Design and development of relational repetitive group sampling plans - (RRGS - 1)

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Abstract
Repetitive Group Sampling Plans are widely used in industries for inspection of lots comprises of components or products. In RGS plans if the sample result does not lead to either acceptance or rejection of the lot then it is discarded and a new sample is drawn and the procedure is continued until the lot is sentenced. However in modern industries if the lots are not accepted based on the few sample results then there may be heavy loss in terms of money, huge cost in terms of sampling inspection, etc. Hence producer may face severe risk during such circumstances. Also customers may lose hope on the producer and the relationship between them may be at stake. The major setback of RGS while implementation is, it is not advantageous when the lot size is small. Hence to offset the disadvantages, new algorithm called as Relational Repetitive Group Sampling is given and new types of Relational Repetitive Group Sampling Plans are developed. It is termed as RRG Inspections plans and the efficiency measures are derived. Tables are constructed to facilitate quality control engineers.

Keywords: Design, development, relational repetitive, plans

Introduction
In industries Single Sampling Plans for attributes quality characteristics are widely used because of its simplicity. However when the lot is not accepted on the basis of sample result then there is some suspicion about the quality of the product and hence the producer may face high risk due to rejection of products or lots. Therefore, double sampling, sequential sampling plans, etc., were developed which has its own advantages and disadvantages. Since single sampling plan procedure is easy to apply in shaft floor, Robert Sherman (1965) [10] has developed Repetitive Group Sampling plans based on SSP procedure. The operating procedure of RGS plan is very simple, draw a random sample of size n from the lot and find the number of defective units d. If d ≤ c₁, accept the lot. If d > c₂, then reject the lot. If d lies in between c₁ and c₂, then repeat the inspection until a decision is made on the lot. Thus the parameters of RGS are n, c₁ and c₂.

In RGS plans if the sample result does not leads to either acceptance or rejection of the lot then it is discarded and new sample is drawn and it is continued until the lot is sentenced. If the decision is not made based on the sample results, then inspection continues, again and again sample is discarded. However in modern industries if the lots are not accepted based on the few sample results then there may be heavy loss in terms of money, high cost in sampling inspection, etc. which may affect the production cost. During such circumstances the producer may face severe risk in terms of sampling cost, production cost and finally competitive cost. Also customers may lose hope on the producer and the relationship between them may be at stake. The main drawback of RGS is, it is not an advantage plan for small lots since the sampling algorithm of RGS goes infinite number of iterations. Hence to offset the disadvantages of RGS a novel RRGS plans are developed.

In order to satisfy the need of the consumer and at the same time to maintain the quality of the products, if the lots are not accepted then immediately the sampling inspection parameter n of RRGS is being linearly related with number of repetition within the same lot while sampling continues. Quality control engineers need special type of repetitive sampling plans in order to achieve quality criteria for small lots.
During the Relational Repetitive Group Sampling inspection if the first sample result does not lead to the acceptance then immediately the sample size is linearly increased in the next coming sampling inspections and this should be continued until the lot is either accepted or rejected based on the new protocol. Hence based on new sampling procedure the efficiency measures are derived and given. In this paper an attempt has been made to develop new type of Relational Repetitive Group Sampling plans and is termed as RRGS sampling plans.

In RGS plans the repetition continues until the lot is either accepted or rejected and sometimes the inspection continues for a large number of trials. There is no pressure or criteria on the producer in terms of sampling plan parameters. Quality of the lot may be at stake, since same sample size is constantly repeated with same acceptance numbers. Hence to maintain the quality of the products and also to give pressure on the producer during inspection, if the lots are not accepted based on first sample, then immediately the sampling inspection parameter especially the sample size is linearly increased in relation with number of repetitions. Thus pressure is given to the producer by increasing the sample size during the inspection. Therefore the lot will be either accepted or rejected at the earliest within smaller number of trials and inspection. Hence time and cost may be reduced while implementing the new RRGS inspection plans. The number of trials is reduced during the sampling and consequently the sampling protocol is also reduced.

Even though many authors have contributed to RGS plans there is a research gap in pressurizing the producer to maintain the quality of the lots. Hence new RRGS plans are developed and accordingly the designing procedure is presented. Based on the designing procedure tables are constructed for easy implementation in industries.


Formulation and Operating Procedure of RRGS

In this new RRGS plans, the sample size is linearly increased in relation to number of repetition during inspection. For instance, the first sample size is n, if the second repetition occurs then the second sample size is 2 times of n, if the third repetition continues then the third sample size is 3n and this is continues until the lot is either accepted or rejected with the same acceptance number. If the sample size is increased linearly then the probability acceptance of the lot will be lesser. This will create very high pressure on the producer to maintain the quality of the lots.

Let
\[ n_1 = n = \text{first sample size} \]
\[ n_k = n^*k = \text{subsequent sample sizes } [ k = 2,3,4,...] \]
\[ d = \text{number of defectives} \]
\[ c_1 = \text{first acceptance number} \]
\[ c_2 = \text{second acceptance number} \]
\[ k = \text{Maximum number of iteration allowed} \]

Algorithm to sentence the lot
1. Draw a random sample of size \( n_1 = n \).
2. Count the number of defectives. Let it be \( d \).
3. If \( d \leq c_1 \), accept the Lot.
4. If \( d > c_2 \), then reject the Lot.
5. If \( c_1+1 \leq d \leq c_2 \), then discard the first sample result and go to next step inspection.
6. Draw another sample of size \( n_2 = 2n \).
7. Count the number of defectives. Let it be \( d \).
8. If \( d \leq c_1 \), accept the Lot.
9. If \( d > c_2 \), then reject the Lot.
10. If \( c_1+1 \leq d \leq c_2 \), then discard the second sample result and go to next step inspection.
11. In general, Draw another sample of size \( n_k = k*n \). Where \( k \) is number of allowable repetition
12. Repeat steps 2 to 5 until the lot is either accepted or rejected.
13. If the lot is not accepted or rejected till \( k \), inspection is stopped and then the lot is sentenced to screening.
14. All the rejected lots are screened 100%.

Selection of \( k \): The repetition parameter can be chosen such that \( k^* = N/n \). Where \( N \) is the lot size. Also \( k \geq 1 \).

Theorem
The Operating Characteristics function of RRGS plans is given as
\[ P_a (p) = P_{a1} [d \leq C_1] + P_{a1} [C_1+1 \leq d \leq C_2] P_{a2} [d \leq C_1] + P_{a1} [C_1+1 \leq d \leq C_2] P_{a2} [d \leq C_1] + \ldots P_{a1} [C_1+1 \leq d \leq C_2] P_{a2} [d \leq C_1] \ldots \ldots \ldots P_{a_{k-1}} [C_1+1 \leq d \leq C_2] P_{a_k} [d \leq C_1]. \]

Proof
Let \( n_1 = n = \text{First Sample Size} \)
\[ n_2 = 2n = \text{Second repetitive Sample Size} \]
\[ n_k = nk = k^* \text{ repetitive Sample Size} \]
Let \( C_1 = \text{First acceptance number} \)
\[ C_2 = \text{Second acceptance number} \]
\[ d = \text{Number of defective} \]
\[ P_{a1} (p) = \text{Probability of accepting the lot at the 1 repetition} \]
\[ P_{a1} [C_1+1 \leq d \leq C_2] P_{a2} (p) = \text{Probability of accepting the lot at the 2^nd repetition} \]
\[ P_{a_{k-1}} [C_1+1 \leq d \leq C_2] P_{a_k} (p) = \text{Probability of accepting the lot at the k^th repetition} \]
The Lot may be accepted in the following cases:
Case(i). In the first trial, the probability of acceptance is \( P_{a1} [d \leq C_1] \).
Where \( n_1 = n \)
Case (ii) Similarly in the second \( P_{a1} [C_1+1 \leq d \leq C_2] \) and \( P_{a2} [d \leq C_1] \).
Where \( n_2 = 2^n \)
Case (iii) \( P_{a1} [C_1+1 \leq d \leq C_2] \) and \( P_{a2} [C_1+1 \leq d \leq C_2] \) and \( P_{a3} [d \leq C_1] \).
Where \( n_3 = 3^n \)
In general
Case (k) \( P_{a1} [C_1+1 \leq d \leq C_2] \) and \( P_{a2} [C_1+1 \leq d \leq C_2] \) and \( P_{a3} [d \leq C_1] \) and \( \ldots \) and \( P_{a_{k-1}} [C_1+1 \leq d \leq C_2] P_{a_k} [d \leq C_1] \).
Where \( n_k = k^n \)
Case(i), Case(ii), \ldots, are mutually exclusive events. Therefore by addition theorem on Probability we get,
\[ P_a (p) = P_{a1} [d \leq C_1] + P_{a1} [C_1+1 \leq d \leq C_2] P_{a2} [d \leq C_1] + P_{a1} [C_1+1 \leq d \leq C_2] P_{a2} [d \leq C_1] + \ldots P_{a1} [C_1+1 \leq d \leq C_2] P_{a2} [d \leq C_1] \ldots \ldots \ldots P_{a_{k-1}} [C_1+1 \leq d \leq C_2] P_{a_k} [d \leq C_1]. \]

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Generalization of OC function:
Let

\[ P_{a,nk} = P_{nk}[d \leq C_1] \]

\[ P_{r,nk-1} = P_{nk-1}[C_1 + 1 \leq d \leq C_2] \]

In general

\[ P_{a}(p) = P_{a,n1} + P_{r,n1}(P_{a,n2}) + P_{r,n1}P_{r,n2}(P_{a,n3}) + \cdots \]

\[ + P_{r,n1}P_{r,n2} \cdots \cdots P_{r,n(k-1)}(P_{a,nk}) \]

\[ P_{a}(p) = P_{a,n1} + \sum_{i=2}^{k} \prod_{i=2}^{k} P_{a,i}P_{r,i-1} \]

Average sample number
ASN is defined as the expected number of units required to make a unique decision on the lots.

\[ \text{ASN} = n_1 + n_2 \]

\[ P_{a, n1} \text{ [ } C_1 + 1 \leq d \leq C_2 \text{ ] } + n_3 \text{ [ } C_1 + 1 \leq d \leq C_2 \text{ ] } + \cdots + n_k \text{ [ } C_1 + 1 \leq d \leq C_2 \text{ ] } \]

Average outgoing quality
AOQ = \[ p \rightleftharpoons \]

\[ \text{AOQ} = p ( P_{a} [ d \leq C_1 ] + P_{a} [ C_1 + 1 \leq d \leq C_2 ] P_{a} [ d \leq C_1 ] + P_{a} [ C_1 + 1 \leq d \leq C_2 ] P_{a} [ d \leq C_1 ] + P_{a} [ C_1 + 1 \leq d \leq C_2 ] P_{a} [ d \leq C_1 ] + \cdots P_{a} [ C_1 + 1 \leq d \leq C_2 ] P_{a} [ d \leq C_1 ] \]

| Table 1: Values of Probability of acceptance of the lot for the known sample size and fraction defects with parameter k and c=0 and c= 2 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | n=100 | n=200 | n=300 | n=400 | n=500 |
| | k=1 | k=2 | k=3 | k=1 | k=2 | k=3 | k=1 | k=2 | k=3 | k=1 | k=2 | k=3 |
| p | 0.0011 | 0.0010 | 0.0008 | 0.0069 | 0.0064 | 0.0058 | 0.0459 | 0.0447 | 0.0435 | 0.0946 | 0.0934 | 0.0923 |
| p | 0.0020 | 0.0018 | 0.0016 | 0.0128 | 0.0124 | 0.0120 | 0.0731 | 0.0722 | 0.0714 | 0.1602 | 0.1594 | 0.1586 |
| p | 0.0030 | 0.0028 | 0.0026 | 0.0256 | 0.0252 | 0.0248 | 0.1176 | 0.1168 | 0.1161 | 0.2557 | 0.2549 | 0.2541 |
| p | 0.0040 | 0.0038 | 0.0036 | 0.0404 | 0.0400 | 0.0397 | 0.1720 | 0.1713 | 0.1706 | 0.4013 | 0.4005 | 0.4000 |
| p | 0.0050 | 0.0048 | 0.0046 | 0.0552 | 0.0548 | 0.0545 | 0.2348 | 0.2341 | 0.2334 | 0.5061 | 0.5053 | 0.5045 |

Illustration: Determine the RRGS Sampling Plans for the process average 0.01% with probability of acceptance 95%. Solution: It is given that,

\[ p_1 = 0.01 \% \rightleftharpoons \text{AQL} = p_1 = 0.0001, 1 - \alpha = 0.95 \]

For practical reason if k=2. Then from table (1), the required sample size is \( n=200 \). Hence the parameters of RRGS sampling plans are \( n=200, k=2, c_1=0, c_2=2 \).

Sampling Procedure
1. Draw a random sample of size \( n_1 = n = 200 \).

2. Count the number of the defectives in the sample. Let it be \( d \).

3. Decision Rule
   - If \( d = 0 \), accept the lot.
   - If \( d > 2 \), reject the lot.
   - If \( 1 \leq d \leq 2 \), go to next step by discarding the first sample.

4. Draw another sample of size \( n_2 = k^n = 2^n = 400 \).

5. Count the number of the defectives in the sample. Let it be \( d \).

6. Decision Rule
   - If \( d = 0 \), accept the lot.
   - If \( d > 2 \), reject the lot.
   - If \( 1 \leq d \leq 2 \), reject the lot since the maximum iteration criteria \( k = 2 \).

7. Screen the entire rejected lot.
Summary
In this paper, if the lots are not accepted then immediately the sampling inspection parameter ‘n’ of RRGS is being linearly related with number of repetition within the same lot while sampling continues. The New algorithm makes the lot either accepted or rejected at the earliest with smaller number of trials and inspection. Hence the advantage is time and cost may be reduced while implementing the new RRGS inspection plans. It is found that when quality deteriorates then there is a decrease in the probability of acceptance of the lot.

Fig 1: Probability of acceptance for the known AQL values
Interpretation: From figure (1), one can find the probability acceptance rapidly decreases over the increase in the sample size. Hence it is clear that when the quality deteriorates a high pressure is created for producer during the inspection.

Fig 2: Probability of acceptance for different values of k
Interpretation: From figure (2), one can find that the OC values decreases rapidly even for small increase in the allowable repetition index. Hence during designing of sampling protocol it is advisable to have small value of k.

References
