$ (Uche and Ezepue, 1991) [16]. The distribution is

$$P[n_{I1}(t), n_{i2}(t), ..., n_{im}(t)] = \frac{\left(\sum_{j=1}^{m} n_{ij}(t)\right)!}{\prod_{j=1}^{m} (n_{ij}(t))!} \prod_{j=1}^{m} p_{ij}^{n_{ij}(t)}(t) \text{ ....(16)}$$

Using the likelihood function according to Lindgreen (1993) [10], equation (14) becomes

$$\hat{p}_{ij}(t) = \frac{n_{ij}(t)}{n_j(t)} \quad \forall i, j = 1,2,...,m \text{ ....(17)}$$

By stationarity assumption, the maximum likelihood estimates of p_{ij} are obtained by pooling equation (17)

$$\hat{p}_{ij} = \frac{\sum_{t=1}^{T} n_{ij}(t)}{\sum_{t=1}^{T} n_j(t)} \text{ ....(18)}$$

Where $t = 1,2,...,T$

Data

Table 1: Academic staff flow based on promotion and wastages for 2012/2013 session

	1	2	3	4	5	6	7	W_i	R_i	$n_i(t)$
1	102	0	0	0	0	0	0	0	0	102
2	0	189	5	0	0	0	0	1	0	195
3	0	0	221	6	0	0	0	4	0	231
4	0	0	0	269	17	0	0	3	0	289
5	0	0	0	0	279	15	0	0	1	295
6	0	0	0	0	0	101	5	2	2	110
7	0	0	0	0	0	0	233	2	6	241

Table 2: Academic staff flow based on promotion and wastage for 2013/2014 session

	1	2	3	4	5	6	7	W_i	R_i	$n_i(t)$
1	88	1	0	0	0	0	0	2	0	91
2	0	170	10	0	0	0	0	2	0	182
3	0	0	221	12	0	0	0	4	0	237
4	0	0	0	311	14	0	0	1	0	326
5	0	0	0	0	264	7	0	1	2	274
6	0	0	0	0	0	95	2	0	2	99
7	0	0	0	0	0	0	0	201	1	2

Table 3: Academic staff flow based on promotion and wastage for 2014/2015 session

	1	2	3	4	5	6	7	W_i	R_i	$n_i(t)$
1	117	0	0	0	0	0	0	2	0	119
2	0	177	9	0	0	0	0	2	0	188
3	0	0	251	15	0	0	0	6	0	272

4	0	0	0	292	25	0	0	2	1	320
5	0	0	0	0	326	9	0	0	0	335
6	0	0	0	0	0	125	2	1	1	129
7	0	0	0	0	0	0	200	1	3	204

Table 4: Academic staff flow based on promotion and wastages for 2015/2016

	1	2	3	4	5	6	7	W_i	R_i	$n_i(t)$
1	94	3	0	0	0	0	0	1	0	98
2	0	152	13	0	0	0	0	1	0	166
3	0	0	242	9	0	0	0	4	0	255
4	0	0	0	266	21	0	0	0	1	288
5	0	0	0	0	353	12	0	2	2	269
6	0	0	0	0	0	111	4	0	1	116
7	0	0	0	0	0	0	210	1	2	213

Table 5: Cumulative Academic staff flow for all the sessions

	1	2	3	4	5	6	7	W_i	R_i	$n_i(t)$
1	362	43	0	0	0	0	0	5	0	410
2	0	651	74	0	0	0	0	6	0	731
3	0	0	891	88	0	0	0	16	0	995
4	0	0	0	1101	114	0	0	6	2	1223
5	0	0	0	0	1137	126	0	3	7	1273
6	0	0	0	0	0	407	33	4	10	454
7	0	0	0	0	0	0	783	19	48	862

Results and Discussions

From the pooled academic staff flow based on promotion and withdrawal, the transition probability matrix P is obtained

$$
\begin{array}{c} \\ \\ \\ p = \end{array}
\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array}
\begin{array}{ccccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 & \mathrm{W} & \mathrm{r} \\
0.8829 & 0.1049 & 0 & 0 & 0 & 0 & 0 & 0{,}0122 & 0 \\
0 & 0.8906 & 0.1012 & 0 & 0 & 0 & 0 & 0.0082 & 0 \\
0 & 0 & 0.8955 & 0.0884 & 0 & 0 & 0 & 0.0161 & 0 \\
0 & 0 & 0 & 0.9002 & 0.0931 & 0 & 0 & 0.0049 & 0.0016 \\
0 & 0 & 0 & 0 & 0.8932 & 0.0990 & 0 & 0.0024 & 0.0055 \\
0 & 0 & 0 & 0 & 0 & 0.8965 & 0.0727 & 0.0088 & 0.0220 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.9084 & 0.0220 & 0.0696
\end{array}
$$

Putting the transition probability matrix P in canonical form, we have

$$
P = \begin{array}{c} \\ W \\ R \\ . \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array}
\begin{array}{ccccccccccc}
\mathrm{W} & \mathrm{r} & & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\
1 & 0 & . & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & . & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
. & . & . & . & . & . & . & . & . & . \\
0.0122 & 0 & . & 0.8829 & 0.1049 & 0 & 0 & 0 & 0 & 0 \\
0.0082 & 0 & . & 0 & 0.8906 & 0.1012 & 0 & 0 & 0 & 0 \\
0.0161 & 0 & . & 0 & 0 & 0.8955 & 0.0884 & 0 & 0 & 0 \\
0.0049 & 0.0016 & . & 0 & 0 & 0 & 0.9002 & 0.0931 & 0 & 0 \\
0.0024 & 0.0055 & . & 0 & 0 & 0 & 0 & 0.8932 & 0.0990 & 0 \\
0.0088 & 0.0220 & . & 0 & 0 & 0 & 0 & 0 & 0.8965 & 0.0727 \\
0.0022 & 0.0696 & . & 0 & 0 & 0 & 0 & 0 & 0 & 0.9084
\end{array}
$$

Table 6: Prediction of Dropout and Retirement Rates in Next Seven Years.

	1		**2**		**3**		**4**		**5**		**6**		**7**	
SESSION	**D**	**R**	**D**	**R**	**D**	**R**	**D**	**R**	**D**	**R**	**D**	**R**	**D**	**R**
2016//2017	0.0122	0.0000	0.0082	0.0000	0.0161	0.0000	0.0049	0.0016	0.0024	0.0055	0.0880	0.0220	0.0220	0.0696
2017/2018	0.0238	0.0000	0.0171	0.0000	0.0310	0.0001	0.0095	0.0036	0.0054	0.0126	0.0183	0.0468	0.0420	0.1328
2018/2019	0.0258	0.0000	0.0266	0.0000	0.0447	0.0004	0.0140	0.0060	0.0090	0.0214	0.0269	0.0694	0.0435	0.1374
2019/2020	0.0369	0.0000	0.0364	0.0000	0.0574	0.0009	0.0183	0.0009	0.0132	0.0319	0.0371	0.0976	0.0600	0.1896
2020/2021	0.0475	0.0000	0.0464	0.0000	0.0691	0.0016	0.0226	0.0127	0.0180	0.0441	0.0475	0.1267	0.0750	0,2370
2021/2022	0.0579	0.0000	0.0565	0.0001	0.0800	0.0025	0.0267	0.0171	0.0233	0.0578	0.0579	0.1562	0.0886	0.2800
2022/2023	0.0682	0.0000	0.0666	0.0003	0.0901	0.0037	0.0311	0.0244	0.0291	0.0730	0.0682	0.1858	0.1010	0.3191

Where D = Dropout and R = Retired

Table 7: Ex Pected Length of Stay

$$(1-Q)^{-1} = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array}\begin{bmatrix} 8.5397 & 8.1884 & 7.9293 & 7.0240 & 6.1230 & 5.8568 & 4.6484 \\ 0 & 9.1408 & 8.8521 & 7.8409 & 6.8351 & 6.5280 & 5.1890 \\ 0 & 0 & 9.5694 & 8.4763 & 7.3890 & 7.0677 & 5.6094 \\ 0 & 0 & 0 & 10.0200 & 8.7343 & 8.3549 & 6.6310 \\ 0 & 0 & 0 & 0 & 9.3633 & 8.9562 & 7.1082 \\ 0 & 0 & 0 & 0 & 0 & 9.6618 & 7.6683 \\ 0 & 0 & 0 & 0 & 0 & 0 & 10.9170 \end{bmatrix}$$

$$(1-Q)^{-1}e = \begin{bmatrix} 48.3096 \\ 44.3859 \\ 38.118 \\ 33.7402 \\ 25.4277 \\ 17.3301 \\ 10.917 \end{bmatrix}$$

Table 8: Dropout and Retirement Rates In The Long Run

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array}\begin{bmatrix} D & R \\ 0.5029 & 0.4971 \\ 0.4441 & 0.5559 \\ 0.4013 & 0.5987 \\ 0.2895 & 0.7195 \\ 0.2577 & 0.7423 \\ 0.2427 & 0.7573 \\ 0.2402 & 0.7598 \end{bmatrix}$$

From the transition probability matrix, the major diagonal elements (repetition probability) of matrix are relatively large, while the upper diagonal elements (promotion probabilities) are relatively small, indicating that on average, an insignificant proportion of staff in each grade is promoted every year. This could be that either the academic staff is not actively involved in research to meet conditions/requirements for promotion or the management deliberately stagnate some staff. In table 6, it is observed that the dropout increased across the session but decreased across the grade, up to grade 5. It is also observed there is no or very insignificant proportion of retired staff across the grade and across the session up to grade 4, indicating that retirement occurs mainly at grades 5-7 because of age or number of years of service.

Data on academic staff flow in the university were obtained from personnel department of the university from 2012/2013 to 2015/2016 academic sessions. The data are presented in tables 1-4. The sessions fall within the period when the retirement age is 65years for grades 1-5 and 70years for grades 6-7(professorial cadre). The maximum career length for university system in Nigeria does not exceed 35years. From table 7, the expected length of stay in the system, we observed that grades from grades 1-3 violate the maximum career length of 35years. This could be attributed to the fact that many of them in grades 1-3 could have entered the system at very young age and would be Associate professors or professors. Grades 4-7 do not violate the maximum career length. From table 8, the rate of dropout is higher in grades 1-3. The reason could be that those in grades 1-3 have greater opportunity for greener pasture due to their young age, while the rate of retirement is higher in grades 4-7. The reason could also be that those staff in grades 4-7 are old and close to their retirement either by age or by years of service.

Conclusion

Hitherto, most authors have been using single absorbing state markov chain to predict manpower system. This single-state absorbing markov chain lumps both the retirement and dropout together as wastages. However, it is imperative to examine the necessity of separating the two as this shows a clearer picture of how wastages occur in the system. In this study, we have been able to separate the two (retired from dropout) using multiple-absorbing states.

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