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Design and development of relational repetitive group sampling plans - (RRGS - 1)

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Abstract
Repetitive Group Sampling Plans are widely used in industries for inspection of lots comprises of components or products. In RGS plans if the sample result does not lead to either acceptance or rejection of the lot then it is discarded and a new sample is drawn and the procedure is continued until the lot is sentenced. However in modern industries if the lots are not accepted based on the few sample results then there may be heavy loss in terms of money, huge cost in terms of sampling inspection, etc. Hence producer may face severe risk during such circumstances. Also customers may lose hope on the producer and the relationship between them may be at stake. The major setback of RGS while implementation is, it is not advantageous when the lot size is small. Hence to offset the disadvantages, new algorithm called as Relational Repetitive Group Sampling is given and new types of Relational Repetitive Group Sampling Plans are developed. It is termed as RRGS Inspection plans and the efficiency measures are derived. Tables are constructed to facilitate quality control engineers.

Keywords: Design, development, relational repetitive, plans

Introduction

In industries Single Sampling Plans for attributes quality characteristics are widely used because of its simplicity. However when the lot is not accepted on the basis of sample result then there is some suspicion about the quality of the product and hence the producer may face high risk due to rejection of products or lots. Therefore, double sampling, sequential sampling plans, etc., were developed which has its own advantages and disadvantages. Since single sampling plan procedure is easy to apply in shaft floor, Robert Sherman (1965) [10] has developed Repetitive Group Sampling plans based on SSP procedure. The operating procedure of RGS plan is very simple, draw a random sample of size n from the lot and find the number of defective units d. If $d \leq c_1$, accept the lot. If $d > c_2$, then reject the lot. If d lies in between c_1 and c_2, then repeat the inspection until a decision is made on the lot. Thus the parameters of RGS are n, c_1 and c_2.

In RGS plans if the sample result does not leads to either acceptance or rejection of the lot then it is discarded and new sample is drawn and it is continued until the lot is sentenced. If the decision is not made based on the sample results, then inspection continues, again and again sample is discarded. However in modern industries if the lots are not accepted based on the few sample results then there may be heavy loss in terms of money, high cost in sampling inspection, etc. which may affect the production cost. During such circumstances the producer may face severe risk in terms of sampling cost, production cost and finally competitive cost. Also customers may lose hope on the producer and the relationship between them may be at stake. The main drawback of RGS is, it is not an advantage plan for small lots since the sampling algorithm of RGS goes infinite number of iterations. Hence to offset the disadvantages of RGS a novel RRGS plans are developed.

In order to satisfy the need of the consumer and at the same time to maintain the quality of the products, if the lots are not accepted then immediately the sampling inspection parameter n of RRGS is being linearly related with number of repetition within the same lot while sampling continues. Quality control engineers need special type of repetitive sampling plans in order to achieve quality criteria for small lots.

During the Relational Repetitive Group Sampling inspection if the first sample result does not lead to the acceptance then immediately the sample size is linearly increased in the next coming sampling inspections and this should be continued until the lot is either accepted or rejected based on the new protocol. Hence based on new sampling procedure the efficiency measures are derived and given. In this paper an attempt has been made to develop new type of Relational Repetitive Group Sampling plans and is termed as RRGS sampling plans.

In RGS plans the repetition continues until the lot is either accepted or rejected and sometimes the inspection continues for a large number of trials. There is no pressure or criteria on the producer in terms of sampling plan parameters. Quality of the lot may be at stake, since same sample size is constantly repeated with same acceptance numbers. Hence to maintain the quality of the products and also to give pressure on the producer during inspection, if the lots are not accepted based on first sample, then immediately the sampling inspection parameter especially the sample size is linearly increased in relation with number of repetitions. Thus pressure is given to the producer by increasing the sample size during the inspection. Therefore the lot will be either accepted or rejected at the earliest within smaller number of trials and inspection. Hence time and cost may be reduced while implementing the new RRGS inspection plans. The number of trials is reduced during the sampling and consequently the sampling protocol is also reduced.

Even though many authors have contributed to RGS plans there is a research gap in pressurizing the producer to maintain the quality of the lots through the sample size. Hence new RRGS plans are developed and accordingly the designing procedure is presented. Based on the designing procedure tables are constructed for easy implementation in industries.

Soundararajan and Ramaswamy (1984) [14] have constructed tables of sampling plans based on Sherman's procedure. Balamurali *et al*. [2015] [4-5], Singh *et al*. [1989] [13] have developed RGS plan for variable characteristics. Shankar (1993) have developed GERT based RGS plans. Gauri Shankar *et al*. (2001) [11], have developed Two-Stage Conditional Repetitive Group Sampling Plans. Sang-Ho Lee, *et al*. (2012) [8], have developed Repetitive Group Sampling Plans With Double Specification Limits. Senthil Kumar D, *et al*. (2015) [9] have studied constructed table for Repetitive Deferred Variables Sampling (RDVS) Plan Indexed by Quality Levels.

Formulation and Operating Procedure of RRGS

In this new RRGS plans, the sample size is linearly increased in relation to number of repetition during inspection. For instance, the first sample size is n, if the second repetition occurs then the second sample size is 2 times of n, if the third repetition continues then the third sample size is 3n and this is continues until the lot is either accepted or rejected with the same acceptance number. If the sample size is increased linearly then the probability acceptance of the lot will be lesser. This will create very high pressure on the producer to maintain the quality of the lots.

Let

n_1 = n = first sample size
n_k = n*k = subsequent sample sizes [k = 2,3,4,...]
d = number of defectives
c_1= first acceptance number
c_2 = second acceptance number
k = Maximum number of iteration allowed.

Algorithm to sentence the lot

1. Draw a random sample of size n_1=n.
2. Count the number of defectives. Let it be d.
3. If $d \le c_1$, accept the Lot.
4. If $d > c_2$, then reject the Lot.
5. If $c_1+1 \le d \le c_2$, then discard the first sample result and go to next step inspection.
6. Draw another sample of size n_2=2*n.
7. Count the number of defectives. Let it be d.
8. If $d \le c_1$, accept the Lot.
9. If $d > c_2$, then reject the Lot.
10. If $c_1+1 \le d \le c_2$, then discard the second sample result and go to next step inspection.
11. In general, Draw another sample of size n_k=k*n. Where k is number of allowable repetition
12. Repeat steps 2 to 5 until the lot is either accepted or rejected.
13. If the lot is not accepted or rejected till k, inspection is stopped and then the lot is sentenced to screening.
14. All the rejected lots are screened 100%.

Selection of k: The repetition parameter can be chosen such that k=N/n, Where N is the lot size. Also $k \ge 1$.

Theorem

The Operating Characteristics function of RRGS plans is given as

P_a (p) = P_{n1} [d≤C_1] + P_{n1} [C_1+1≤d≤C_2]P_{n2} [d≤C_1] + P_{n1} [C_1+1≤d≤C_2]P_{n2} [C_1+1≤d≤C_2]P_{n3} [d≤C_1] +… P_{n1} [C_1+1≤d≤C_2]P_{n2} [C_1+1≤d≤C_2]………P_{nk-1} [C_1+1≤d≤C_2]P_{nk} [d≤C_1].

Proof

Let n_1 = n = First Sample Size
n_2 = 2n = Second repetitive Sample Size…….
n_k = nk = k^{th} repetitive Sample Size
Let C_1 = First acceptance number
C_2 = Second acceptance number
d = Number of defective
P_{n1} (p) =Probability of accepting the lot at the 1 repetition
P_{n1} [C_1+1≤d≤C_2] P_{n2} (p) =Probability of accepting the lot at the 2^{nd}repetition
P_{nk-1} [C_1+1≤d≤C_2]P_{nk} (p) =Probability of accepting the lot at the k^{th} repetition
The Lot may be accepted in the following cases:
Case(i). In the first trial, the probability of acceptance is P_{n1} [d≤C_1]: Where n_1=n
Case (ii) Similarly in the second P_{n1} [C_1+1≤d≤C_2] and P_{n2} [d≤C_1]: Where n_2 = 2*n
Case (iii) P_{n1} [C_1+1≤d≤C_2] and P_{n2} [C_1+1≤d≤C_2] and P_{n3} [d≤C_1]: Where n_3 = 3*n……
In general
Case (k) P_{n1} [C_1+1≤d≤C_2] and P_{n2} [C_1+1≤d≤C_2]………and P_{nk-1} [C_1+1≤d≤C_2]P_{nk} [d≤C_1]

Where n_k = k*n
Case(i), Case(ii),...etc., are all mutually exclusive events. Therefore by addition theorem on Probability we get,

P_a (p) = P_{n1} [d≤C_1] + P_{n1} [C_1+1≤d≤C_2]P_{n2} [d≤C_1] + P_{n1} [C_1+1≤d≤C_2]P_{n2} [C_1+1≤d≤C_2]P_{n3} [d≤C_1] +…… P_{n1} [C_1+1≤d≤C_2]P_{n2} [C_1+1≤d≤C_2]…P_{nk-1} [C_1+1≤d≤C_2]P_{nk} [d≤C_1].

Generalization of OC function:

Let

$$P_{a,n_k} = P_{n_k}[\mathrm{d} \le C_1]$$
$$P_{r,n_{k-1}} = P_{n_{k-1}}[C_1 + 1 \le \mathrm{d} \le C_2]$$

In general

$$P_a(p) = P_{a,n1} + P_{r,n1}(P_{a,n2}) + P_{r,n1}P_{r,n2}\left(P_{a,n3}\right) + \cdots \ldots + P_{r,n1}P_{r,n2} \ldots \ldots . . P_{r,n(k-1)}\left(P_{a,nk}\right)$$

$$P_a(p) = P_{a,n1} + \sum_{i=2}^{k} \prod_{i=2}^{k} P_{(a,ni)P_{r,ni-1}}$$

Average sample number

ASN is defined as the expected number of units required to make a unique decision on the lots.

ASN = $n_1 + n_2 P_{n1} [C_1+1 \le d \le C_2] + n_3 P_{n1} [C_1+1 \le d \le C_2]P_{n2} [C_1+1 \le d \le C_2] + \ldots + n_k P_{n1} [C_1+1 \le d \le C_2]P_{n2} [C_1+1 \le d \le C_2] \ldots\ldots\ldots P_{nk-1} [C_1+1 \le d \le C_2]P_{nk} [d \le C_1]$

Average outgoing quality

AOQ = $p \{ P_{n1} [d \le C_1] + P_{n1} [C_1+1 \le d \le C_2]P_{n2} [d \le C_1] + P_{n1} [C_1+1 \le d \le C_2]P_{n2} [C_1+1 \le d \le C_2]P_{n3} [d \le C_1] + \ldots P_{n1} [C_1+1 \le d \le C_2]P_{n2} [C_1+1 \le d \le C_2] \ldots\ldots\ldots P_{nk-1} [C_1+1 \le d \le C_2]P_{nk} [d \le C_1] \}$

Advantages of RRGS Plans

1. Since RRGS is truncated at k^{th} sample, the decision is arrived earlier than infinite number of sample inspections.
2. Since sample size is linearly increased with same acceptance number c_1 and c_2, the lot will be rejected very soon if the quality deteriorate within the lot.
3. The RRGS is effective when the lot size is not very large.
4. The parameter of the sampling plans are c_1,c_2,k and n only. The subsequent sample sizes n_2, n_3...n_k are all not the parameters since they are all multiples of n. Hence the estimation of parameters with regard to sample size is minimum.

Designing RRGS plans indexed through AQL.

Step 1: Determine the process average from the production process.

Let it be p_1= AQL

Step 2: Let α be the producer risk.

Step 3: Determine the parameters of RRGS such that

$P_a(p_1) \ge 1 - \alpha$

Step 4: $P_{n1} [d \le C_1] + P_{n1} [C_1+1 \le d \le C_2]P_{n2} [d \le C_1] + P_{n1} [C_1+1 \le d \le C_2]P_{n2} [C_1+1 \le d \le C_2]P_{n3} [d \le C_1] + \ldots P_{n1} [C_1+1 \le d \le C_2]P_{n2} [C_1+1 \le d \le C_2] \ldots\ldots\ldots P_{nk-1} [C_1+1 \le d \le C_2]P_{nk} [d \le C_1] \ge 1-\alpha$.

Step 5: Construct the required tables for various AQL.

Tables are being constructed for selection of RRGS-1 indexed through AQL, LQL, IQL, Mintan method, crossover point, through MAPD, etc.,

Table 1: Values of Probability of acceptance of the lot for the known sample size and fraction defectives with parameter k and c_1=0 and c_2 = 2

	n=100			n=200			n=300			n=400			n=500		
p	**k=1**	**k=2**	**k=3**	**k=1**	**k=2**	**k=3**	**k=1**	**k=2**	**k=3**	**k=1**	**k=2**	**k=3**	**k=1**	**k=2**	**k=3**
0.0001	0.9900	0.9802	0.9704	0.9802	0.9608	0.9417	0.9704	0.9417	0.9138	0.9608	0.9230	0.8867	0.9512	0.9047	0.8603
0.0002	0.9802	0.9608	0.9417	0.9608	0.9230	0.8867	0.9417	0.8867	0.8346	0.9230	0.8516	0.7851	0.9047	0.8178	0.7382
0.0003	0.9704	0.9417	0.9138	0.9417	0.8867	0.8346	0.9138	0.8346	0.7613	0.8867	0.7851	0.6935	0.8603	0.7382	0.6307
0.0004	0.9608	0.9230	0.8867	0.9230	0.8516	0.7851	0.8867	0.7851	0.6935	0.8516	0.7230	0.6108	0.8178	0.6650	0.5361
0.0005	0.9512	0.9047	0.8603	0.9047	0.8178	0.7382	0.8603	0.7382	0.6307	0.8178	0.6650	0.5361	0.7771	0.5978	0.4532
0.0006	0.9417	0.8867	0.8346	0.8867	0.7851	0.6935	0.8346	0.6935	0.5725	0.7851	0.6108	0.4689	0.7382	0.5361	0.3810
0.0007	0.9323	0.8690	0.8095	0.8690	0.7535	0.6511	0.8095	0.6511	0.5186	0.7535	0.5602	0.4087	0.7008	0.4796	0.3185
0.0008	0.9230	0.8516	0.7851	0.8516	0.7230	0.6108	0.7851	0.6108	0.4689	0.7230	0.5129	0.3549	0.6650	0.4280	0.2649
0.0009	0.9138	0.8346	0.7613	0.8346	0.6935	0.5725	0.7613	0.5725	0.4231	0.6935	0.4689	0.3071	0.6307	0.3810	0.2192
0.001	0.9047	0.8178	0.7382	0.8178	0.6650	0.5361	0.7382	0.5361	0.3810	0.6650	0.4280	0.2649	0.5978	0.3383	0.1805
0.0011	0.8957	0.8013	0.7156	0.8013	0.6374	0.5016	0.7156	0.5016	0.3424	0.6374	0.3901	0.2277	0.5663	0.2997	0.1479
0.0012	0.8867	0.7851	0.6935	0.7851	0.6108	0.4689	0.6935	0.4689	0.3071	0.6108	0.3549	0.1952	0.5361	0.2649	0.1208

Illustration: Determine the RRGS Sampling Plans for the process average 0.01% with probability of acceptance 95%.

Solution: It is given that,

$$p_1 = 0.01\% \Longrightarrow AQL = p_1 = 0.0001, 1 - \alpha = 0.95$$

For practical reason if k=2. Then from table (1), the required sample size is n=200. Hence the parameters of RRGS sampling plans are

$$\begin{pmatrix} n = 200 \\ k = 2 \\ c_1 = 0 \\ c_2 = 2 \end{pmatrix}$$

Sampling Procedure

1. Draw a random sample of size n_1 = n =200.
2. Count the number of the defectives in the sample. Let it be d.
3. **Decision Rule**
   - If d = 0, accept the lot.
   - If d > 2, reject the lot.
   - If 1≤ d ≤ 2, go to next step by discarding the first sample.
4. Draw another sample of size n_2=k*n_1 = k*n = 2*200 = 400.
5. Count the number of the defectives in the sample. Let it be d.
6. **Decision Rule**
   - If d = 0, accept the lot.
   - If d >2, reject the lot.
   - If 1≤ d ≤ 2, reject the lot since the maximum iteration criteria k = 2.
7. Screen the entire rejected lot.

Summary
In this paper, if the lots are not accepted then immediately the sampling inspection parameter 'n' of RRGS is being linearly related with number of repetition within the same lot while sampling continues. The New algorithm makes the lot either accepted or rejected at the earliest within smaller number of trials and inspection. Hence the advantage is time and cost may be reduced while implementing the new RRGS inspection plans. It is found that when quality deteriorates then there is a decrease in the probability of acceptance of the lot.

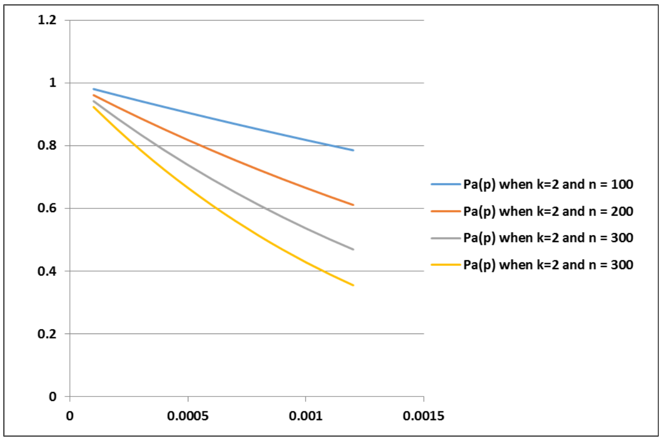

Fig 1: Probability of acceptance for the known AQL values
Interpretation: From figure (1), one can find the probability acceptance rapidly decreases over the increase in the sample size. Hence it is clear that when the quality deteriorates a high pressure is created for producer during the inspection.

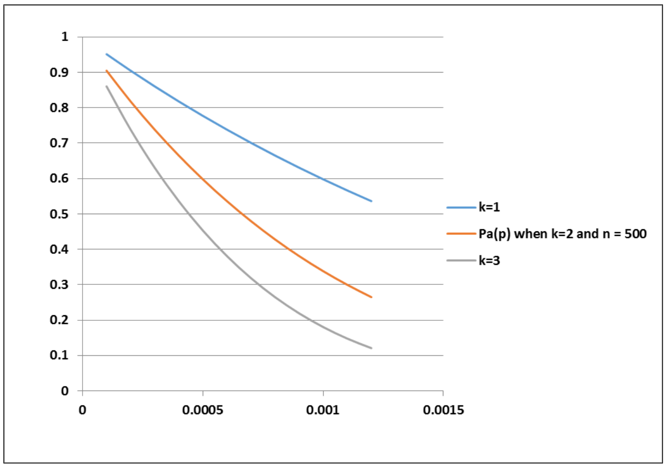

Fig 2: Probability of acceptance for different values of k
Interpretation: From figure (2), one can find that the OC values decreases rapidly even for small increase in the allowable repetition index k. Hence during designing of sampling protocol it is advisable to have small value of k.

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