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Measure of slope rotatability for second order response surface designs using a pair of symmetrical unequal block arrangements with two unequal block sizes

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Abstract
A measure enables to assess the degree of slope rotatability for a given response surface design. In this paper, measure of slope rotatability for second order response surface designs using a pair of symmetrical unequal block arrangements with two unequal block sizes is recommended.

Keywords: Response surface designs, measure of slope rotatability for second order response surface designs, symmetrical unequal block arrangements with two unequal block sizes

1. Introduction

Response surface methodology is used in experiments in which the main interests are to determine the relationship between the response and the setting of a group of experimental factors. The concept of rotatability, which is very important in response surface designs, was proposed by Box and Hunter (1957) [1]. Das and Narasimham (1962) [3] constructed rotatable designs through balanced incomplete block designs. Narasimham *et al*. (1983) [5] constructed second order rotatable designs through a pair of balanced incomplete block designs.

A design is said to be rotatable if the variance of the response estimate is a function only of the distance of the point from the design center. The study of rotatable designs is mainly emphasized on the estimation of differences of yields and its precision. Estimation of differences in responses at two points in the factor space will often be of great importance. If differences in responses at two points close together are of interest then estimation of local slope (rate of change) of the response is required. Estimation of slopes occurs frequently in practical situations. For instance, there are cases in which we want to estimate rate of reaction in chemical experiment, rate of change in the yield of a crop to various fertilizer doses and rate of disintegration of radioactive material in an animal, etc. (Park, 1987) [6].

Hader and Park (1978) [4] proposed slope rotatability for second order response surface designs and constructed slope rotatable central composite designs. Victorbabu and Narasimham (1991a) [13] suggested conditions for slope rotatability in any general second order response surface designs and constructed second order slope rotatable designs (SOSRD) using balanced incomplete block designs (BIBD). Victorbabu and Narasimham (1991b) [14] constructed a new unified method of SOSRD through a pair of incomplete block designs. Victorbabu and Narasimham (1993a) [15] constructed three level SOSRD using balanced incomplete block designs. Victorbabu and Narasimham (1993b) [16] studied SOSRD using pairwise balanced designs (PBD). Victorbabu (2002) [8] constructed SOSRD using symmetrical unequal block arrangements (SUBA) with two unequal block sizes. Victorbabu (2005, 06) [9] constructed modified SOSRD using central composite designs (CCD) and BIBD. Victorbabu and Narayanarao (2006) [10, 17] constructed SOSRD through a pair of suitably chosen SUBA with two unequal block sizes. Victorbabu (2007) [11] suggested review on second order slope rotatable designs. Victorbabu (2019) [12] suggested a new method of SOSRD using PBIBD. Park and Kim (1992) [7] introduced measure of slope rotatability for second order response surface experimental designs. Victorbabu and Surekha (2011, 12a, 12b, 12c, 13, and 16) [18- 23] studied measure of slope rotatability for second order response surface designs using CCD,

BIBD, PBD, SUBA with two unequal block sizes and partially balanced incomplete block type designs, pair of BIBD respectively. Victorbabu and Chiranjeevi (2018) [24] studied measure of rotatability for second order response surface designs using SUBA with two unequal block sizes when r >3λ is suggested. Chiranjeevi and Victorbabu (2019) [2] suggested measure of slope rotatability for second order response surface designs using supplementary difference sets.
In this paper, a new measure of slope rotatability for second order response surface designs using a pair of SUBA with two unequal block sizes is suggested.

2. Conditions for second order slope rotatable designs

Suppose we want to use the second order response surface designs D= $((x_{iu}))$ to fit the surface,

$$Y_u = b_0 + \sum_{i=1}^{v} b_i x_{iu} + \sum_{i=1}^{v} b_{ii} x_{iu}^2 + \sum_{i<j}\sum b_{ij} x_{iu} x_{ju} + e_u \tag{1}$$

where x_{iu} denotes the level of the i^{th} factor (i=1,2,…,v) in the u^{th} run (u=1,2,…,N) of the experiment and the e_u's are uncorrelated random errors with mean zero and variance σ^2. A second order response surface design D is said to be SOSRD if the variance of estimated of first order partial derivative of $Y_u(x_1, x_2, ..., x_v)$ with respect to each of independent variable (x_i) is only a function of the distance $\left(d^2 = \sum x_i^2\right)$ of the point $(x_1, x_2, ..., x_v)$ from the origin (center) of the design. Such a spherical variance function for estimation of slopes in the second order response surface is achieved. If the design points satisfy the following conditions. (cf. Hader and Park (1978) [4], Victorbabu and Narasimham (1991a) [13]).

$$\sum x_{iu} = 0, \sum x_{iu} x_{ju} = 0, \sum x_{iu} x_{ju}^2 = 0, \sum x_{iu} x_{ju} x_{ku} = 0,$$
$$\sum x_{iu}^3 = 0, \sum x_{iu} x_{ju}^3 = 0, \sum x_{iu} x_{ju} x_{ku}^2 = 0, \sum x_{iu} x_{ju} x_{ku} x_{lu} = 0.$$

for i≠j≠k≠l; (2)

(i) $\sum x_{iu}^2 = \text{constant} = N\lambda_2$; (ii) $\sum x_{iu}^4 = \text{constant} = cN\lambda_4$; for all i (3)

$$\sum x_{iu}^2 x_{ju}^2 = \text{constant} = N\lambda_4; \text{ for all } i \neq j \tag{4}$$

$$\frac{\lambda_4}{\lambda_2^2} > \frac{v}{(c+v-1)} \tag{5}$$

$$\lambda_4\left[v(5-c)-(c-3)^2\right] + \lambda_2^2\left[v(c-5)+4\right] = 0 \tag{6}$$

where c, λ_2 and λ_4 are constants and the summation is over the design points.
The variances and co-variances of the estimated parameters are,

$$V(\hat{b}_0) = \frac{\lambda_4(c+v-1)\sigma^2}{N[\lambda_4(c+v-1) - v\lambda_2^2]},$$

$$V(\hat{b}_i) = \frac{\sigma^2}{N\lambda_2},$$

$$V(\hat{b}_{ij}) = \frac{\sigma^2}{N\lambda_4},$$

$$V(\hat{b}_{ii}) = \frac{\sigma^2}{(c-1)N\lambda_4}\left[\frac{\lambda_4(c+v-2)-(v-1)\lambda_2^2}{\lambda_4(c+v-1)-v\lambda_2^2}\right],$$

$$Cov(\hat{b}_0, \hat{b}_{ii}) = \frac{-\lambda_2\sigma^2}{N[\lambda_4(c+v-1) - v\lambda_2^2]},$$

$$Cov(\hat{b}_{ii},\hat{b}_{jj})=\frac{(\lambda_2^2-\lambda_4)\sigma^2}{(c-1)N\lambda_4[\lambda_4(c+v-1)-v\lambda_2^2]}$$ and other covariances vanish. (7)

Therefore the conditions (2) to (7) give a set of conditions for slope rotatability in any general second order response surface design.

3. Second order slope rotatable designs using a pair of SUBA with two unequal block sizes (cf. Victorbabu and Narayanarao (2006) [10, 17])

Result: The method of construction of SOSRD using a pair of SUBA is given the following result. If $D_1=(v,b_1,r_1,k_{11},k_{12},b_{11},b_{12},\lambda_1)$, $k_1=\sup(k_{11},k_{12})$, $b_{11}+b_{12}=b_1$ with $r_1 \underset{>}{<} c\lambda_1$ and $D_2=(v,b_2,r_2,k_{21},k_{22},b_{21},b_{22},\lambda_2)$, $k_2=\sup(k_{21},k_{22})$, $b_{21}+b_{22}=b_2$ with $r_2 \underset{<}{>} c\lambda_2$ be a pair of SUBA with two unequal block sizes respectively. Let $2^{t(k_1)}$ and $2^{t(k_2)}$ denote fractional factorial design replicates of 2^{k_1} and 2^{k_2} with ± 1 levels. Then the design points $[1-(v,b_1,r_1,k_{11},k_{12},b_{11},b_{12},\lambda_1)]2^{t(k_1)}U[a-(v,b_2,r_2,k_{21},k_{22},b_{21},b_{22},\lambda_2)]2^{t(k_2)}U(n_0)$ give a v-dimensional SOSRD in $N=b_1 2^{t(k_1)}+b_2 2^{t(k_2)}+n_0$ design points, where a^2 is a positive real root of the biquardratic equation

$$\begin{aligned}
&\left[2^{2t(k_2)}N\{\lambda_2^2(5v-9)-r_2^2+r_2\lambda_2(6-v)\}+2^{3t(k_2)}r_2^2\{vr_2-5v\lambda_2+4\lambda_2\}\right]a^8+\\
&\left[2^{t(k_1)+2t(k_2)+1}r_1r_2(vr_2-5v\lambda_2+4\lambda_2)\right]a^6+\\
&\begin{bmatrix}2^{t(k_1)+t(k_2)}N\{\lambda_1\lambda_2(10v-18)+(6-v)(r_1\lambda_2+r_2\lambda_1)-2r_1r_2\}+\\ 2^{t(k_1)+2t(k_2)}r_2^2(vr_1-5v\lambda_1+4\lambda_1)+2^{2t(k_1)+t(k_2)}r_1^2(vr_2-5v\lambda_2+4\lambda_2)\end{bmatrix}a^4+\\
&\left[2^{2t(k_1)+t(k_2)+1}r_1r_2(vr_1-5v\lambda_1+4\lambda_1)\right]a^2+\\
&\left[2^{2t(k_1)}N\{\lambda_1^2(5v-9)+r_1\lambda_1(6-v)-r_1^2\}+2^{3t(k_1)}r_1^2\{vr_1-5v\lambda_1+4\lambda_1\}\right]=0
\end{aligned} \quad (8)$$

If at least one positive real root exist for a^2 in equation (8) then the design exists.
Note: Value of SOSRD using a pair of SUBA with two unequal block sizes can be obtained by solving the above equation (8).

4. Conditions of Measure of Slope Rotatability for Second Order Response Surface Designs
(cf. Park and Kim 1992) [7]
Following Hader and Park (1978) [4], Victorbabu and Narasimham (1991a) [13] and Park and Kim (1992) [7] conditions (2) to (7) give the necessary and sufficient conditions for a measure of slope rotatability for any general second order response surface designs. Further we have,
$V(b_i)$ are equal for i,
$V(b_{ii})$ are equal for i,
$V(b_{ij})$ are equal for i, j, where $i\neq j$,
$Cov(b_i, b_{ii})=Cov(b_i, b_{ij})=Cov(b_{ii}, b_{ij})=Cov(b_{ij}, b_{il})=0$ for all $i\neq j$, $j\neq l$, $l\neq i$. (9)
Park and Kim (1992) [7] proposed that if the conditions in (2) to (7) and (9) are met, then the following measure ($Q_v(D)$) given below can be used to assess the degree slope of rotatability for any general second order response surface design D with v independent variables.

$$Q_V(D)=\frac{1}{2(v-1)\sigma^4}\left\{(v+2)(v+4)\sum_{i=1}^{v}\left[(V(b_i)-\frac{1}{v}\sum_{i=1}^{v}V(b_i))+\frac{(4V(b_{ii})+\sum\limits_{\substack{j=1\\j\neq i}}^{v}V(b_{ij}))-\frac{1}{v}\sum\limits_{i=1}^{v}(4V(b_{ii})+\sum\limits_{\substack{j=1\\j\neq i}}^{v}V(b_{ij}))}{v+2}\right]^2\right.$$

$$+\frac{4}{v(v+2)}\sum_{i=1}^{v}\left((4V(b_{ii})+\sum_{\substack{j=1\\j\neq i}}^{v}V(b_{ij}))-\frac{1}{v}\sum_{i=1}^{v}(4V(b_{ii})+\sum_{\substack{j=1\\j\neq i}}^{v}V(b_{ij}))\right)^2+2\sum_{i=1}^{v}\left[\left[4V(b_{ii})-\frac{(4V(b_{ii})+\sum_{\substack{j=1\\j\neq i}}^{v}V(b_{ij}))}{v}\right]^2+\sum_{\substack{j=1\\j\neq i}}^{v}\left[V(b_{ij})-\frac{(4V(b_{ij})+\sum_{\substack{j=1\\j\neq i}}^{v}V(b_{ij}))}{v}\right]^2\right]$$

$$+4(v+4)\left[4Cov^2(b_i,b_{ii})+\sum_{\substack{j=1\\j\neq i}}^{v}Cov^2(b_i,b_{ij})\right]+4\sum_{i=1}^{v}\left[4\sum_{\substack{j=1\\j\neq i}}^{v}Cov^2(b_{ii},b_{jj})+\sum_{\substack{j<1\\j,l\neq i}}^{v}\sum Cov^2(b_{ij},b_{il})\right]\Bigg\}$$

where $Q_v(D)$ is the proposed measure of slope rotatability. Further, it is greatly simplified to $Q_v(D)=\frac{1}{\sigma^4}\left[4V(b_{ii})-V(b_{ij})\right]^2$.

5. Study of measure of slope rotatability for second order response surface designs using a pair of SUBA with two unequal block sizes

The proposed method of measure of slope rotatability for second order response surface designs using a pair of SUBA with two unequal block sizes is suggested in this section. Let $D_1=(v,b_1,r_1,k_{11},k_{12},b_{11},b_{12},\lambda_1)$, $k_1=\sup(k_{11},k_{12})$, $b_{11}+b_{12}=b_1$ with $r_1 \underset{>}{\overset{<}{-}} c\lambda_1$ and $D_2=(v,b_2,r_2,k_{21},k_{22},b_{21},b_{22},\lambda_2)$, $k_2=\sup(k_{21},k_{22}), b_{21}+b_{22}=b_2$ with $r_2 \underset{<}{\overset{>}{-}} c\lambda_2$ be two SUBA with two unequal block sizes respectively. Let $2^{t(k_1)}$ and $2^{t(k_2)}$ denote fractional factorial design replicates of 2^{k_1} and 2^{k_2} with ± 1 levels. The design points achieved from the transpose of incidence matrix of design D_1 by multiplication (see Das and Narasimham 1962) [3] are denoted by $\left[1-(v,b_1,r_1,k_{11},k_{12},b_{11},b_{12},\lambda_1)\right]2^{t(k_1)}$. Let $\left[1-(v,b_1,r_1,k_{11},k_{12},b_{11},b_{12},\lambda_1)\right]2^{t(k_1)}$ are the $b_1 2^{t(k_1)}$ design points generated from D_1 by multiplication. Let $\left[a-(v,b_2,r_2,k_{21},k_{22},b_{21},b_{22},\lambda_2)\right]2^{t(k_2)}$ are the $b_2 2^{t(k_2)}$ design points generated from D_2 by multiplication. Let n_0 be the number of central points. Then with the above design points, we can obtain measure of slope rotatability for second order response surface designs using is given in the following theorem.

Theorem (5.1): The design points, $\left[1-(v,b_1,r_1,k_{11},k_{12},b_{11},b_{12},\lambda_1)\right]2^{t(k_1)}U\left[a-(v,b_2,r_2,k_{21},k_{22},b_{21},b_{22},\lambda_2)\right]2^{t(k_2)}U(n_0)$ give a v-dimensional measure of slope rotatability for second order response surface designs using a pair of SUBA with two unequal block sizes in $N=b_1 2^{t(k_1)}+b_2 2^{t(k_2)}+n_0$ design points with level 'a' pre-fixed

$$c=\frac{r_1 2^{t(k_1)}+r_2 2^{t(k_2)}a^4}{\lambda_1 2^{t(k_1)}+\lambda_2 2^{t(k_2)}a^4}. \quad (10)$$

Proof: For the design points generated from a pair of SUBA with two unequal block sizes, conditions (2) to (5) are true. Conditions in (2) are true obviously. Conditions (3) to (5) are true as fallows.

$$\sum x_{iu}^2=r_1 2^{t(k_1)}+r_2 2^{t(k_2)}a^2=N\lambda_2 \quad (11)$$

$$\sum x_{iu}^4=r_1 2^{t(k_1)}+r_2 2^{t(k_2)}a^4=cN\lambda_4 \quad (12)$$

$$\sum x_{iu}^2 x_{ju}^2=\lambda_1 2^{t(k_1)}+\lambda_2 2^{t(k_2)}a^4=N\lambda_4 \quad (13)$$

Solving equations (12) and (13), we get c.

Measure of slope rotatability for second order response surface designs using a pair of SUBA with two unequal block sizes can be obtained by solving the given equation (cf. Park and Kim 1992) [7].

$$Q_v(D)=\left[\frac{\sum x_{iu}^2}{N}\right]^4\left[4e-V(b_{ij})\right]^2, \tag{14}$$

where

$$e=\frac{\begin{array}{l}(v-1)\left[2^{t(k_1)}\lambda_1 n_0+2^{t(k_1)+t(k_2)}b_2\lambda_1-2r_1r_2 2^{t(k_1)+t(k_2)}a^2\right]+\\ \left[\begin{array}{l}(v-1)\left(2^{t(k_1)+t(k_2)}b_1\lambda_2+2^{2t(k_2)}b_2\lambda_2+2^{t(k_2)}n_0\lambda_2-2^{2t(k_2)}r_2^2\right)+\\ (r_2-\lambda_2)\left(2^{t(k_1)+t(k_2)}b_1+2^{2t(k_2)}b_2+2^{t(k_2)}n_0\right)\end{array}\right]a^4+\\ (r_1-\lambda_1)\left(2^{2t(k_1)}b_1+2^{t(k_1)+t(k_2)}b_2+2^{t(k_1)}n_0\right)+(vb_1\lambda_1-(v-1)r_1^2-b_1\lambda_1)2^{2t(k_1)}\end{array}}{\left[2^{t(k_1)}(r_1-\lambda_1)+2^{t(k_2)}(r_2-\lambda_2)a^4\right]\left[\begin{array}{l}(r_1-\lambda_1)\left(2^{2t(k_1)}b_1+2^{t(k_1)+t(k_2)}b_2+2^{t(k_1)}n_0\right)+\\ \left[\begin{array}{l}(r_2-\lambda_2)\left(2^{t(k_1)+t(k_2)}b_1+2^{2t(k_2)}b_2+2^{t(k_2)}n_0\right)+\\ 2^{t(k_1)+t(k_2)}vb_1\lambda_2+2^{2t(k_2)}vb_2\lambda_2+2^{t(k_2)}vn_0\lambda_2-2^{2t(k_2)}vr_2^2\end{array}\right]a^4+\\ (b_1\lambda_1-r_1^2)v2^{2t(k_1)}+2^{t(k_1)}v\lambda_1 n_0+(b_2\lambda_1-2r_1r_2a^2)v2^{t(k_1)+t(k_2)}\end{array}\right]}$$

Example: We illustrate the measure of slope rotatability for second order response surface designs, for 12- factors with the help of a pair of SUBA with two unequal block sizes with parameters

$D_1=(v=12, b_1=13, r_1=4, k_{11}=3, k_{12}=4, b_{11}=4, b_{12}=9, \lambda_1=1)$

$D_2=(v=12, b_2=26, r_2=6, k_{21}=2, k_{22}=3, b_{21}=6, b_{22}=20, \lambda_2=1)$ is given bellow.

The design points, $\left[1-(12,13,4,3,4,4,9,1)\right]2^{t(4)}U\left[a-(12,26,6,2,3,6,20,1)\right]2^{t(3)}U\left(n_0=1\right)$ will give a measure of slope of rotatability for second order response surface designs using a pair of SUBA with two unequal block sizes in N=417 design points for 12- factors. Here equations (11), (12) and (13) are,

$$\sum x_{iu}^2=64+48a^2=N\lambda_2 \tag{14}$$

$$\sum x_{iu}^4=64+48a^4=cN\lambda_4 \tag{15}$$

$$\sum x_{iu}^2x_{ju}^2=16+8a^4=N\lambda_4 \tag{16}$$

From equations (15) and (16) we get $c=\frac{64+48a^4}{16+8a^4}$, $\lambda_2=\frac{64+48a^2}{417}$ and $\lambda_4=\frac{16+8a^4}{417}$. Substituting λ_2, λ_4 and c in (6) and on simplification we get the following biquardratic equation a^2.

$$265536a^8-786432a^6+90880a^4+786432a^2-649984=0 \tag{17}$$

Alternatively from equation (8) we obtain equation (17) directly. Solving (17), we get a^2= 2.5100, substituting this value of a^2 in (14), (15) and (16) and on simplification we obtained λ_2=0.4424, λ_4= 0.1592 and c=5.5181. Non singularity condition (5) is also satisfied as 0.8135 > 0.7265.

We exemplify the measure of slope rotatability for second order response surface designs using a pair of SUBA with two unequal block sizes for v =12 factors the design points N=417 hence we get slope rotatability value a=1.5843, then the $Q_v(D)=0$. Instead of taking a=1.5843, suppose we take a=1.3, we get c=5.1763, e=0.009371530982 and $Q_v(D)$ = 2.0235×10^{-6}. Here $Q_v(D)$ becomes large it deviates from a second order slope rotatable design.

The following Table 1 gives the values of measure of slope rotatability $(Q_v(D))$ for second order response surface designs using pair of SUBA with two unequal block sizes for n_0 =1, 2, 3, 4, 5 and level ‘a’. It can be verified that the measure of slope rotatability is zero if and only if a design D is a second order slope rotatable design. Measure of slope rotatability becomes large as D deviates from a second order slope rotatable design.

Table 1: Measure of slope rotatability for second order response surface designs using a pair of symmetrical unequal block arrangements with two unequal block sizes

D_1=(v=12, b_1=13, r_1=4, k_{11}=3, k_{12}=4, b_{11}=4, b_{12}=9, λ_1=1), D_2= (v=12, b_2=26, r_2=6, k_{21}=2, k_{22}=3, b_{21}=6, b_{22}=20, λ_2=1)					
a	n_0=1 (N=417)	n_0=2 (N=418)	n_0=3 (N=419)	n_0=4 (N=420)	n_0=5 (N=421)
1.0	7.2455×10^{-6}	1.8700×10^{-6}	2.0231×10^{-6}	1.8139×10^{-6}	1.6347×10^{-6}
1.3	2.0235×10^{-6}	1.3374×10^{-6}	1.4511×10^{-9}	1.2417×10^{-6}	1.0687×10^{-6}
1.6	9.2703×10^{-10}	5.8251×10^{-9}	2.4379×10^{-9}	3.3848×10^{-9}	4.4139×10^{-9}
1.9	1.1008×10^{-7}	1.2247×10^{-7}	1.1149×10^{-7}	1.1212×10^{-7}	1.1270×10^{-7}
2.2	1.7277×10^{-7}	1.8146×10^{-7}	1.7152×10^{-7}	1.7087×10^{-7}	1.7022×10^{-7}
2.5	1.9249×10^{-7}	1.9875×10^{-7}	1.9015×10^{-6}	1.8896×10^{-7}	1.8780×10^{-7}
2.8	1.9535×10^{-7}	7.3812×10^{-7}	1.9258×10^{-7}	1.9120×10^{-7}	1.8984×10^{-7}
3.1	1.9234×10^{-7}	1.9625×10^{-7}	1.4919×10^{-7}	1.8797×10^{-7}	1.8654×10^{-7}
*	1.5843	1.5788	1.5734	1.568	1.5626

D_1=(v=15, b_1=33, r_1=9, k_{11}=5, k_{12}=4, b_{11}=3, b_{12}=30, λ_1=2), D_2= (v=15, b_2=28, r_2=6, k_{21}=5, k_{22}=3, b_{21}=3, b_{22}=25, λ_2=1)					
a	n_0=1 (N=977)	n_0=2 (N=978)	n_0=3 (N=979)	n_0=4 (N=980)	n_0=5 (N=981)
1.0	9.3471×10^{-5}	8.7070×10^{-5}	8.1281×10^{-5}	7.6029×10^{-5}	7.1251×10^{-5}
1.3	5.8155×10^{-5}	5.5249×10^{-5}	$5\text{-}.2545\times10^{-5}$	5.0022×10^{-5}	4.7666×10^{-5}
1.6	3.6505×10^{-6}	3.5739×10^{-6}	3.4994×10^{-6}	3.4269×10^{-6}	3.3563×10^{-6}
1.9	5.3299×10^{-7}	5.2456×10^{-7}	5.1628×10^{-7}	5.0816×10^{-7}	5.0018×10^{-7}
2.2	1.0995×10^{-7}	1.0827×10^{-7}	1.0662×10^{-7}	1.0499×10^{-7}	1.0340×10^{-7}
2.5	2.4652×10^{-8}	2.4226×10^{-8}	2.3807×10^{-8}	2.3395×10^{-8}	2.2989×10^{-8}
2.8	4.5370×10^{-9}	4.4277×10^{-9}	4.3205×10^{-9}	4.2156×10^{-9}	4.1127×10^{-9}
3.1	2.5638×10^{-10}	2.3995×10^{-10}	2.2421×10^{-10}	2.0913×10^{-10}	1.9469×10^{-10}
*	1.4305	1.4279	1.4255	1.4230	1.4206

* Denote the exact slope rotatability values

6. Conclusions

In this paper measure of slope rotatability for second order response surface designs using a pair of SUBA with two unequal block sizes have been proposed which enable us to assess the degree of slope rotatability for second order response surface designs.

It may be pointed out here that the measure of slope rotatability for second order response surface designs using a pair of SUBA with two unequal block sizes with parameters $D_1=(v=12, b_1=13, r_1=4, k_{11}=3, k_{12}=4, b_{11}=4, b_{12}=9, \lambda_1=1)$, $D_2=(v=12, b_2=26, r_2=6, k_{21}=2, k_{22}=3, b_{21}=6, b_{22}=20, \lambda_2=1)$ has only 417 design points for 12 factors where as corresponding measure of slope rotatability for second order response surface designs using a pair of BIBD with parameters $D_1=(v=12, b_1=33, r_1=11, k_1=4, \lambda_1=3)$, $D_2=(v=12, b_2=44, r_2=11, k_2=3, \lambda_2=1)$ of Victorbabu and Surekha (2016) [23] need 881 design points. Similarly, for the measure of slope rotatability for second order response surface designs using a pair of SUBA with two unequal block sizes with parameters $D_1=(v=15, b_1=33, r_1=9, k_{11}=5, k_{12}=4, b_{11}=3, b_{12}=30, \lambda_1=2)$, $D_2=(v=15, b_2=28, r_2=6, k_{21}=5, k_{22}=3, b_{21}=3, b_{22}=25, \lambda_2=1)$ has only 977 design points for 15 factors where as corresponding measure of slope rotatability for second order response surface designs using a pair of BIBD with parameters $D_1=(v=15, b_1=15, r_1=7, k_1=7, \lambda_1=3)$, $D_2=(v=15, b_2=35, r_2=7, k_2=3, \lambda_2=1)$ of Victorbabu and Surekha (2016) [23] need 1241 design points. Thus this new method sometimes lead to measure of slope rotatability for second order response surface designs using a pair of SUBA with two unequal block sizes with lesser number of design points than the measure of slope rotatability for second order response surface design using a pair of BIBD in some cases.

7. References

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