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Analysis of means based on Rayleigh variate

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Abstract
A measurable quality characteristic is assumed to follow a Rayleigh distribution which is an important skewed distribution in statistical inference and related fields. The statistical technique used in illustrating important variations among groups of data employed commonly in quality control, Analysis of Means methodology compares the mean of each group to the overall process mean to detect statistically significant differences is called Analysis of Means. This principle enables us to work out the decision lines. The preferabilility of the proposed Analysis of Means decision lines over that of Ott (1967) based on normal and half logistic distribution is illustrated by some examples.

Keywords: ANOM, confidence interval, Rayleigh

1. Introduction
In classical statistical inference confidence for unknown parameters of a statistical population is an inferential procedure. An application of confidence interval usually the 99.73% confidence interval when the variate follows normal distribution is the origin of the well knownshewart control charts. Construction of control charts using the theory of confidence intervals when the variate follows inverse Gaussian distribution is considered by Edgemen (1989) [1]. The justification Edgemen (1989) [1] is that the appeal to the central limit theorem for non-normal process control charts is not possible as sample size in the control chart analysis is usually 10 or less. Moreover quality variate such as product life is often better modeled by the probability distribution of a normal distribution.

If normal distribution is considered as a central model for any classical inferential procedure similar place can be attributed to exponential distribution in life testing and reliability studies. It is the only model exemplifying constant failure rate of a product which has life and eventual failure or death. On the other hand we know that ageing of the product is a primary criterion that contributes to its failure. Here models that represent aging phenomenon of as product are also equally desirable in problems of quality control and reliability. Among many such models Rayleigh distribution is an ageing model also called an increasing failure rate model. From a different version is generalization of weibull distribution with shape parameter 2 also. Kantam and SriRam (2001) [4] developed control charts to be used when the process variate follows Rayleigh distribution. Similarly study is made in Kantam *et al.* (2006). Kantam *et al.* (2012) [16] developed ANOM procedure for exponential and gamma variate. In this paper we made an attempt to study the ANOM procedure representing a quality characteristic modeled by Rayleigh distribution. The procedure of ANOM is presented in section 2 and respective illustrations are given in section 3 and are compared.

2. Analysis of Means
The Shewart control chart is as common tool of statistical quality control for many practioners. When these charts indicate the presences of an assignable cause (of non random variability) are adjustment of the process is made if the remedy is known. Otherwise the suspected presence of assignable cause is regarded to be an indication of heterogeneity of the sub group statistic for which the control chart is developed. For instance the statistic is sample mean, this leads to heterogeneity of process mean indicating departure from target mean. Such an analysis is generally carried out with the help of the well-known analysis of variance. Ott (1967) [8] developed a procedure called Analysis of Means (ANOM) to divide a collection of a given number of sub group means into categories such that means within a category are

homogeneous and those between categories are heterogeneous, under the assumption that the probability model of the of the variate is normal. In this section we make an attempt to develop the ANOM procedure of Ott (1967) [8] when the data variate is supposed to follow Rayleigh distribution.

Suppose $\bar{x}_1$, $\bar{x}_2$, $\bar{x}_3$,…….., $\bar{x}_k$ are arithmetic means of k sub groups of size 'n' each drawn from an Rayleigh distribution. If these subgroup means are used to develop control charts to assess whether the population from which these subgroups are drawn is operating admissible quality variations depending on the basic population model we may use the control chart constants developed by us or the popular Shewart constants given in any SQC text book. Generally we say that the process is in control if all the subgroup means fall within the control limits. Otherwise we say the process lacks control. If α is the level of significance of the above decisions we can have the following probability statements.

$$P\{ LCL < \bar{x}_i < UCL\} = 1-\alpha, \forall\ i = 1,2,\ldots\ k \qquad (2.1)$$

Using the notation of independent subgroups equation (2.1) becomes

$$P\{LCL < \bar{x}_i < UCL\} = (1-\alpha)^{1/k} \qquad (2.2)$$

With equitailed probability for each subgroup mean we can find two constants say L^* and U^* such that

$$P\{ \bar{x}_i \le L^*\} = \frac{1-(1-\alpha)^{1/K}}{2} \qquad (2.3)$$

$$P\{ \bar{x}_i \le U^*\} = \frac{1+(1-\alpha)^{1/K}}{2} \qquad (2.4)$$

Accordingly, these depend on the subgroup size 'n' and the number of subgroups 'k', We can make use of equation s (2.3) and (2.4)for specified values of 'n' and 'k' in order to get the percentile values called L^*,U^* for various values of'α'.

In the case of normal population L^* and U^* satisfy $U^* = -L^*$.For the skewed populations like Rayleigh distribution we have calculated L^*,U^* separately from the sampling distribution of $\bar{x}_i$, these L^* and U^* calculated for Rayleigh distribution are

$$L^* = \sqrt{\frac{2}{n}\log\left(\frac{2}{1+(1-\alpha)^{1/k}}\right)}, \quad U^* = \sqrt{-\frac{2}{n}\log\left(\frac{1-(1-\alpha)^{1/k}}{2}\right)}$$

$$P\{ L^*\hat{\sigma} < \bar{x}_i < U^*\hat{\sigma} \} = (1-\alpha)^k, \text{ where } \hat{\sigma} = \sqrt{\frac{\sum X_i^2}{2n}}$$

$$P\{ h_\alpha.\sqrt{\sum X_i^2} < \bar{x}_i < h_{1-\alpha}.\sqrt{\sum X_i^2} \} = (1-\alpha)^k \qquad (2.5)$$

$$\text{Where } h_\alpha = \sqrt{\frac{1}{n^2}\log\left(\frac{2}{1+(1-\alpha)^{1/k}}\right)}, \quad h_{1-\alpha} = \sqrt{-\frac{1}{n^2}\log\left(\frac{1-(1-\alpha)^{1/k}}{2}\right)}$$

$\because \bar{x}_i \in$(LDL, UDL) and are tabulated in respective Tables (2.1), (2.2), (2.3), (2.4), (2.5), (2.6). The LDL and UDL of Rayleigh distribution are given by

$$\therefore\ LDL = h_\alpha.\sqrt{\sum X_i^2}, \quad UDL = h_{1-\alpha}.\sqrt{\sum X_i^2}$$

A control chart for averages giving "In control" indicates that all the subgroup means though vary among themselves are homogeneous in some sense. This is exactly the null hypothesis in an analysis of variance technique. Hence the constants tables of (2.1), (2.2), (2.3),(2.4), (2.5),(2.6). can be used as an alternative to analysis of variance technique Which are presented at the end. For a normal population one can use the tables of Ott (1967). For Rayleigh population our Tables can be used. We therefore present below some examples for which the 'goodness of fit' of an appropriate model is assessed with Q-Q plot technique (strength of linearity between observed and theoretical quintiles of a model) and tested homogeneity of means involved in each case are given in table 3.1

Table 2.1: Rayleigh distribution constants of Analysis of means (1-α=0.9986)

						1-α =0.99865									
n → k↓	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**15**	**20**	**30**	**40**	**60**
2	0.0260 2.7020	0.1612 2.8273	0.0528 2.8981	0.0799 2.9473	0.0581 2.9849	0.0622 3.0153	0.0557 3.0407	0.0551 3.0626	0.0522 3.0818	0.0509 3.0988	0.0421 3.1636	0.0401 3.2087	0.0335 3.2713	0.0317 3.3150	0.0266 3.3756
3	0.0047 2.2062	0.02121 2.3085	0.0150 2.3663	0.0122 2.4065	0.0106 2.4371	0.0095 2.4620	0.0086 2.4827	0.0080 2.500	0.0075 2.5163	0.0071 2.5302	0.0067 2.5831	0.0055 2.6199	0.0039 2.6710	0.0033 2.7067	0.0027 2.7561
4	0.018374 1.9106	0.0130 1.9992	0.0106 2.0493	0.0092 2.0841	0.0082 2.1106	0.0075 2.1321	0.0069 2.1501	0.0065 2.1656	0.0061 2.1791	0.0058 2.1912	0.0047 2.2370	0.0041 2.2689	0.0034 2.3131	0.0029 2.3440	0.0024 2.3869
5	0.0164 1.7089	0.0116 1.7881	0.0095 1.8329	0.0082 1.8640	0.0073 1.8878	0.0067 1.9070	0.0062 1.9231	0.0058 1.9972	0.0055 1.9491	0.0052 1.9599	0.0042 2.0008	0.0037 2.0294	0.0030 2.0689	0.0025 2.0966	0.0021 2.1349
6	0.0150 1.56	0.2122 1.6323	0.0461 1.6732	0.0856 1.7016	0.0562 1.7339	0.0633 1.7409	0.0552 1.7555	0.0553 1.7682	0.0521 1.7793	0.0509 1.7891	0.0421 1.8265	0.0401 1.8526	0.0335 1.8886	0.0317 1.9139	0.0266 1.9489
7	0.013889 1.4443	0.2205 1.5113	0.0452 1.5491	0.0864 1.5754	0.0559 1.5955	0.6109 1.6117	0.0551 1.6253	0.0553 1.6370	0.0521 1.6473	0.0509 1.6564	0.0420 1.6910	0.0401 1.7151	0.0335 1.7486	0.0317 1.7719	0.0266 1.8043
8	0.012993 1.3510	0.2280 1.4136	0.0444 1.4490	0.0872 1.4736	0.0557 1.4924	0.0635 1.5076	0.0551 1.5204	0.0554 1.5313	0.0521 1.5409	0.0509 1.5494	0.0420 1.5818	0.0401 1.6044	0.0335 1.6356	0.0317 1.6575	0.0266 1.6878
9	0.0122 1.2737	0.2348 1.3328	0.0438 1.3662	0.0878 1.3894	0.0555 1.4071	0.0637 1.4214	0.0550 1.4334	0.0554 1.4437	0.0521 1.4528	0.0509 1.4608	0.0420 1.4913	0.0401 1.5126	0.0335 1.5421	0.0317 1.5627	0.0266 1.5913
10	0.0116 1.2083	0.2411 1.2644	0.0432 1.2961	0.0884 1.3181	0.0553 1.3349	0.0638 1.3485	0.0550 1.3599	0.0554 1.3696	0.0520 1.3782	0.0509 1.3858	0.0420 1.4148	0.0401 1.4350	0.0335 1.4630	0.0317 1.4824	0.0266 1.5096

Table 2.2: Rayleigh distribution constants of Analysis of means (1-α=0.9973)

							1-α =0.9973								
n → k↓	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**15**	**20**	**30**	**40**	**60**
2	0.0367 2.3018	0.1917 2.7019	0.0685 2.7758	0.0993 2.8272	0.0738 2.8664	0.07814 2.8980	0.0703 2.9244	0.0693 2.9472	0.0658 2.9671	0.06401 2.9848	0.0530 3.0519	0.0505 3.0987	0.0422 3.1634	0.040 3..2086	0.0336 3.2712
3	0.0300 2.0988	0.2122 2.2061	0.0652 2.2665	0.1018 2.3084	0.0729 2.3404	0.0786 2.3662	0.0701 2.3878	0.0694 2.4064	0.0658 2.4226	0.0641 2.4371	0.0530 2.4919	0.0505 2.5301	0.0422 2.5830	0.040 2.6198	0.0336 2.6709
4	0.0232 1.9106	0.2411 1.9992	0.0611 2.0493	0.1051 2.0841	0.0717 2.1106	0.0793 2.1321	0.0698 2.1501	0.0696 2.1656	0.0657 2.1791	0.0641 2.1912	0.0530 2.2370	0.0505 2.2689	0.0422 2.3131	0.04 2.3440	0.0336 2.3869
5	0.0232 1.6257	0.2411 1.7088	0.0611 1.7556	0.1051 1.7881	0.0717 1.8128	0.0793 1.8328	0.0698 1.8496	0.0696 1.8640	0.0657 1.8766	0.0641 1.8877	0.0530 1.9302	0.0505 1.9598	0.0422 2.0007	0.04 2.0293	0.0336 2.0689
6	0.1536 1.4841	1.1329 1.5510	0.4590 1.6026	0.7708 1.6323	0.626 1.6549	0.7239 1.6731	0.6969 1.6884	0.7317 1.7016	0.7329 1.7135	0.7494 1.7232	0.8084 1.7620	0.8261 1.7891	0.8855 1.8264	0.9026 1.8524	0.9596 1.8886
7	0.0196 1.3740	0.2623 1.4442	0.0586 1.4838	0.1074 1.5112	0.0709 1.5321	0.07968 1.5490	0.0696 1.5632	0.0696 1.5753	0.06566 1.5859	0.0642 1.5954	0.0530 1.6313	0.0505 1.6563	0.0422 1.6909	0.0400 1.7151	0.0337 1.7485
8	0.0183 1.2853	0.2712 1.3509	0.0576 1.3871	0.1083 1.4136	0.0707 1.4332	0.07985 1.4490	0.0695 1.4622	0.0697 1.4736	0.0656 1.4835	0.0642 1.4924	0.0530 1.5260	0.0505 1.5493	0.0422 1.5817	0.0400 1.6043	0.0335 1.6356
9	0.0173 1.2118	0.2793 1.2737	0.0568 1.3085	0.1091 1.3327	0.0704 1.3512	0.079998 1.3661	0.0695 1.3786	0.0697 1.3893	0.0656 1.3987	0.0642 1.407	0.0530 1.4387	0.0505 1.4607	0.0422 1.4912	0.040 1.5126	0.0336 1.5420
10	0.0164 1.1496	0.2867 1.2083	0.0561 1.2414	0.1098 1.2643	0.0702 1.2819	0.0801 1.2960)	0.0694 1.3078	0.0698 1.3180	0.0656 1.3269	0.0641 1.3348	0.0530 1.3649	0.0505 1.3858	0.0422 1.4147	0.0400 1.4349	0.0336 1.4629

Table 2.3: Rayleigh distribution constants of Analysis of means (1-α=0.95)

						1-α=0.95									
n → k↓	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**15**	**20**	**30**	**40**	**60**
2	0.0367 2.3018	0.1917 2.4472	0.0685 2.5285	0.0993 2.5847	0.0737 2.6275	0.0781 2.6619	0.0703 2.6907	0.0693 2.7154	0.0658 2.7370	0.0641 2.7561	0.0530 2.8287	0.0505 2.8791	0.0422 2.9487	0.040 2.9970	0.0336 3.0639
3	0.0300 1.8794	0.2122 1.9982	0.0652 2.0646	0.1018 2.1104	0.0728 2.1453	0.0786 2.1735	0.0700 2.1970	0.0694 2.2171	0.0658 2.2347	0.0641 2.2504	0.0530 2.3096	0.0505 2.3508	0.0422 2.4076	0.040 2.4471	0.0336 2.5017
4	0.0259 1.6276	0.2280 1.7305	0.0629 1.7880	0.1037 1.8277	0.0722 1.8579	0.0790 1.8823	0.0699 1.9026	0.0695 1.9201	0.0657 1.9353	0.0641 1.9489	0.0530 2.000	0.0505 2.0358	0.0422 2.0850	0.040 2.1192	0.0336 2.1665
5	0.0232 1.4558	0.2411 1.5478	0.0611 1.5992	0.1051 1.6347	0.0717 1.6617	0.0793 1.6836	0.0698 1.7017	0.0695 1.7174	0.0657 1.731	0.0641 1.7431	0.0530 1.7890	0.0505 1.8211	0.0422 1.8649	0.040 1.8955	0.0336 1.9378
6	0.0212 1.3289	0.2523 1.4129	0.0597 1.4599	0.1063 1.4923	0.0713 1.5170	0.0795 1.5369	0.0697 1.5535	0.0696 1.5677	0.0657 1.5802	0.0642 1.5913	0.0530 1.6332	0.0505 1.6623	0.0422 1.7024	0.040 1.7303	0.0336 1.7690
7	0.0196 1.2304	0.2623 1.3081	0.0586 1.3516	0.1074 1.3816	0.0710 1.4045	0.0797 1.4229	0.0696 1.4382	0.0696 1.4514	0.0657 1.4630	0.0642 1.473	0.0530 1.5120	0.0505 1.5389	0.0422 1.5761	0.040 1.6020	0.0336 1.6377
8	0.0184 1.1509	0.2712 1.2236	0.0576 1.2643	0.1083 1.2924	0.0707 1.3137	0.0798 1.3310	0.0695 1.3453	0.0697 1.3577	0.0656 1.3685	0.0642 1.3781	0.0530 1.4144	0.0505 1.4396	0.0422 1.4743	0.040 1.4985	0.0336 1.532
9	0.0173 1.0851	0.2793 1.1536	0.0568 1.1920	0.1091 1.2185	0.0704 1.2386	0.080 1.2548	0.0694 1.2684	0.0697 1.2805	0.0656 1.2902	0.0642 1.2993	0.0530 1.3335	0.0505 1.3572	0.0422 1.3900	0.040 1.4128	0.0336 1.4444
10	0.0164 1.0294	0.2867 1.0944	0.0561 1.1308	0.1098 1.1559	0.0702 1.1750	0.0801 1.1904	0.0694 1.2033	0.0698 1.2144	0.0656 1.2240	0.0642 1.2326	0.0530 1.2650	0.0505 1.2876	0.0422 1.3187	0.040 1.3403	0.0336 1.3702

Table 2.4: Rayleigh distribution constants of Analysis of means (1-α=0.90)

						1-α= 0.90									
n → k↓	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**15**	**20**	**30**	**40**	**60**
2	0.2265 1.7308	0.4791 1.9139	0.2696 2.0149	0.3116 2.0839	0.2594 2.1362	0.2596 2.1781	0.2403 2.2129	0.2337 2.2427	0.2235 2.2686	0.2168 2.2916	0.1798 2.3780	0.1710 2.4376	0.1432 2.5192	0.1356 2.5756	0.1137 2.6530
3	0.1849 1.4132	0.5302 1.5627	0.2562 1.6451	0.3196 1.7016	0.2561 1.7442	0.2613 1.7784	0.2396 1.8068	0.2341 1.8311	0.2233 1.8523	0.2170 1.8711	0.1798 1.9417	0.1711 1.9903	0.1432 2.0569	0.1356 2.1030	0.1138 2.1662
4	0.1601 1.2239	0.5545 1.3533	0.2472 1.4247	0.3254 1.4736	0.2538 1.5105	0.2625 1.5401	0.2390 1.5648	0.2343 1.5858	0.2232 1.6041	0.2170 1.6204	0.1798 1.6815	0.1711 1.7236	0.1432 1.7814	0.1356 1.8212	0.1138 1.8760
5	0.1432 1.0947	0.6024 1.2104	0.2404 1.2743	0.3299 1.3180	0.2520 1.3511	0.2634 1.378	0.2386 1.3996	0.2345 1.4184	0.2231 1.4348	0.2170 1.4493	0.1798 1.5040	0.1711 1.5417	0.1432 1.5933	0.1356 1.6289	0.1137 1.6779
6	0.1307 0.9993	0.6305 1.1050	0.2350 1.1633	0.3337 1.2032	0.2506 1.2333	0.2642 1.2575	0.2383 1.278	0.2347 1.2948	0.2230 1.3098	0.2171 1.3230	0.1798 1.3730	0.1711 1.4073	0.1432 1.4545	0.1356 1.4870	0.1138 1.5317
7	0.1210 0.9252	0.6553 1.0230	0.2305 1.0770	0.3369 1.1139	0.2494 1.1419	0.2647 1.1642	0.2380 1.1828	0.2349 1.1987	0.2229 1.2126	0.2171 1.2249	0.1798 1.2711	0.1711 1.3029	0.1432 1.3460	0.1356 1.3767	0.1138 1.4181
8	0.1132 0.8654	0.6775 0.9569	0.2267 1.0074	0.3398 1.0420	0.2484 1.0681	0.2653 1.0890	0.2377 1.1064	0.2350 1.1213	0.2229 1.1343	0.2171 1.1458	0.1798 1.189	0.1711 1.2188	0.1432 1.2596	0.1356 1.2878	0.1137 1.3265
9	0.1068 0.8159	0.6978 0.9022	0.2238 0.9498	.3423 0.9824	0.2475 1.0070	0.2658 1.0268	0.2375 1.0432	0.2351 1.0572	0.2228 1.0694	0.2172 1.0803	0.1798 1.1210	0.1711 1.1491	0.1432 1.1876	0.1356 1.2141	0.1138 1.2507
10	0.1013 0.7740	0.7164 0.8559	0.2204 0.9011	0.3445 0.9320	0.2467 0.95535	0.2662 0.9741	0.2373 0.9896	0.2352 1.0029	0.2228 1.0145	0.2172 1.0248	0.1798 1.0635	0.1711 1.0901	0.1432 1.1266	0.1356 1.1518	0.1138 1.1865

Table 2.5: Rayleigh distribution constants of Analysis of means (1-α=0.85)

						1-α =0.85									
n → k↓	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**15**	**20**	**30**	**40**	**60**
2	0.2792 1.6094	0.5340 1.8010	0.3164 1.9067	0.3565 1.9790	0.3007 2.0336	0.29911 2.0773	0.2778 2.1137	0.2697 2.1447	0.2581 2.1717	0.2504 2.1956	0.2077 2.2855	0.1976 2.3473	0.1655 2.4319	0.1566 2.4901	0.1315 2.5702
3	0.2279 1.3140	0.5909 1.4705	0.3007 1.5568	0.3657 1.6159	0.2969 1.6604	0.3010 1.6961	0.2769 1.7258	0.2702 1.7511	0.2579 1.7731	0.2505 1.7927	0.2077 1.8661	0.1976 1.9166	0.1655 1.9856	0.1566 2.0332	0.1315 2.0985
4	0.1974 1.1380	0.6350 1.2735	0.2901 1.3482	0.3723 1.3994	0.2943 1.4380	0.3023 1.4689	0.2763 1.4946	0.2704 1.5165	0.2578 1.5356	0.2506 1.5525	0.2077 1.6161	0.1976 1.6598	0.1655 1.7196	0.1566 1.7608	0.1315 1.8174
5	0.1765 1.0179	0.6714 1.1390	0.2821 1.2059	0.3776 1.2517	0.2922 1.2862	0.3034 1.3138	0.2758 1.3368	0.2707 1.3564	0.2577 1.3735	0.2506 1.3886	0.2076 1.4454	0.1976 1.4846	0.1654 1.5380	0.1566 1.5749	0.1315 1.6255
6	0.1612 0.9292	0.7027 1.0398	0.2758 1.1009	0.3819 1.1426	0.2905 1.1743	0.3043 1.1993	0.2754 1.2203	0.2709 1.2382	0.2576 1.2538	0.2507 1.2676	0.2076 1.3195	0.1976 1.3552	0.1654 1.4040	0.1566 1.4377	0.1315 1.4839
7	0.1493 0.8603	0.7303 0.9627	0.2705 1.0191	0.3856 1.0578	0.2892 1.0870	0.3050 1.1104	0.2750 1.1298	0.2711 1.1463	0.2575 1.1608	0.2507 1.1736	0.2076 1.2216	0.1976 1.2547	0.1654 1.2999	0.1566 1.3311	0.1314 1.3738
8	0.1396 0.8047	0.7551 0.9005	0.2660 0.95337	0.3888 0.9895	0.2879 1.0168	0.3057 1.0387	0.2748 1.0568	0.2712 1.0723	0.2575 1.0858	0.2507 1.0978	0.2076 1.1428	0.1976 1.1737	0.1654 1.2159	0.1566 1.2451	0.1314 1.2851
9	0.1316 0.7587	0.7777 0.8490	0.2621 0.8988	0.3917 0.9329	0.2869 0.9586	0.3062 0.9792	0.2745 0.9963	0.2713 1.0110	0.2574 1.0238	0.2507 1.0350	0.2076 1.0774	0.1976 1.1065	0.1654 1.1464	0.1566 1.1738	0.1314 1.2116
10	0.1249 0.7197	0.7984 0.8054	0.2587 0.85272	0.3943 0.88506	0.2859 0.9095	0.3067 0.9290	0.2743 0.9453	0.2714 0.9591	0.2573 0.9712	0.2508 0.9819	0.2076 1.0221	0.1977 1.0497	0.1654 1.0876	0.1566 1.1136	0.1314 1.1494

Table 2.6: Rayleigh distribution constants of Analysis of means (1-α=0.80)

							1-α=0.80								
n → k↓	1	2	3	4	5	6	7	8	9	10	15	20	30	40	60
2	0.3245 1.5174	0.5780 1.7151	0.3553 1.8245	0.3934 1.8992	0.3349 1.9557	0.3317 2.0008	0.3088 2.0384	0.2995 2.0704	0.2868 2.0983	0.2781 2.1229	0.2308 2.2155	0.2195 2.2791	0.1839 2.3660	0.1740 2.4258	0.1461 2.5078
3	0.2650 1.2390	0.6397 1.4004	0.3378 1.4897	0.4035 1.5507	0.3307 1.5968	0.3338 1.6337	0.3078 1.6643	0.3000 1.6904	0.2866 1.7132	0.2782 1.7333	0.2308 1.8090	0.2196 1.8609	0.1839 1.9319	0.1740 1.9807	0.1461 2.0476
4	0.2295 1.073	0.6874 1.2127	0.3259 1.2901	0.4109 1.3430	0.3277 1.3829	0.3353 1.4148	0.3071 1.4413	0.3003 1.4639	0.2864 1.4837	0.2783 1.5011	0.2308 1.5666	0.2196 1.6116	0.1839 1.6730	0.1740 1.7153	0.1461 1.7733
5	0.2053 0.9597	0.72686 1.0847	0.3169 1.1539	0.4166 1.2012	0.3255 1.2369	0.3365 1.2654	0.3066 1.2892	0.3006 1.3094	0.2863 1.3270	0.2789 1.3426	0.2307 1.4012	0.2196 1.4414	0.1838 1.4964	0.1741 1.5342	0.1461 1.5861
6	0.1874 0.8761	0.76076 0.9902	0.3098 1.0533	0.4214 1.0965	0.3236 1.1291	0.3374 1.1552	0.3061 1.1768	0.3008 1.1953	0.2862 1.2114	0.2784 1.2257	0.2307 1.2791	0.2196 1.3158	0.1839 1.3660	0.1741 1.4006	0.1461 1.4479
7	0.1735 0.8111	0.79065 0.9167	0.3038 0.9752	0.4254 1.0152	0.3220 1.0454	0.3382 1.0695	0.3058 1.0895	0.3001 1.1067	0.2861 1.1216	0.2785 1.1347	0.2307 1.1842	0.2196 1.2182	0.1839 1.2647	0.1741 1.2967	0.1461 1.3405
8	0.1623 0.7587	0.81749 0.8575	0.2988 0.9122	0.4290 0.9496	0.3207 0.9778	0.3389 1.0004	0.3054 1.0192	0.3011 1.0352	0.2860 1.0492	0.2785 1.0615	0.2307 1.1078	0.21962 1.1396	0.18388 1.1830	0.1741 1.2129	0.1461 1.2539
9	0.1068 0.7153	0.69780 0.8085	0.2233 0.8601	0.3423 0.8953	0.2475 0.9219	0.2658 0.9432	0.2375 0.9609	0.2351 0.9760	0.2228 0.9892	0.2172 1.0007	0.1797 1.0444	0.1711 1.0744	0.1432 1.1153	0.1356 1.1436	0.1137 1.1822
10	0.1452 0.6786	0.8644 0.7670	0.2906 0.8159	0.4351 0.8494	0.3185 0.8746	0.3401 0.89480	0.3049 0.9116	0.3014 0.9259	0.2859 0.9384	0.2786 0.9494	0.2306 0.9908	0.2196 1.0192	0.18389 1.0581	0.1741 1.0849	0.1461 1.1215

3. Illustrations

Example 1: Wadsworth (1986) [11]: Consider the following data of 25 observations on manufacture of metal products that suspect variations in iron content of raw material supplied by five suppliers. Five ingots were randomly selected from each of the five suppliers. The following table contains the data for the iron determinations on each ingots in percent by weight.

Supplier				
1	2	3	4	5
3.46	3.59	3.51	3.38	3.29
3.48	3.46	3.64	3.40	3.46
3.56	3.42	3.46	3.37	3.37
3.39	3.49	3.52	3.46	3.32
3.40	3.50	3.49	3.39	3.38

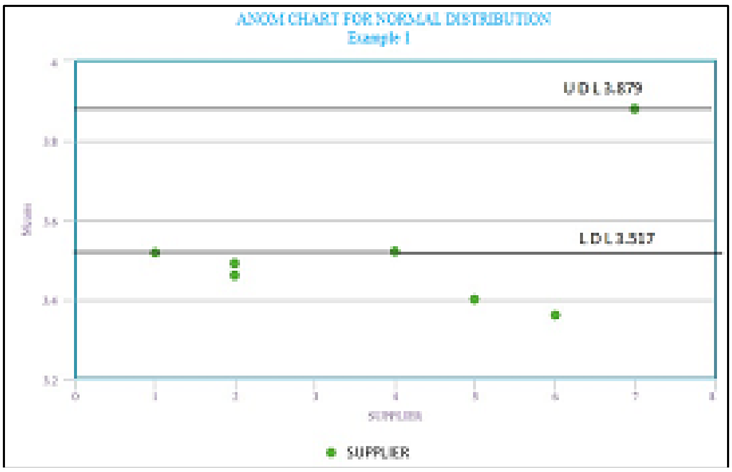

Fig 1.1

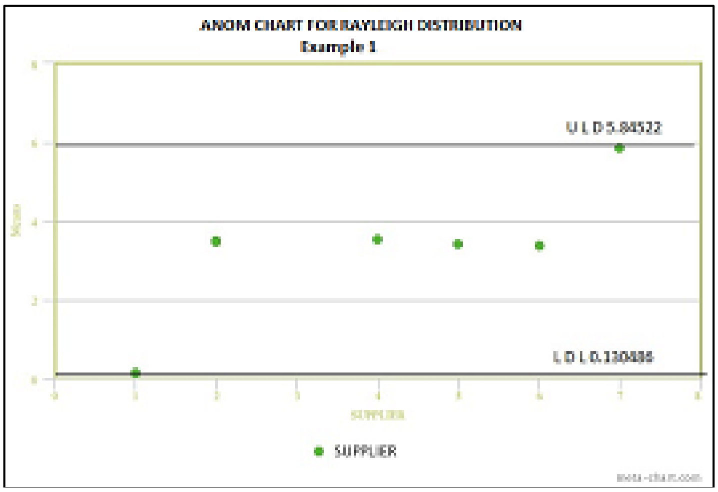

Fig 1.2

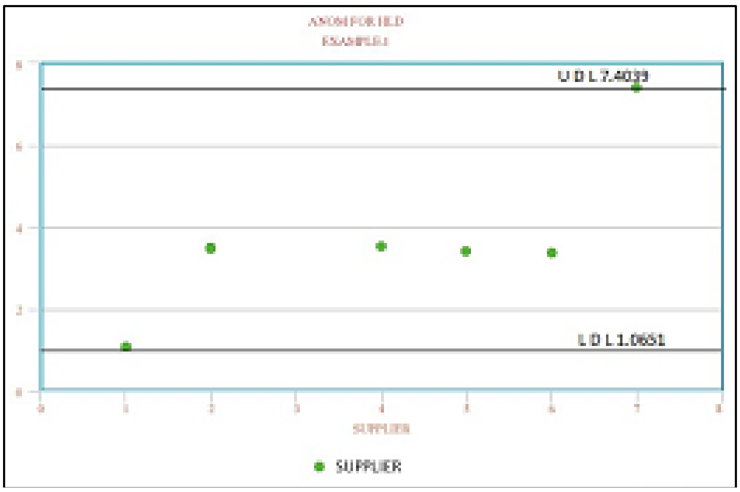

Fig 1.3

Example 2: Three bands of brand are tested with the following results. Test whether the lives of these brands of batteries are different at 5% batteries are under study. It is suspected that the life (in weeks) of the three brands is different. Five batteries of each level of significance.

Weeks of Life		
Brand1	**Brand2**	**Brand3**
100	76	108
96	80	100
92	75	96
96	84	98
92	82	100

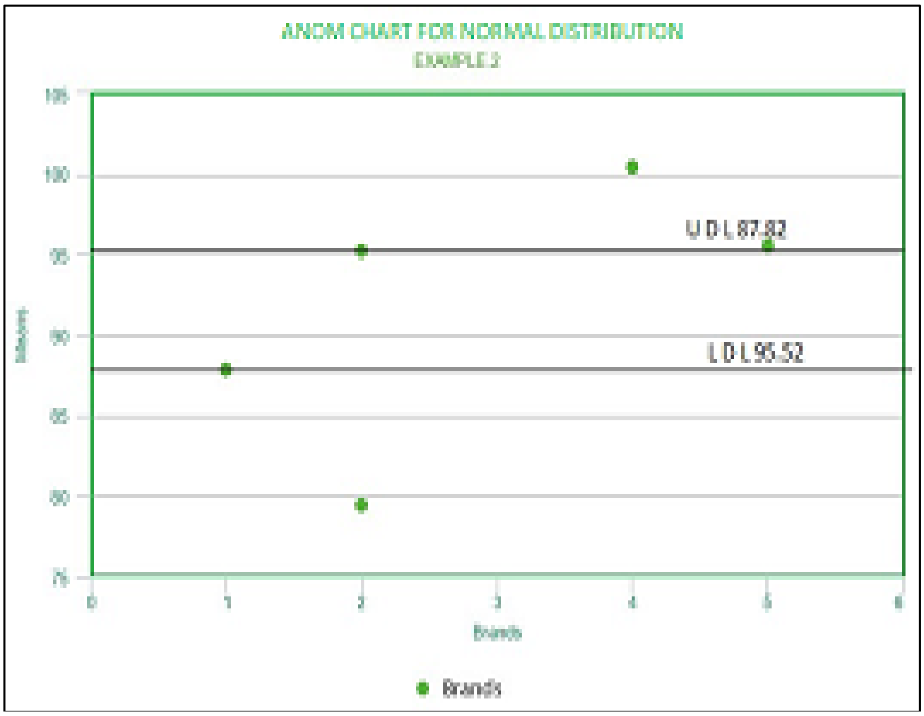

Fig 2.1

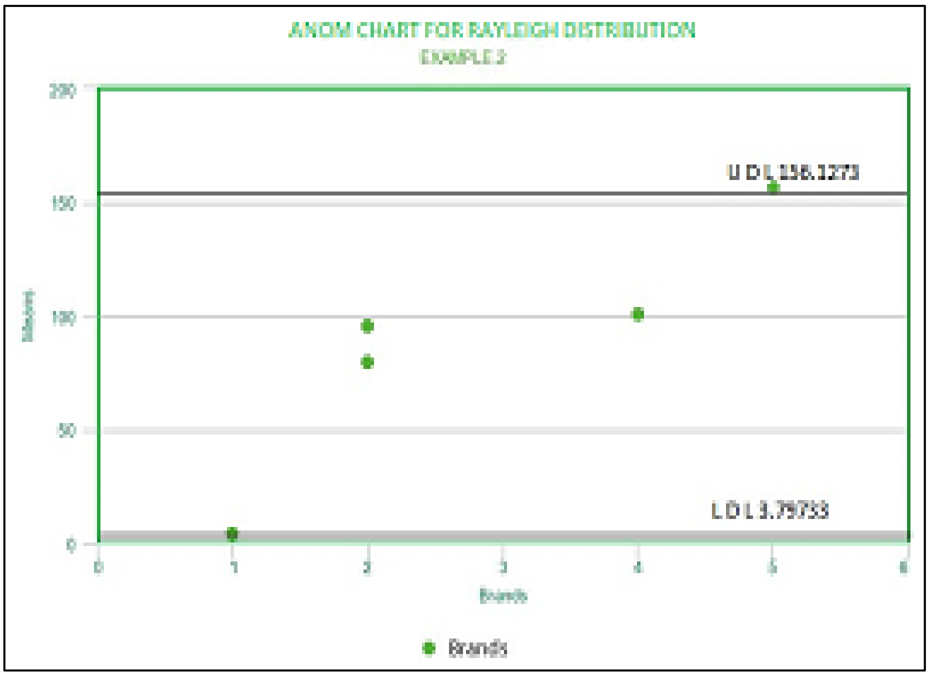

Fig 2.2

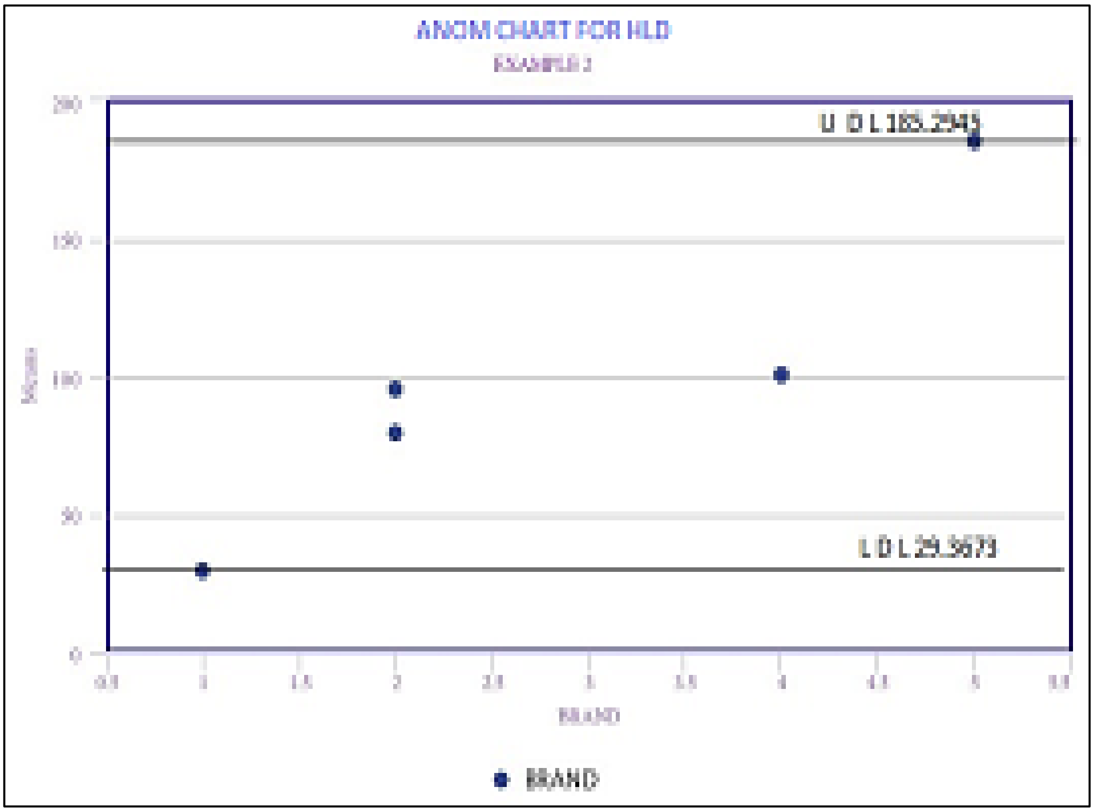

Fig 2.3

Example 3: Four catalysts that may affect the concentration of one component in a three- component liquid mixture are being investigated. The following concentrations are obtained. Test whether the four catalysts have the same affect on the concentration at 5% level of significance.

Catalyst			
1	2	3	4
58.2	56.3	50.1	52.9
57.2	54.5	54.2	49.9
58.4	57.0	55.4	50.0
55.8	55.3	54.9	51.7

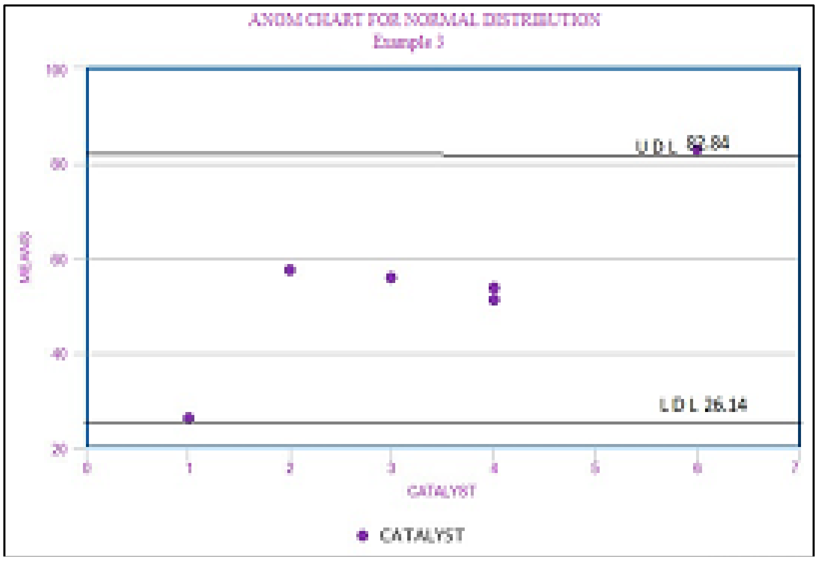

Fig 3.1

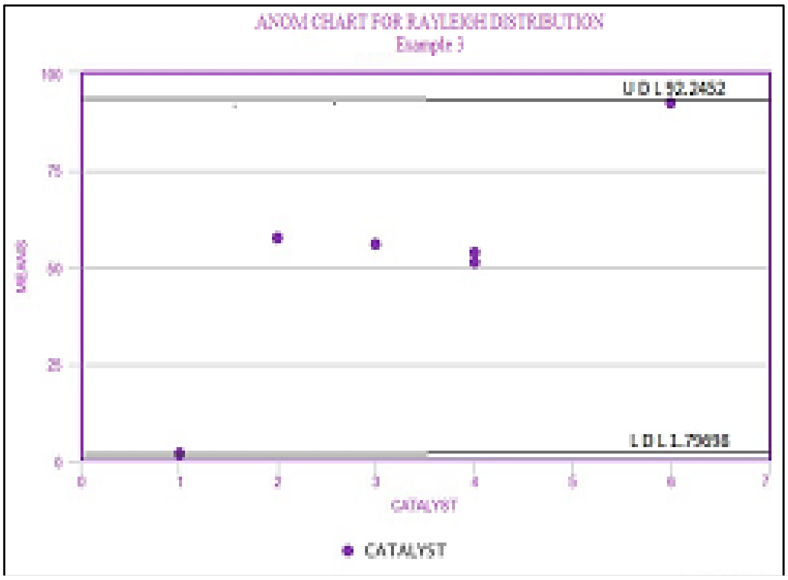

Fig 3.2

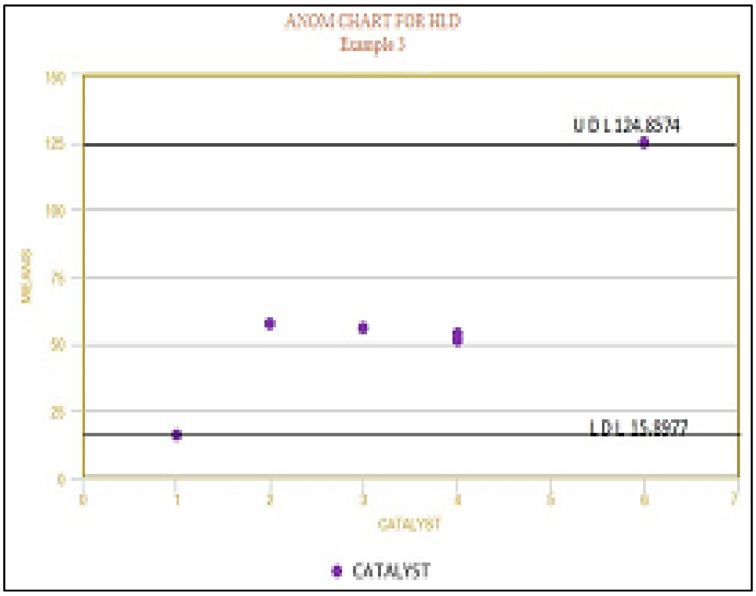

Fig 3.3

The goodness of the of data in these three examples as revealed by Q-Q plot (correlation coefficient) are summarized in the following table, which shows that Rayleigh is a better model, exhibiting significant linear relation between sample and population quantiles.

Table 3.1

	Rayleigh	Normal	HLD
Example 1	0.8369	0.2067	0.6088
Example 2	0.5489	0.4149	0.0862
Example 3	0.9738	0.4447	0.9897

Treating these observations in the data as a single sample, we have calculated the decision limits for the Normal distribution, Rayleigh distribution and HLD and have given them in the following tables respectively.

Table 3.2

	Normal Distribution				
	No. of Counts				
	[LDL,UDL](Ott 1967)	In	P=in/k	Out	Q= Out/k
Example 1 n=5,k=5,= 0:05	[3.517,3.879]	3	0.6	2	0.6
Example 2 n=5,k=3,= 0:05	[87.82,95.52]	2	0.7	1	0.3
Example 3 n=4,k=4,= 0:05	[26.14,82.84]	2	0.5	2	0.5.

Table 3.3

	Rayleigh				
	No. Of Counts				
	[LDL,UDL]	In	P=in/k	Out	Q= Out/k
Example 1 n=5,k=5,= 0:05	[0.130486,5.84522]	5	1	0	0
Example 2 n=5,k=3,= 0:05	[3.79733,156.1273]	3	1	0	0
Example 3 n=4,k=4,= 0:05	[1.79698,92.2452]	4	1	0	0.

Table 3.4

	HLD				
	No. Of Counts				
	[LDL,UDL]	In	P=in/k	Out	Q= Out/k
Example 1 n=5,k=5,= 0:05	[1.0651,7.4039]	5	1	0	0
Example 2 n=5,k=3,= 0:05	[29,3673,185.2945]	3	1	0	0
Example 3 n=4,k=4,= 0:05	[15.8977,124.8574]	4	1	0	0.

Summary and Conclusions

From the above tables (4.2), (4.3) and (4.4) and Figures 1.1, 1.2, 1.3; 2.1,2.2,2.3;3.1,3.2,3.3., we observe that the control points for Rayleigh, half-logistic distributions all points fall within the decision lines when compared to normal distribution. Therefore, either of Rayleigh, half-logistic distributions are preferable than normal distribution.

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