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A stochastic model of urban characterization of a select district in Kerala

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Abstract
Urbanization is a process in which more people move from rural areas to urban centers by anticipating better physical infrastructure and enhanced quality of life. Urban life is meant to easy accessibility to the modern life style and provide gleaming standard of living. A stochastic model is a tool for estimating probability distributions of potential outcomes by allowing for random variation in one or more inputs over time. A Markov chain is a stochastic model describing a sequence of possible events in which the probability of each event depends only on the state attained in the previous event. Here a stochastic model of urban characterization of a select district, Ernakulum in Kerala is analyzed by considering 56 urban places in the district through different time points.

Keywords: Markov chain, urbanization, probability, city size, projection

Introduction
Over a five years or ten years period the demographic situation of a country is usually affected to a greater degree by changes in proportional distribution of its people than its growth. The rapidity and the magnitude of differential impact of modern development are so complex that the real mechanics of adjustment of people to economic opportunities comes, more from the distribution by movement than by natural growth. This problem has become serious concern of the planners because an unplanned concentration of people in cities may take away much of the benefits accrued by planned development. An essential characteristic of city environment is that it is suspectable to change. The spatial distribution of people in a country reflects its economic activity, material possession and prosperity. Almost half of the world's population is now urban, a percentage that rises to 75% in the case of the developed countries. An urbanization process has taken place throughout the length of the 20th century. This phenomenon, at least as far as developed countries is concerned, has been accomplished by significant process of industrialization and economic growth, which although showing an irregular pattern over the course of the years, nevertheless demonstrate a clearly increasing tendency. Prasanth & Praseeja [1] explained, that the city can spread out uniformly and entire land is available for residential buildings. That is, no estuaries, mountains, national parks or lakes are there considerably large to affect our assumption. The city population and the rank of the city are closely related and such a relation is represented by rank size rule. Also Prasanth & Praseeja [2] compared the nature of urbanization in Kerala and Tamil Nadu empirically. Archana & Prasanth [3] explained City Expansion and Urbanization of Kerala has marked unique features. Towards the centre of the city it is seen that the population density has an increasing tendency. In other words we can say that all the cities have an outer expanding nature and this expansion is circular. Pradeep & Prasanth [4] detailed that the slum dwellers have an association turning out to be a determinant factor for stalling of urbanization. Irma G Adelman [5] detailed the stochastic nature of the size distribution within industries. He suggested that the changes in the size distribution of a group of business firms over time might be analyzed in terms of a stochastic model and such model might be used to the size distribution which would be attained in long run dynamic equilibrium- that is the steady state. Fernando Sans [6] *et al*., analyzed the evolution of the Spanish urban structure during the period 1900-1999.

Preston and Bell [7] proposed a stochastic model to analyze the changing size distributions of large firms in the food processing and chain store food distributing industries during 1948-1958.Narayanaswami [8] studied the city size mobility and reveals a fund of evidence on the nature of urban growth in the period 1901 to 1971.

Methodology

In the analysis of urbanization, the total population is usually dichotomized into urban – rural classification. In the recent past urbanization in developing countries is weighted to big cities and there is no known or established pattern of distribution of people. The urban population can be polytomised by stratifying the cities according to the number of inhabitants. The hidden difference in the distribution can be identified by classifying the population in places by different sizes. Abundant empirical measures are being employed to know the hierarchical classification of cities by sizes. In this analysis the Markove model is employed to know the mobility of the cities by size. The essential ingredient of any stochastic model of mobility is thus a probabilistic description of how movement takes place from one class to another. The inherent uncertainty behaviour in these situations means that, the future development of the mobility process cannot be predicted with certainty but in only in terms of probability. The assumption is made that “the *chance of moving depends only on the present class and not on the past*”. If movement can be regarded as taking place at discrete points in time the appropriate model becomes a simple Markove chain.

Urban area is defined as: ‘All places, with municipality, corporation contentment or notified town area Committee, and the all other places, which satisfy the following criteria, i.e., a minimum population engaged in non–agricultural pursuits and at least 5% of male working population engaged in nonagricultural pursuits, and a density of population of at least 400 people per sq.km’. In India, the rank of the population growth was higher in the urban areas than the rural areas. Urbanization is the process of concentration with respect to the size of the urban center.

City size represents the number of population in the city. The causes of changes in the city size are random in nature. The city size variable is treated as a stochastic variable. Most of the city size distribution analysis are concerned either to identify the increases in the number of urban places or to identify the differences in the size classes at a particular point of time or a period of time.

Modeling is a fundamental aspect of the design process of a complex system, as it allows the designer to compare different architectural choices as well as predict the behavior of the system under varying inputs. A Markov chain essentially consists of a set of transitions, which are determined by some probability distribution, that satisfy the Markov property. The probability distribution is obtained by observing transitions from the current time to the next. The defining characteristic of a Markov chain is that no matter *how* the process arrived at its present state, the possible future states are fixed. This illustrates the Markov property, the unique characteristic of Markov processes that renders them memory less. This typically leaves them unable to successfully produce sequences in which some underlying trend would be expected to occur

Let $\{X_n\}$ be a sequence of random variables (r.v.) taking the values n ε I, assume I is finite. The sequence $\{X_n\}$ is said to be a Markov Chain (MC) if for all $i_0, i_1, \ldots, i_{n+1}$ ε I and for all n, $P(X_{n+1}= i_{n+1} / X_0= i_0, \ldots, X_n= i_n) = P(X_{n+1}= i_{n+1} / X_n= i_n)$. This property is called Markov property. The space I is called the state space and i_j’s are called the state of Markov chain. Since the state space I is assumed finite, we get a finite MC. Markov property can be interpreted to mean that the conditional distribution of X_{n+1} given $X_0, X_1, \ldots X_n$ depends only on X_n assuming n as time, X_n as the present value, $X_{n=1}$ the future value and $X_0,X_1,\ldots X_{n-1}$ as the past values, of a certain sequence of observations. Markov property implies that the future value depends only on the present value and not on the past values. Thus the future value depends only on the latest known value and not on the earlier observations. $P(X_0 = i_0)$ is called the initial probability and $P(X_j= i_j/ X_{j-1} =i_{j-1})$, for all j=1,2,3…, n+1 are called one-step probability of the MC. These probabilities determine the joint distribution of any finite number of r.v.’s of the MC. sequence. Hence the probability distribution of MC is completely determined if we are given the initial probability distribution and one – step transition probabilities.

If $P (X_{n+1}= i_{n+1} / Xn= i_n)$ depends only on (i_{n+1}, i_n) and not on n, the transition probabilities are said to be stationary. Then we can denote it as $Pi_{n+1}, i_{n.}$ in this case $P(X_n= j / X_{n-1} = i) =P (X_1 = j / X_0 = i)$

$= P_{ij}$, (i,j ε I),specify the one-step probability matrix (TPM). If $P = (p_{ij})$, P is called the one-step TPM. We have $0 \le P_{ij} \le 1$, $\Sigma P_{ij} = 1.$ Hence the elements of P are non- negative and the row-sum equals unity, then the matrix is called a stochastic matrix. Let $P_{ij}^{(n)}=P(X= j/X_0= i) = P(X_{n+m}= j / X_m= i)$, for all m=1,2,… Then the matrix with (i,j)thelement as $(P_{ij}(n))$ is called n-step TPM and is denoted by P(n). It is a stochastic matrix. The unconditional probability $P(X_n=j) = P_j^{(n)} = \Sigma P(X_n= j/ X_0 =i)$, (sum on i) which is equal to $\Sigma P(X_n= j/ X_0 =i)P(X_0=i)= \Sigma P_i^{(0)} P_{ij}^{(n)}$ where $P_i^{(0)}$is the initial probability distribution of i. $P^{(n)}$ is the row vector of $P_j(n)$’s. Thus knowing the initial state and the n-step transition probabilities, the distribution of values after n additional units of time can be determined. Then for MC with stationary probabilities $P_{ij}^{(m+n)}=\Sigma P_{ik}^{(n)}P_{kj}^{(m)}$, n≥0,m≥k, k ε I., or $P^{(m+n)} = P^{(m)} P^{(n)}$. This equation is called Chapman- Kolmogrove equation. The problem of obtaining n-step transition probabilities is same as the problem of finding the nth power of P. Let the urban population of a country be divided into m size classes, where m stands for the states of a MC, that is, the different size classes are termed as the states of a MC. As population growth takes place, cities move from one state to another at different points of observation. These points can be taken as the natural points of censes dates which are normally conducted in most of the countries within a period of ten years. At different points of time the changes in city size distribution can be represent by the TPM, P with $0 \le P_{ij} \le 1$, $\Sigma P_{ij}= 1$, i,j =1,2,…m. Since, the only present state of the process determine the future, $P(X_n=j) = \Sigma P(X_{n-1} =i)P(X_{n=j / Xn-1} =i) = \Sigma P_i^{(n-1)}P_{ij}= P_j^{(n)}$. Here, the size of the cities is projected using Markov chain model.

Empirical Analysis

The data for this analysis are taken from the Census of India (1981, 1991, 2001& 2011) [9 -12]. The population of 56 urban cities in Ernakulum district Kerala state of India is considered. Indian census has a history of more than hundred years. The period of observation is confined to 1981 to 2011 omitting the date from the initial period of census for obvious reasons of coverage. As we are dealing with ‘the number of urban places’, the quality will be of very high order, compare to any other demographic characteristics and at least in this respect, we are in a confident position to exploit the census materials as applicable to our study.

The urban classes are classified into six classes as, below 5000, 5000-10000, 10000-20000, 20000-50000, 50000-100000, and above 100000. The number of places belonging to the class, ‘below 5000’ are mostly a subjective classification by census directors as mentioned above and their inclusion and declassification at different census points poses certain difficulties in constructing the transition probabilities. As they are very few in number and their inclusion as urban places is mostly of subjective nature, this category has omitted from our study. The places included in the analysis are so selected that a place in 1981 continues to hold the status of urban throughout the period of observation. Size of these cities is projected up to 2050 using Markov chain model. The population distribution of 56 urban cities in Ernakulum district Kerala state from Census of India (1981, 1991, 2001 & 2011) is in table 1.

Table 1: Distribution of Ernakulum city urban population

Sl No	Name of the place	Population 1981	Population 1991	Population 2001	Population 2011
1.	Kochi	504589	564589	625522	633553
2.	Maradu	31995	34995	40,993	44704
3.	Kumbalangy	32310	38310	40331	42.367
4.	Thrippunithura	47078	51078	59884	69390
5.	Kumbalam	20400	25400	27549	29193
6.	Thiruvankulam	11412	18412	21717	23160
7.	Mulavukad	18322	22322	22,845	21833
8.	Kureekkad	7100	7800	9,730	13348
9.	Eranelloor	20990	24990	28223	33829
10.	Vazhakkala	14975	18975	43078	51242
11.	Puthuvype	20400	21000	22109	23717
12.	Mulamthuruthy	20107	21807	23615	25852
13.	Kakkanad	18500	19800	22486	25531
14.	Manakunnam	23880	29880	33523	39538
15.	Elamkunnapuzha	20013	24013	26092	26997
16.	Kanayannur	6750	7650	9443	9308
17.	Kadamakkudy	9023	11023	15,823	16295
18.	Njarackal	16803	19803	24166	23760
19.	Puthencruz	13000	18000	21054	22378
20.	Amballur	7900	9800	10568	11358
21.	Eloor	30485	34485	30,092	31468
22.	Kunnathunad	13958	18958	20500	22881
23.	Kalamassery	45342	54342	63,176	71038
24.	Varappuzha	20114	22514	24,524	26750
25.	Choornikkara	30837	34837	36,998	43207
26.	Alangad	32956	38956	40,585	47329
27.	Aluva	19,774	24,774	24,108	22428
28.	Kadungalloor	18433	25433	35,451	39666
29.	Kottuvally	33457	34457	37,884	42922
30.	Edathala	51397	56397	67,137	77811
31.	Chowwara	8864	9864	13,603	14933
32.	Paravur	21906	27906	30,056	31503
33.	Karumalloor	22896	25896	26856	29805
34.	Vengola	18689	21689	26034	32697
35.	Chengamanad	21621	24621	29,775	29576
36.	Chennamangalam	24,825	26,825	28,147	29326
37.	Marampilly	15975	18975	20071	23272
38.	Thekkumbhagom	8967	9867	10023	10798
39.	Vadakkekara	16965	18965	20099	20571
40.	Vadakkumbhagom	7354	10354	11584	11727
41.	Nedumbassery	24089	24689	28607	29706
42.	Perumbavoor	23667	24667	26547	28110
43.	Angamaly	30,009	30,391	33,424	33465
44.	Velloorkunnam	9235	10235	10363	11576
45.	Kizhakkumbhagom	9146	9246	10038	10791
46.	Muvattupuzha	25595	27595	29246	30397
47.	Moothakunnam	23698	24698	27293	27488
48.	Perumbavoor	8986	7986	8026	9185
49.	Puthenvelikkara	25686	29686	32213	33372
50.	Kalady	18865	19865	20407	20380
51.	Koovappady	20658	24658	26525	39339
52.	Mattoor	11689	15689	17862	18890
53.	Cheranallur	18407	21407	26,330	30594
54.	chelamattom	11569	12569	15,366	16844
55.	Vazhakulam	12054	12546	14206	18358
56.	Kothamangalam	35235	35535	37173	38837
	Total	1638950	1861224	2089080	2202068

Census of India 1981, 1991, 2001 & 2011 Kerala, Final population Totals

Bivariate frequency distribution of urban places in Ernakulum district -Kerala with respect to their sizes in 1991- 2001 (Table 2), 2001-2011 (Table 3). The data in (Table 2 & Table 3) are used to form the transition probability matrix (TPM) of cities in Ernakulum district -Kerala 1991- 2001 (Table 4) and 2001-2011 (Table 5).

Table 2: Bi-variate frequency distribution of urban places in Ernakulum district with respect to their sizes in 1991- 2001

2001→1991 ↓	5000-10000	10000- 20000	20000- 50000	50000- 100000	Greater than 100000	Total
5000-10000	8	1	1	0	0	10
10000-20000	0	9	5	0	0	14
20000-50000	0	0	28	1	0	29
50000-100000	0	0	0	2	0	2
Greater than 100000	0	0	0	0	1	1
Total	8	10	34	3	1	56

Table 3: Bi-variate frequency distribution of urban places in Ernakulum district with respect to their sizes in 2001- 2011

2011 → 2001 ↓	5000-10000	10000-20000	20000-50000	50000-100000	Greater than 100000	Total
5000-10000	4	2	2	0	0	8
10000-20000	0	9	1	0	0	10
20000-50000	0	0	33	1	0	34
50000-100000	0	0	0	3	0	3
Greater than 100000	0	0	0	0	1	1
Total	4	11	36	4	1	56

Table 4: Transition probability matrix (TPM) of cities in Ernakulum district 1991- 2001

1991‹ 1981$	5000- 10000	10000- 20000	20000-50000	50000-100000	Greater than 100000	Total
5000-10000	0.8	0.1	0.1	0	0	1
5000-10000	0	0.64	0.36	0	0	1
20000-50000	0	0	0.97	0.03	0	1
50000-100000	0	0	0	1	0	1
Greater than 100000	0	0	0	0	1	1

Table 5: Transition probability matrix (TPM) of cities in Ernakulum district with respect to their sizes in 2001- 2011

2001 ‹ 1991$	5000-10000	10000-20000	20000-50000	50000-100000	Greater than100000	Total
5000-10000	0.5	0.25	0.25	0	0	1
5000-10000	0	0.9	0.1	0	0	1
20000-50000	0	0	0.97	0.03	0	1
50000-100000	0	0	0	1	0	1
Greater than 100000	0	0	0	0	1	1

Projection of urban population in Ernakulum district by using Chapman – Kolmogorov equation

$X^{(1)} = P X^{(0)}$, where P is the initial probability matrix in (Table 4), $X^{(0)}$ is the initial frequency in each group, $X^{(1)}$ is the first estimated frequency.

So,

$X^{(i+1)} = P^{(i+1)} X^{(i)}$, i = 1,2,3…, where $X^{(i)}$ is the ith estimated frequency and $P^{(i)}$ is the ith power of initial TPM, P.

Here we get, $X^{(0)} = (10,14,29,2,1)$ from (Table 2).

The initial probability matrix P is from Table 4 and calculates the powers of P, Table 6. Hence we get the TPM P, P^2, P^3… P^6respectively.

Table 6. The TPM P, P^2, P^3… P^6.

0.8	0.1	0.1	0	0
0	0.64	0.36	0	0
0	0	0.97	0.03	0
0	0	0	1	0
0	0	0	0	1

P =

0.64	0.144	0.213	0.003	0
0	0.409	0.579	0.011	0
0	0	0.941	0.059	0
0	0	0	1	0
0	0	0	0	1

P^2 =

0.512	0.156	0.322	0.009	0
0	0.262	0.709	0.028	0
0	0	0.912	0.087	0
0	0	0	1	0
0	0	0	0	1

P^3 =

0.409	0.151	0.420	0.019	0
0	0.167	0.782	0.049	0
0	0	0.885	0.114	0
0	0	0	1	0
0	0	0	0	1

P^4=

0.167	0.087	0.66	0.082	0
0	0.028	0.824	0.147	0
0	0	0.783	0.216	0
0	0	0	1	0
0	0	0	0	1

P^5=

0.262	0.120	0.570	0.046	0
0	0.068	0.833	0.097	0
0	0	0.832	0.167	0
0	0	0	1	0
0	0	0	0	1

P^6= Then the Projection of urban population of Ernakulum district in Kerala is,
$X^{(0)}$ = (10,14,29,2,1), $X^{(1)}$ =$PX^{(0)}$ = (12,20,29,3,1), $X^{(2)}$ = (13,24,26,3,1), $X^{(3)}$ = (20,25,24,3,1), $X^{(4)}$ = (21,29,22,3,1), $X^{(5)}$ = (24,32,19,3,1), $X^{(6)}$ = (25,33,16,3,1)

Table 7: Projected of urban population of Ernakulum district in Kerala

Year	Urban classes				
	5000-10000	**10000- 20000**	**20000-50000**	**50000-100000**	**Greater than 100000**
1991	10	14	29	2	1
2001	12	20	29	3	1
2011	13	24	26	3	1
2021	20	25	24	3	1
2031	21	29	22	3	1
2041	24	32	19	3	1
2051	25	33	16	3	1

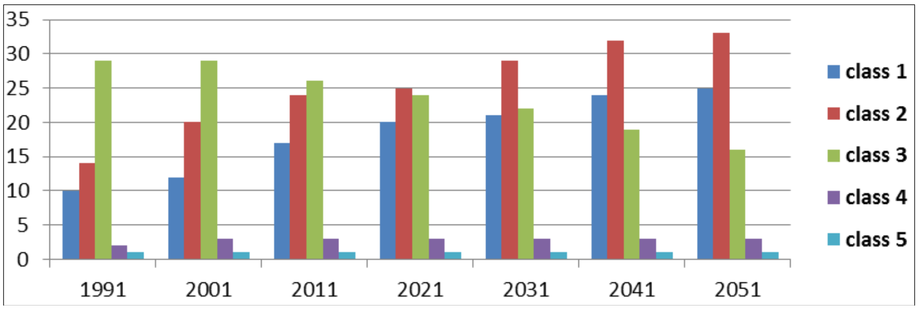

Fig 1: Projected urban population up to 2051 of Ernakulum district in Kerala

Here Class 1: 5000-10000, Class 2: 10000- 20000, Class 3: 20000-50000, Class 4: 50000-100000, Class 5: Greater than 100000. As Prasanth & Praseeja[1] explained the city can spread out uniformly and entire land is available for residential buildings and as Archana & Prasanth[3] explained City Expansion and Urbanization of Kerala, towards the centre of the city it is seen that the population density has an increasing tendency. In other words we can say that all the cities have an outer expanding nature and this expansion is circular. So with respect to time new urban cities are formulating and many existing cities are showing a clear change in the city size & its population and hence shifting the classes from one to other. This fact is clear from the figure 1 & table 7, as at 1991 in class1, there exist only 10 cities where as at 2050 it shows 25 cities in the same. More than that at 1991 there are only 56 urban cities in the Ernakulum district & it comes to a total of 78 in the projected scores of 2050. Excluding the newly formulate such cities & comparing the existing 56 city sizes with the expected one by using chi-square statistics the Projection of urban population in Ernakulum district by using Chapman – Kolmogorov equation is a good fit and hence the model is suitable for such data analysis.

Table 8: Inference about the goodness of Markove model (Ernakulum district urban population)

Year	Calculated χ^2	Table χ^2 (a =0.01)	d.f	p-value	Inference
1991	5.40	9.21	2	0.07	Model is suitable for these data
2001	7.20	9.21	2	0.027	
2011	8.10	9.21	2	0.018	

Conclusions

The projected urban population (Table 7 and Figure 1) shows the number of cities increases in class 5000-10000 & 10000-20000 and decreases in class 20000-50000 with respect to migration and reconstruction of the area of cities. There is a consistency in higher city size classes. The Markove Model is suitable for the study of city size distribution. The real data analysis by using the Ernakulum district's urban population in different decades shows the suitability of the model. More than an empirical analysis this is to establish the suitability of such a stochastic model in these types of studies. This study point outs the migration and reconstruction of the area of cities. That implies, still there exists many significant anomalies, which should be identified and analyzed by the government.

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