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On a statistical control of a technological process described by a two-dimensional normal distribution

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Abstract
In the papera methodology for controlling a technological process described by a two-dimensional normal distribution using new control charts is given, where the control value is Kolmogorov's statistics ρ_x and ρ_y and a correlation coefficient (r_{xy}). Using these control charts shows the control of the machine producing the shotgun shells (weapons or other shotgun shells). Control charts are based on Kolmogorov's consentcriterion and Fisher's criterion of linear dependence of the components of a random normal vector.

Keywords: Statistical hypothesis, kolmogorov's criterion, fisher's criterion, ρ_x-charts, ρ_y-charts, r_{xy}-charts

1. Introduction

The need for active methods of preventing defects in the manufacturing process has led to the development of methods and quality management systems based on methods of mathematical statistics. In this case, one of the most important problems is the control of the machine. In the paper,as an example, we will consider one of such machines manufacturing parts and show the statistical control of this process.

Let the shotgun shells (weapon shells, cylinder shells, etc.) with bore nominal diameter $d = 30mm$ be manufactured on a central lathe. The control of the bore diameter of the shotgun shells is carried out in two mutually perpendicular planes. Deviations from the nominal in these planes are denoted by X and Y. In [1] it was proved that these random variables are distributed normally with defined parameters and are linearly dependent.

Machine control described by a two-dimensional random vector(X, Y):

$$(X, Y) \sim N\left(\mu_{ox}, \mu_{oy}, \sigma_{ox}^2, \sigma_{oy}^2, \rho_{xy}^o\right).$$

In practice, these five parameters are taken as technical standards. Here μ_{ox}, μ_{oy} are mathematical expectations and $\sigma_{ox}^2, \sigma_{oy}^2$ are variances of X and Y respectively, ρ_{xy}^o is the correlation coefficient of X and Y.

If at certain moments $j = 1, 2, \dots, t$ we take a sample (or instantaneous sample) from (X, Y) with volume n:

$$(X_j, Y_j) = \{(x_{j1,} y_{j1}); (x_{j2}, y_{j2}), \dots, (x_{jn}, y_{jn})\},$$

evaluating the five parameters, we check the shape of the two-dimensional density concerning a given standard, and if one observes that the shape is almost preserved, then we say that the process is in a statistically controlled state. Note that deviations from the standard, can probably due to random reasons acting on the machine. Otherwise, we say that the process exits a statistically controlled state and is unpredictable, because of non-random factors which is affecting to the machine.

Mathematical verification of the above leads to the verification of statistical hypotheses

D)$H_0: F(x,y) = \Phi(x,y,\mu_{ox},\mu_{oy},\sigma_{ox}^2,\sigma_{oy}^2,\rho_{xy}^o)$;
$H_1: F(x,y) \neq \Phi(x,y,\mu_{ox},\mu_{oy},\sigma_{ox}^2,\sigma_{oy}^2,\rho_{xy}^o)$,

where $\Phi(x,y,\mu_{ox},\mu_{oy},\sigma_{ox}^2,\sigma_{oy}^2,\rho_{xy}^o)$ is the distribution function of the random normal vector (X,Y).
This task, in turn, is equivalent to testing the following three hypotheses;

A) $H_0: F(x,+\infty) = \Phi(x,\mu_{ox},\sigma_{ox}^2)$;
$H_1: F(x,+\infty) \neq \Phi(x,\mu_{ox},\sigma_{ox}^2)$.

B) $H_0: F(+\infty,y) = \Phi(y,\mu_{oy},\sigma_{oy}^2)$;
$H_1: F(+\infty,y) \neq \Phi(y,\mu_{oy},\sigma_{oy}^2)$.

C) $H_0: \rho_{xy} \geq \rho_{xy}^o$;
$H_1: \rho_{xy} < \rho_{xy}^o$.

Indeed, if D is valid, then knowing the density of (X,Y) one can find the densities of X and Y. Moreover,

$X \sim N(x,\mu_{ox},\sigma_{ox}^2)$ and $Y \sim N(y,\mu_{oy},\sigma_{oy}^2)$.

Using the conditional densities $f(^y/_x)$ and $f(^x/_y)$ [2] one may prove that X and Y are linearly dependent, i.e. C is valid, where f is the density function of the Gaussian distribution).
The inverse problem: "From A, B and C it follows D" is not always solvable. But our (X,Y) is distributed normally, therefore X and Y are linearly dependent and D is valid.
In practice, using of control charts (CC) is convenient and visual (see, for example, [3]) instead of testing hypotheses. Therefore, here we use the CC obtained in [4] and [5] to solve problems A, B and C. Further, we will give the basic rules arising from the proved theorems in [4] and [5].

2. Materials and Methods

The main characteristics of CC are LCL (lower control limit), UCL (upper control limit), CL (central line) and control value $K_j = K(X_j,Y_j), j = 1,2,\dots,t$. It is known that accuracy of CC are carried out through power functions and LCL and UCL can be found if one knows the distribution of the control value.
In [4] a CC was obtained using the Kolmogorov'sconsent criterion to verify the normality of the general set. Using these results, we construct ρ_x and ρ_y charts here to verify the normality of the sets X and Y.
In [5], based on the Fisher criterion, the characteristics of the ρ_{xy} chart was obtained, which checks the linear dependence of $X$and Y.
Here, using these results, we give rules for solving the above problem.
First, we give a rule for constructing a ρ-chart (ρ_y-chartcan be constructed analogously).
At the moment of times $j = 1,2,\dots,t$, we take samples (or instantaneous samples) $X_j = (x_{j1},x_{j2},\dots,x_{jn})$ and compose the variational series $X_j^* = (x_{j1}^*,x_{j2}^*,\dots,x_{jn}^*)$. For the control value, we select the Kolmogorov statistics

$$\rho_j(X) = \max_{1\leq k\leq n}\left|\Phi(x_{jk}^*,\bar{X}_j,S_{jx}^2) - \frac{2k-1}{2n}\right| + \frac{1}{2n}. \quad (1)$$

where $\Phi(x_{jk}^*,\bar{X}_j,S_{jx}^2) = \Phi(T_k^*), T_k^* = \frac{x_{jk}^*-\bar{X}_j}{S_{jx}^2}, \Phi(T_k^*) \sim N(0,1)$ is the standard normal distribution, $\bar{X}_j$ and S_{jx}^2 are mean and corrected standard deviation of sample $X_j, j = 1,2,\dots,t$.
The control limits of the ρ_x-chart was defined by the formula [4]:

$$UCL_{\rho_x} = \frac{k_{1-\alpha}}{\sqrt{n}} \quad (2)$$

Here $k_{1-\alpha}$ is the quantile of the distribution of statistics of Kolmogorov-Smirnov type [6]. This quantile is different from the quantile of the Kolmogorov statistics and affects the value of UCL_{ρ_x}. We will discuss about this during considering a problem with a machine producing shotgun shells.
By replacing X with Y and the corresponding quantities, we obtain formulas of the type (1) and (2) for the ρ_y-chart.
For the ρ_{xy}-chart, the control value will be the sample correlation coefficient determined by the formula:

$$r_j(x,y) = \frac{\sum_{i=1}^n (x_{ji}-\bar{X}_j)(y_{ji}-\bar{Y}_j)}{nS_n(X_j)S_n(Y_j)}, j = 1,2,\dots,t. \quad (3)$$

The control limits of the ρ_{xy}-chart are defined as follows (see [5]):

$$LCL_r = th(z_1); UCL_r = th(z_2), \quad (4)$$

where $z_1 = argth(\rho^0) - \frac{\psi(1-\alpha)}{\sqrt{n-3}}$, $z_2 = argth(\rho^0) + \frac{\psi(1-\alpha)}{\sqrt{n-3}}$, $th(z) = \frac{e^{2z}-1}{e^{2z}+1}$, $argth(\rho^0) = \frac{1}{2}\ln\frac{1+\rho^0}{1-\rho^0}$, $\psi(p)$ is p-quantile of the normal distribution-$N(0,1)$.(see[7]).

Remark 1. Since in problem C) the hypothesis
$H_0: \rho_{xy} \geq \rho_{xy}^o$,
is being tested, therefore, the ρ_{xy}-chart will be single-sided, i.e., in (4)$UCL_r = 1$.
The validity of the inequality $LCL_r < r_j < 1$ is sufficient to verify the linear dependence of X and Y. In this case, the power function of the ρ_{xy}-chart is determined by the following formula [5]:

$$G(r) = P(r \leq LCL) = \Phi(\frac{argth(LCL)-a(r)}{\sigma}), \quad (5)$$

where $a(r) \approx \frac{1}{2}\ln\frac{1+r}{1-r} + \frac{r}{2(n-3)}$, $\sigma \approx \frac{1}{\sqrt{n-3}}$; $n \geq 50$.

3. Results

First, using a perfectly working standard machine, we take samples of volume $n = 150$:
$(X_j, Y_j) = \{(x_{j1,}y_{j1}); (x_{j2}, y_{j2}), \dots, (x_{j150}, y_{j150})\}, j = 1$, then we find five parameters of the random vector (X, Y). Moreover, the nominal size of shotgun shells bore diameter is $d = 30$mm. Using these statistical observations the following parameters $\mu_{ox} = 7.82$mk, $\mu_{oy} = 7.86$mk, $\sigma_{ox} = S_{ox} = 4.86$mk, $\sigma_{oy} = S_{oy} = 5.50mk$, $\rho_{xy}^0 = 0.5$ were calculated.

Remark 2. In the case if there is no standard machine, we can get samples on a computer according to demand.
We find the control limits of ρ_x and ρ_y-charts for fixed $\alpha = 0.05, n = 150$. Moreover, the quantile $k_{1-\alpha}$ can be found from [6]. Calculations by (2) show that $UCL_\rho = 0.07$.

Remark 3. Ifwe find the value of $k_{1-\alpha}$ using the Kolmogorov function, then we would have $UCL_\rho = 0.11$. In this case, one would make a mistake $(0.11 - 0.07) * 100\% = 4\%$
in regulating the process.
The control limits of the ρ_{xy}-chart is found by formula (4). According to Remark 1 we have $UCL_r = 1$. The calculations show that $LCL_r = 0.39$.
Our CC are ready to control the machine producing shotgun shells. Next, at single moments $j = 1, 2, \dots, t, (t = 8)$ we take instantaneous samples with a volume of $n \leq 150$ and calculate the control quantities $\rho_j(x)$, $\rho_j(y)$ and $r_j(x, y)$ using formulae (1) and (3). Then we check CC inequality
$\rho_j(x) < 0.07$, $\rho_j(y) < 0.07$, and $0.39 < r_j(x, y) < 1$
on the diagram.
The validity of these inequalities means that at the moment *t*the machine producesthe high-quality shotgun shells.
At the same time, at single moments $j = 1,2, \dots, t$ we simultaneously find the values of the power function $G(r)$ in percent according to formula (5). A large percentage of the value of $G(r)$ means that one needs to stop the machine and looks for the causes of this signal.

Remark 4. Since our work is of a scientific and explanatory nature, therefore, further we take samples (or instant samples) of a different nature and check the standardity of the shotgun shells.

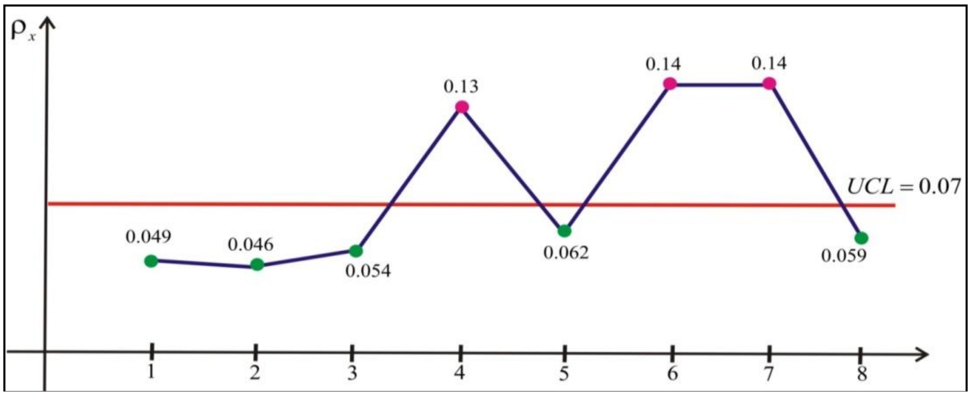
4. Discussion

Let that at single moments $j = 1, 2, \dots, 8$ from (X, Y) be taken instant samples with different volumes $n \leq 150$, found the values of the control values and the power function of the ρ_{xy}-charts. The results are provided in the following table.

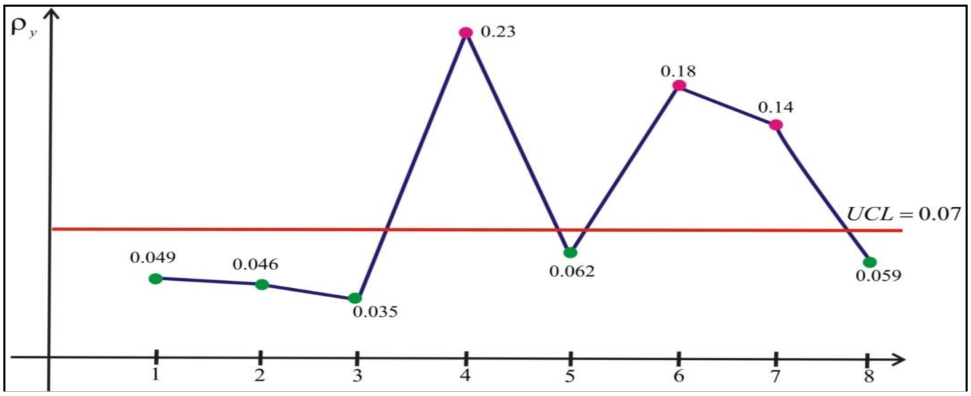
Table 1. The results are provided in the following table.

j	1	2	3	4	5	6	7	8
n	150	150	150	60	150	50	25	150
ρ_x	0.049	0.046	0.054	0.13	0.062	0.14	0.14	0.059
ρ_y	0.049	0.046	0.035	0.23	0.062	0.18	0.14	0.059
r_{xy}	0.60	0.99	0.47	0.18	0.99	0.02	0.11	0.56
$G(r_{xy})$	0.2%	0%	18%	96%	0%	99%	91%	1.4%

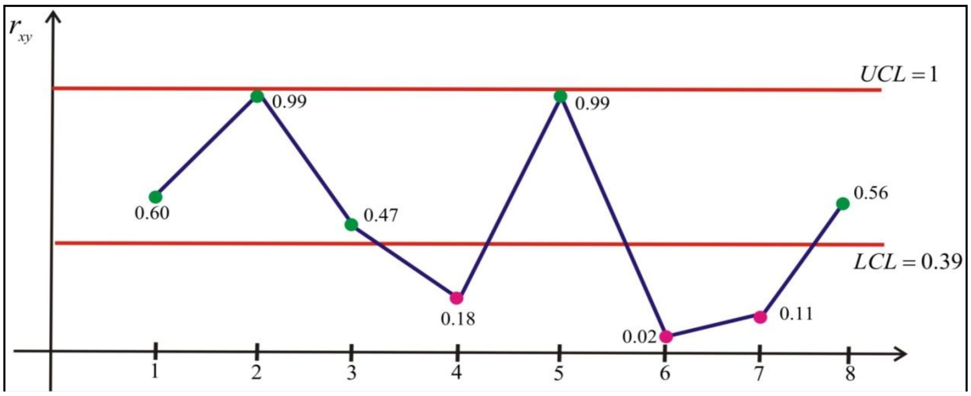
Now we give a graphical solution to the problem of monitoring the operation of the machine. In other words, we will construct diagrams of ρ_x, ρ_y and r_{xy} charts. If the values of the control values of ρ_j lower than $UCL_{\rho_x} = UCL_{\rho_y}$ =0.07,while the values of r_j greater than $LCL_r = 0.38$ and less than 1, then the shotgun shells qualities are high. Otherwise, one needs to find the causes and stabilize them.
Further, using table 1 and tusing founded CC characteristicswe construct diagrams that show the progress of working of the machine.



ρ_x-chart



ρ_y-chart



r_{xy}-chart

5. Conclusions

The diagrams of ρ_x, ρ_y and r_{xy}-charts show that at the moments$j = 1, 2, 3, 5$ and 8 the process is in a statistically controlled state. In other cases, together with specialists, one has to look for the reasons for leaving the process of the controlled state.

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