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Measure of modified slope rotatability for second order response surface designs using a pair of balanced incomplete block designs

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Abstract
In this paper, a study on measure of modified slope rotatability for second order response surface designs using a pair of balanced incomplete block designs is suggested which enables us to assess the degree of modified slope rotatability for a given response surface designs and variance of the estimated responses are also obtained.

Keywords: Second order response surface designs, modified slope rotatability, measure of slope rotatability

1. Introduction

Investigation of input-output relationship is a useful activity in many situations. Fitting input-output relations to unorganised data involves complex computations and control of precession of estimates of response at desired points is not possible. An alternative is to use for fitting planned data obtained through appropriate designs. The concept of rotatability, which is very important in second order response surface, was introduced by Box and Hunter (1957) [1] with the property that the variances of estimates of response at points equidistant from the centre of the design are all equal. Das and Narasimham (1962) [2] constructed rotatable designs through balanced incomplete block designs (BIBD). Hader and Park (1978) [4] introduced slope rotatable central composite designs (SRCCD). Victorbabu and Narasimham (1991a) [6] studied second order slope rotatable designs (SOSRD) and constructed SOSRD using BIBD. Victorbabu and Narasimham (1991b) [7] constructed SOSRD through a pair of incomplete block designs. Park and Kim (1992) [5] studied measure of slope rotatability for second order response surface experimental designs. Victorbabu and Surekha (2012a, 12b, 12c, 2016) [16, 17, 18, 19] studied different measures measure of slope rotatability for second order response surface designs using BIBD, PBD and SUBA with two unequal block sizes and a pair of BIBD respectively. Das *et al.* (1999) [3] introduced modified response surface designs. Victorbabu (2005) [8] introduced modified SRCCD. Victorbabu (2006a) [9] suggested modified second order slope rotatable designs using BIBD. Victorbabu (2006b) [10] studied modified second order rotatable designs and second order slope rotatable designs using a pair of BIBD. Victorbabu (2007) [11] suggested a review on SOSRD. Victorbabu and Jyostna (2016) [19] studied measure of rotatability for second order response surface designs using a pair of BIBD. Victorbabu and Jyostna (2020a, 20b, 20c) [13, 14, 15] studied measure of modified slope rotatability for second order response surface designs using central composite designs, BIBD and pairwise balanced design respectively. In this paper, a study on measure of modified slope rotatability for second order response surface design using a pair of BIBD is suggested.

2. Conditions for second order slope rotatable designs

Suppose we want to use the second order response surface design $D=((x_{iu}))$ to fit the surface,

$$Y_u = b_0 + \sum_{i=1}^{v} b_i x_{iu} + \sum_{i=1}^{v} b_{ii} x_{iu}^2 + \sum_{i=1}^{v}\sum_{j=1}^{v} b_{ij} x_{iu} x_{ju} + e_u \quad (1)$$

Where x_{iu} denotes the level of the

i^{th} factor $(i=1,2,\ldots,v)$ in the u^{th} run $(u=1,2,\ldots,N)$

of the experiment, e_u's are uncorrelated random errors with mean zero and variance σ^2 is said to be SOSRD if the variance of the estimate of first order partial derivative of $Y_u(x_1,x_2,\ldots,x_v)$ with respect to each of independent variables (x_i) is only a function of the distance $(d^2=\sum_{i=1}^{v} x_i^2)$ of the point $(x_1,x_2,\ldots,x_v)$ from the origin of the design.

Following Box and Hunter (1957) [1], Hader and Park (1978) [4] and Victorbabu and Narasimham (1991) [6] the general conditions for second order slope rotatability can be obtained as follows. To simplify the fit of the second order polynomial from design points 'D' through the method of least squares, we impose the following simple symmetry conditions on D to facilitate easy solutions of the normal equations:

1. $\sum x_{iu}=0,\ \sum x_{iu}x_{ju}=0,\ \sum x_{iu}x_{ju}^2=0,\ \sum x_{iu}x_{ju}x_{ku}=0,\ \sum x_{iu}^3=0,\ \sum x_{iu}x_{ju}^3=0,$

   $\sum x_{iu}x_{ju}x_{ku}^2=0,\ \sum x_{iu}x_{ju}x_{ku}x_{lu}=0;$ for $i\neq j\neq k\neq l$

2. (i) $\sum x_{iu}^2=\text{constant}=N\lambda_2;$

   (ii) $\sum x_{iu}^4=\text{constant}=cN\lambda_4;$ for all i

3. $\sum x_{iu}^2x_{ju}^2=\text{constant}=N\lambda_4;$ for $i\neq j$ (2)

where c, λ_2 and λ_4 are constants.

The variances and covariance's of the estimated parameters are

$$V(\hat{b}_0)=\frac{\lambda_4(c+v-1)\sigma^2}{N\left[\lambda_4(c+v-1)-v\lambda_2^2\right]},$$

$$V(\hat{b}_i)=\frac{\sigma^2}{N\lambda_2},$$

$$V(\hat{b}_{ij})=\frac{\sigma^2}{N\lambda_4},$$

$$V(\hat{b}_{ii})=\frac{\sigma^2}{(c-1)N\lambda_4}\left[\frac{\lambda_4(c+v-2)-(v-1)\lambda_2^2}{\lambda_4(c+v-1)-v\lambda_2^2}\right],$$

$$Cov(\hat{b}_0,\hat{b}_{ii})=\frac{-\lambda_2\sigma^2}{N[\lambda_4(c+v-1)-v\lambda_2^2]},$$

$$Cov(\hat{b}_{ii},\hat{b}_{jj})=\frac{(\lambda_2^2-\lambda_4)\sigma^2}{(c-1)N\lambda_4[\lambda_4(c+v-1)-v\lambda_2^2]} \qquad (3)$$

And other covariances vanish.

An inspection of the variance of $\hat{b}_0$ shows that a necessary condition for the existence of a non-singular second order design is

4. $$\frac{\lambda_4}{\lambda_2^2}>\frac{v}{(c+v-1)} \qquad (4)$$

For the second order model

$$\frac{\partial\hat{Y}}{\partial x_i}=\hat{b}_i+2\hat{b}_{ii}x_{iu}+\sum_{j\neq i}\hat{b}_{ij}x_{ju} \qquad (5)$$

$$V\left(\frac{\partial \hat{Y}}{\partial x_i}\right) = V(\hat{b}_i) + 4x_{iu}^2 V(\hat{b}_{ii}) + \sum_{j \neq i} x_{ju}^2 V(\hat{b}_{ij}) \quad (6)$$

The condition for right hand side of the equation (2.6) to be a function of $d^2 = \sum_{i=1}^{v} x_i^2$ alone (For slope rotatability) is

On simplification of (7), we get,

$$5.\,[v(5-c)-(c-3)^2]\,\lambda_4 + [v(c-5)+4]\,\lambda_2^2 = 0 \quad (8)$$

Therefore 1, 2 and 3 of (2), (4) and (8) give a set of conditions for slope rotatability in any general second order response surface design (cf. Hader and Park (1978) [4], Victorbabu and Narasimham (1991) [6].

3. Conditions for modified second order slope rotatable designs

Following Hader and Park (1978) [4], Victorbabu and Narasimham (1991) [6], equations 1, 2 and 3 of (2), (3), (4) and (8) give the necessary and sufficient conditions for modified second order slope rotatable designs (cf. Das *et al*. 1999, Victorbabu, 2005a, 06a) [3, 8].

The usual method of construction of SOSRD is to take combinations with unknown constants, associate a 2^v factorial combinations or a suitable fraction of it with factors each at ± 1 levels to make the level codes equidistant. All such combinations form a design. Generally SOSRD need at least five levels (suitably coded) at $0, \pm 1, \pm a$ for all factors ($(0,0,\ldots,0)$ – chosen center of the design, unknown level 'a' to be chosen suitably to satisfy slope rotatability). Generation of design points this way ensures satisfaction of all the conditions even though the design points contain unknown levels. Alternatively by putting some restrictions indicating some relation among $\sum x_{iu}^2, \sum x_{iu}^4$ and $\sum x_{iu}^2 x_{ju}^2$ some equations involving the unknowns are obtained and their solution gives the unknown levels. In SOSRD the restrictions used is $4V(\hat{b}_{ii}) = V(\hat{b}_{ij})$ *viz.* equation (8). Other restrictions are also possible though it seems, not exploited well. We shall investigate the restriction $\left(\sum x_{iu}^2\right)^2 = N \sum x_{iu}^2 x_{ju}^2$ i.e., $(N\lambda_2)^2 = N(N\lambda_4)$ i.e., $\lambda_2^2 = \lambda_4$

to get modified SOSRD. By applying the new restriction in equation (8), we get $c=1$ or $c=5$. The non-singularity condition (4) leads to $c=5$. It may be noted $\lambda_2^2=\lambda_4$ and $c=5$ are equivalent conditions. The variances and covariances of the estimated parameters are,

$$V(\hat{b}_0) = \frac{(v+4)\sigma^2}{4N},$$

$$V(\hat{b}_i) = \frac{\sigma^2}{N\sqrt{\lambda_4}},$$

$$V(\hat{b}_{ij}) = \frac{\sigma^2}{N\lambda_4},$$

$$V(\hat{b}_{ii}) = \frac{\sigma^2}{4N\lambda_4}$$

$$\mathrm{Cov}(\hat{b}_0, \hat{b}_{ii}) = \frac{-\sigma^2}{4N\sqrt{\lambda_4}} \text{ and other covariances are zero.} \quad (9)$$

$$V\left(\frac{\partial \hat{Y}}{\partial x_i}\right) = \left[\frac{\sqrt{\lambda_4}+d^2}{N\lambda_4}\right]\sigma^2 \quad (10)$$

4. Conditions of measure of slope rotatability for second order response surface designs Following Hader and Park (1978) [4], Victorbabu and Narasimham (1991) [6], Park and Kim (1992) [5], equations (2), (3), (4) and (8) give the necessary and sufficient conditions for a measure of slope rotatability for any general second order response surface designs. Further we have,

$V(b_i)$ are the same for all i,

$V(b_{ii})$ are the same for all i,

$V(b_{ij})$ are the same for all i, j, where $i \neq j$,

$Cov(b_i,b_{ii}) = Cov(b_i,b_{ij}) = Cov(b_{ii},b_{ij}) = Cov(b_{ij},b_{il})$ for all $i \neq j \neq l$.

The measure of slope rotatability for second order response surface design can be obtained by using the following equation (cf. Park and Kim, 1992, page 398) [5].

$$Q_v(D)=\frac{1}{2(v-1)\sigma^4}\left\{(v+2)(v+4)\sum_{i=1}^{v}\left[\left(v(b_i)-\frac{1}{v}\sum_{i=1}^{v}v(b_i)\right)+\frac{\left(4v(b_{ii})+\sum\limits_{\substack{j=1\\ j\neq i}}^{v}v(b_{ij})\right)-\frac{1}{v}\sum\limits_{i=1}^{v}\left(4v(b_{ii})+\sum\limits_{\substack{j=1\\ j\neq i}}^{v}v(b_{ij})\right)}{v+2}\right]^2\right.$$

$$+\frac{4}{v(v+2)}\sum_{i=1}^{v}\left(\left(4v(b_{ii})+\sum_{\substack{j=1\\ j\neq i}}^{v}v(b_{ij})\right)-\frac{1}{v}\sum_{i=1}^{v}\left(4v(b_{ii})+\sum_{\substack{j=1\\ j\neq i}}^{v}v(b_{ij})\right)\right)^2+2\sum_{i=1}^{v}\left[\left[4v(b_{ii})-\frac{4v(b_{ii})+\sum\limits_{\substack{j=1\\ j\neq i}}^{v}v(b_{ij})}{v}\right]^2+\sum_{\substack{j=1\\ j\neq i}}^{v}\left[v(b_{ij})-\frac{4v(b_{ii})+\sum\limits_{\substack{j=1\\ j\neq i}}^{v}v(b_{ij})}{v}\right]^2\right]$$

$$\left.+4(v+4)\left[4cov^2(b_i,b_{ii})+\sum_{\substack{j=1\\ j\neq i}}^{v}cov^2(b_i,b_{ij})\right]+4\sum_{i=1}^{v}\left[4\sum_{\substack{j=1\\ j\neq i}}^{v}cov^2(b_{ii},b_{ij})+\sum_{\substack{j<l\\ j,l\neq i}}^{v}\sum cov^2(b_{ij},b_{il})\right]\right\}$$

where $Q_v(D)$ is the measure of slope-rotatability. It can be verified that $Q_v(D)$ is zero if and only if a design D is slope-rotatable. $Q_v(D)$ becomes larger as D deviates from a slope-rotatable design. Further $Q_v(D)$ is simplified to $Q_v(D)=\frac{1}{\sigma^4}\left[4V(b_{ii})-V(b_{ij})\right]^2$.

5. Modified second order slope rotatable designs using a pair of BIBD: The method of construction of modified SOSRD using a pair of BIBD is given in the following result (cf. Victorbabu 2006) [9].

If $D_1=(v, b_1, r_1, k_1, \lambda_1)$ and $D_2=(v, b_2, r_2, k_2, \lambda_2)$ are two BIBD's, $2^{t(k1)}$ and $2^{t(k2)}$ Resolution V fractional replicates of 2^{k1} and 2^{k2} factorials with levels ± 1 then the design points, $[1-(v, b_1, r_1, k_1, \lambda_1)]2^{t(k_1)} \cup [a-(v, b_2, r_2, k_2, \lambda_2)]2^{t(k_2)} \cup (n_0)$

give a v-dimensional modified SOSRD in

$$N=\frac{(r_1 2^{t(k_1)}+r_2 2^{t(k_2)}a^2)^2}{\lambda_1 2^{t(k_1)}+\lambda_2 2^{t(k_2)}a^4}$$ design points if,

$$a^4=\frac{(5\lambda_1-r_1)}{(r_2-5\lambda_2)}2^{t(k_1)-t(k_2)},$$

$$n_0=\left(\frac{(r_1 2^{t(k_1)}+r_2 2^{t(k_2)}a^2)^2}{\lambda_1 2^{t(k_1)}+\lambda_2 2^{t(k_2)}a^4}\right)-b_1 2^{t(k_1)}-b_2 2^{t(k_2)}$$ and n_0 turns out to be an integer.

6. Measure of slope rotatability for second order response surface designs using a pair of BIBD

The result of measure of slope rotatability for second order response surface designs using a pair of BIBD is suggested here (cf. Victorbabu and Surekha, 2016) [19]. Let $D_1=(v, b_1, r_1, k_1, \lambda_1)$ and $D_2=(v, b_2, r_2, k_2, \lambda_2)$ are two BIBDs. Then the design points, $[1\text{-}(v, b_1, r_1, k_1, \lambda_1)]2^{t(k_1)} \cup [a\text{-}(v, b_2, r_2, k_2, \lambda_2)]2^{t(k_2)} \cup (n_0)$

will give a v-dimensional measure of slope rotatability for second order response surface designs using pair of BIBD in $N = b_1 2^{t(k_1)} + b_2 2^{t(k_2)} + n_0$ design points, as follows

$$Q_v(D)=\left[\frac{\sum x_{iu}^2}{N}\right]^4 \left[4e\text{-}V(b_{ij})\right]^2$$

Where

$$e = V(b_{ii}) = \frac{\begin{gathered}(v\text{-}1)\left[2^{t(k_1)}\lambda_1 n_0 + 2^{t(k_1)+t(k_2)} b_2\lambda_1 \text{-} 2r_1r_2\, 2^{t(k_1)+t(k_2)} a^2\right] + \\ \left[\begin{gathered}(v\text{-}1)\left(2^{t(k_1)+t(k_2)} b_1\lambda_2 + 2^{2t(k_2)} b_2\lambda_2 + 2^{t(k_2)} n_0\lambda_2 - 2^{2t(k_2)} r_2^2\right) + \\ (r_2-\lambda_2)\left(2^{t(k_1)+t(k_2)} b_1 + 2^{2t(k_2)} b_2 + 2^{t(k_2)} n_0\right)\end{gathered}\right] a^4 + \\ (r_1-\lambda_1)\left(2^{2t(k_1)} b_1 + 2^{t(k_1)+t(k_2)} b_2 + 2^{t(k_1)} n_0\right) + \left(vb_1\lambda_1 - (v\text{-}1)r_1^2 - b_1\lambda_1\right)2^{2t(k_1)}\end{gathered}}{\left[2^{t(k_1)}(r_1-\lambda_1) + 2^{t(k_2)}(r_2-\lambda_2)a^4\right]\left[\begin{gathered}(r_1-\lambda_1)\left(2^{2t(k_1)} b_1 + 2^{t(k_1)+t(k_2)} b_2 + 2^{t(k_1)} n_0\right) + \\ \left[\begin{gathered}(r_2-\lambda_2)\left(2^{t(k_1)+t(k_2)} b_1 + 2^{2t(k_2)} b_2 + 2^{t(k_2)} n_0\right) + \\ 2^{t(k_1)+t(k_2)} vb_1\lambda_2 + 2^{2t(k_2)} vb_2\lambda_2 + 2^{t(k_2)} vn_0\lambda_2 - 2^{2t(k_2)} vr_2^2\end{gathered}\right] a^4 + \\ (b_1\lambda_1 - r_1^2)v2^{2t(k_1)} + 2^{t(k_1)} v\lambda_1 n_0 + (b_2\lambda_1 - 2r_1r_2a^2)v2^{t(k_1)+t(k_2)}\end{gathered}\right]}$$

If $Q_v(D)$ is zero, if and only if, a design 'D' is slope-rotatable. $Q_v(D)$ becomes larger as 'D' deviates from a Slope Rotatable Design (cf. Park and Kim (1992) [5], Surekha and Victorbabu (2016) [19].

7. Measure of modified slope rotatability for second order response surface designs using a pair of BIBD

The proposed measure of modified slope rotatability for second order response surface designs using a pair of BIBD is suggested here.

Let $D_1=(v, b_1, r_1, k_1, \lambda_1)$ and $D_2=(v, b_2, r_2, k_2, \lambda_2)$ are two BIBDs. Let $[1\text{-}(v, b_1, r_1, k_1, \lambda_1)]$ denote the design points generated from the transpose of the incidence matrix of the design D_1. Let $[1\text{-}(v, b_1, r_1, k_1, \lambda_1)]2^{t(k_1)}$ are the $b_1 2^{t(k1)}$ design points generated from the design D_1 by multiplication (see Das and Narasimham 1962) [2] and $[a\text{-}(v, b_2, r_2, k_2, \lambda_2)]2^{t(k_2)}$ are the $b_2 2^{t(k2)}$ design points generated from design D_2 by multiplication. N_0 be the number of central points. Repeat the set of $b_2 2^{t(k2)}$ design points generated from design $D_2 n^2$ times. Then with the above design points along with n_0 central points we can construct a measure of modified SOSRD which is given below.

Then the design points, $[1\text{-}(v, b_1, r_1, k_1, \lambda_1)]2^{t(k_1)} \cup n_a[a\text{-}(v, b_2, r_2, k_2, \lambda_2)]2^{t(k_2)} \cup (n_0)$

Generated from pair of BIBD in $N = \dfrac{(r_1 2^{t(k_1)} + r_2 n_a 2^{t(k_2)} a^2)^2}{\lambda_1 2^{t(k_1)} + \lambda_2 n_a 2^{t(k_2)} a^4}$ design points, will give a v-dimensional measure of modified slope rotatability for second order response surface designs using a pair of BIBD with

$$a^4 = \frac{(5\lambda_1\text{-}r_1)}{(r_2\text{-}5\lambda_2)n_a} 2^{t(k_1)\text{ - }t(k_2)},$$

$$n_0 = \left(\frac{(r_1 2^{t(k_1)} + r_2 n_a 2^{t(k_2)} a^2)^2}{\lambda_1 2^{t(k_1)} + \lambda_2 n_a 2^{t(k_2)} a^4}\right) - b_1 2^{t(k_1)} - n_a b_2 2^{t(k_2)}$$

and n_0 turns out to be an integer.

(Alternatively N may be obtained directly as $N = b_1 2^{t(k_1)} + n_a b_2 2^{t(k_2)} + n_0$ design points) For the above design points generated from a pair of BIBDs, conditions (2) to (5) are true. Conditions in (2) are true obviously. Conditions (3) to (5) are true as follows.

$$\sum x_{iu}^2 = r_1 2^{t(k_1)} + r_2 n_a 2^{t(k_2)} a^2 = N\lambda_2 \quad (11)$$

$$\sum x_{iu}^4 = r_1 2^{t(k_1)} + r_2 n_a 2^{t(k_2)} a^4 = cN\lambda_4 \quad (12)$$

$$\sum x_{iu}^2 x_{ju}^2 = \lambda_1 2^{t(k_1)} + \lambda_2 n_a 2^{t(k_2)} a^4 = N\lambda_4 \quad (13)$$

from equations (11), (12) and (13), and on simplification, we get

$$a^4 = \frac{(5\lambda_1 - r_1)}{(r_2 - 5\lambda_2) n_a} 2^{t(k_1) - t(k_2)},$$

$$\lambda_2 = \frac{r_1 2^{t(k_1)} + r_2 n_a 2^{t(k_2)} a^2}{N} \text{ and } \lambda_4 = \frac{\lambda_1 2^{t(k_1)} + \lambda_2 n_a 2^{t(k_2)} a^4}{N}.$$

To obtain measure of modified slope rotatability for second order response surface designs using a pair of BIBD we investigate the restriction $(\sum x_{iu}^2)^2 = N \sum x_{iu}^2 x_{ju}^2$ i.e., $(N\lambda_2)^2 = N(N\lambda_4)$ i.e., $\lambda_2^2 = \lambda_4$ (cf. Victorbabu, 2006) [9] and on simplification, we get,

$$n_0 = \left(\frac{(r_1 2^{t(k_1)} + r_2 n_a 2^{t(k_2)} a^2)^2}{\lambda_1 2^{t(k_1)} + \lambda_2 n_a 2^{t(k_2)} a^4} \right) - b_1 2^{t(k_1)} - n_a b_2 2^{t(k_2)}.$$

$$Q_v(D) = \left[\frac{\sum x_{iu}^2}{N} \right]^4 \left[4e - v(\hat{b}_{ij}) \right]^2$$

$$\text{where } e = V(\hat{b}_{ii}) = \frac{b_1 2^{t(k_1)} + n_a b_2 2^{t(k_2)} + n_0}{4[r_1^2 2^{2t(k_1)} + 2r_1 r_2 2^{t(k_1)+t(k_2)} n_a a^2 + r_2^2 2^{2t(k_2)} n_a^2 a^4]} \qquad (\text{since } \lambda_2^2 = \lambda_4)$$

Measure of modified slope rotatability for second order response surface designs using a pair of BIBDs is

$$Q_v(D) = \left[\frac{r_1 2^{t(k_1)} + r_2 n_a 2^{t(k_2)} a^2}{N} \right]^4 \left[4e - \frac{1}{\lambda_1 2^{t(k_1)} + \lambda_2 n_a 2^{t(k_2)} a^4} \right]^2$$

The following Table gives the values of measure of modified slope rotatability for second order response surface designs using various parameters of pair of BIBDs for different level of 'a'. Variances of the estimated responses are also obtained.

Table 1: Values of measure of modified slope rotatability for second order response surface designs using a pair of BIBD.

$D_1=(7,7,3,3,1)$, $D_2=(7,21,6,2,1)$, $N=216$, $a=1.4142$, $n_a=1$, $n_0=76$		
a	$Q_v(D)$	$V(\partial\hat{y}/\partial x_i)$
1.0	2.6461×10^{-7}	$0.0196\sigma^2 + 0.0937 d^2\sigma^2$
*1.4142	0	$0.0139\sigma^2 + 0.0417 d^2\sigma^2$
1.5	1.8651×10^{-10}	$0.0128\sigma^2 + 0.0355 d^2\sigma^2$
2.0	2.1312×10^{-5}	$0.008\sigma^2 + 0.0289 d^2\sigma^2$

2.5	4.2965×10^{-4}	$0.0053\sigma^2+0.0071d^2\sigma^2$
3.0	3.7527×10^{-3}	$0.0037\sigma^2+0.0036d^2\sigma^2$
3.5	0.0192	$0.0028\sigma^2+0.0021d^2\sigma^2$
4.0	0.0718	$0.0021\sigma^2+0.0013d^2\sigma^2$

$D_1=(8,14,7,4,3)$, $D_2=(8,28,7,2,1)$, $N=448, a=2, n_a=1, n_0=112$		
a	$Q_v(D)$	$V(\hat{\partial y}/\partial x_i)$
1.0	7.1978×10^{-8}	$0.0066\sigma^2+0.022d^2\sigma^2$
1.5	1.2796×10^{-11}	$0.0057\sigma^2+0.0146d^2\sigma^2$
*2.0	0	$0.0045\sigma^2+0.0089d^2\sigma^2$
2.5	8.46103×10^{-7}	$0.0033\sigma^2+0.0054d^2\sigma^2$
3.0	2.0933×10^{-7}	$0.0024\sigma^2+0.0034d^2\sigma^2$
3.5	4.1081×10^{-7}	$0.0019\sigma^2+0.0022d^2\sigma^2$
4.0	5.9998×10^{-7}	$0.0014\sigma^2+0.0014d^2\sigma^2$

$D_1=(9,9,8,8,7)$, $D_2=(9,36,8,2,1)$, $N=784, a=3.4641, n_a=1, n_0=64$		
a	$Q_v(D)$	$V(\hat{\partial y}/\partial x_i)$
1.0	4.4234×10^{-8}	$0.0017\sigma^2+0.0026d^2\sigma^2$
1.5	8.1934×10^{-9}	$0.0017\sigma^2+0.0023d^2\sigma^2$
2.0	6.77604×10^{-10}	$0.0011\sigma^2+0.0019d^2\sigma^2$
2.5	7.9964×10^{-9}	$0.0013\sigma^2+0.0015d^2\sigma^2$
3.0	5.3637×10^{-9}	$0.0013\sigma^2+0.0013d^2\sigma^2$
*3.4641	0	$0.0016\sigma^2+0.001d^2\sigma^2$
3.5	1.43603×10^{-9}	$0.0007\sigma^2+0.001d^2\sigma^2$
4.0	1.9886×10^{-9}	$0.0009\sigma^2+0.0007d^2\sigma^2$

$D_1=(10,15,6,4,2)$, $D_2=(10,45,9,2,1)$, $N=588, a=1.4142, n_a=1, n_0=168$		
a	$Q_v(D)$	$V(\hat{\partial y}/\partial x_i)$
1.0	9.0481×10^{-8}	$0.0069\sigma^2+0.0337d^2\sigma^2$
*1.4142	0	$0.006\sigma^2+0.0208d^2\sigma^2$
1.5	1.1253×10^{-9}	$0.0057\sigma^2+0.0188d^2\sigma^2$
2.0	1.2046×10^{-9}	$0.0042\sigma^2+0.0102d^2\sigma^2$
2.5	1.3815×10^{-8}	$0.003\sigma^2+0.0057d^2\sigma^2$
3.0	7.1568×10^{-8}	$0.0022\sigma^2+0.0033d^2\sigma^2$
3.5	1.4554×10^{-7}	$0.0016\sigma^2+0.002d^2\sigma^2$
4.0	2.1513×10^{-7}	$0.0013\sigma^2+0.0013d^2\sigma^2$

$D_1=(12,33,11,4,3)$, $D_2=(12,44,11,3,2)$, $N=1584, a=1.4142, n_a=2, n_0=704$		
a	$Q_v(D)$	$V(\hat{\partial y}/\partial x_i)$
1.0	1.9682×10^{-10}	$0.0028\sigma^2+0.0128d^2\sigma^2$
*1.4142	0	$0.0019\sigma^2+0.0057d^2\sigma^2$
1.5	4.3819×10^{-8}	$0.0017\sigma^2+0.0064d^2\sigma^2$
2.0	1.4232×10^{-7}	$0.0011\sigma^2+0.003d^2\sigma^2$
2.5	1.7259×10^{-8}	$0.0007\sigma^2+0.001d^2\sigma^2$
3.0	2.6789×10^{-8}	$0.0005\sigma^2+0.0005d^2\sigma^2$

3.5	3.40102×10^{-8}	$0.0004\sigma^2+0.0003d^2\sigma^2$
4.0	3.9329×10^{-8}	$0.0003\sigma^2+0.0002d^2\sigma^2$

D_1=(13,13,4,4,1), D_2=(13,26,6,3,1), N=800, a=1, n_a=2, n_0=384		
a	$Q_v(D)$	$V(\partial\hat{y}/\partial x_i)$
*1.0	0	$0.0063\sigma^2+0.0313d^2\sigma^2$
1.5	1.6606×10^{-10}	$0.0036\sigma^2+0.0102d^2\sigma^2$
2.0	9.4204×10^{-9}	$0.0021\sigma^2+0.004d^2\sigma^2$
2.5	3.0721×10^{-8}	$0.0014\sigma^2+0.0018d^2\sigma^2$
3.0	3.0194×10^{-4}	$0.0004\sigma^2+0.0009d^2\sigma^2$
3.5	1.0166×10^{-4}	$0.0007\sigma^2+0.0005d^2\sigma^2$
4.0	7.6861×10^{-8}	$0.0006\sigma^2+0.0003d^2\sigma^2$

D_1=(15,15,7,7,3), D_2=(15,35,7,3,1), N=1792, a=2, n_a=2, n_0=552		
a	$Q_v(D)$	$V(\partial\hat{y}/\partial x_i)$
1.0	7.8384×10^{-9}	$0.0016\sigma^2+0.0057d^2\sigma^2$
1.5	7.9975×10^{-13}	$0.0014\sigma^2+0.0037d^2\sigma^2$
*2.0	0	$0.0011\sigma^2+0.0022d^2\sigma^2$
2.5	3.1036×10^{-9}	$0.0008\sigma^2+0.0014d^2\sigma^2$
3.0	1.3083×10^{-8}	$0.0013\sigma^2+0.0009d^2\sigma^2$
3.5	2.5676×10^{-8}	$0.0005\sigma^2+0.0005d^2\sigma^2$
4.0	3.7499×10^{-8}	$0.0004\sigma^2+0.0004d^2\sigma^2$

D_1=(16,16,10,10,6), D_2=(16,80,15,3,2), N=2800, a=2.8284, n_a=1, n_0=112		
a	$Q_v(D)$	$V(\partial\hat{y}/\partial x_i)$
1.0	1.4642×10^{-9}	$0.0007\sigma^2+0.0014d^2\sigma^2$
1.5	1.4448×10^{-11}	$0.0006\sigma^2+0.0001d^2\sigma^2$
2.0	8.2363×10^{-10}	$0.0006\sigma^2+0.0009d^2\sigma^2$
2.5	4.0765×10^{-10}	$0.0002\sigma^2+0.0007d^2\sigma^2$
*2.82843	0	$0.0006\sigma^2+0.0004d^2\sigma^2$
3.0	1.6778×10^{-10}	$0.0004\sigma^2+0.0005d^2\sigma^2$
3.5	2.7831×10^{-9}	$0.0003\sigma^2+0.0004d^2\sigma^2$
4.0	7.8525×10^{-9}	$0.0002\sigma^2+0.0003d^2\sigma^2$

*indicates exact modified slope rotatability value using a pair of BIBDs

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