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Group action on prw-locally closed sets in topology

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Abstract
The Objective of this paper is to introduce group acting on $prwlc$- sets in topological spaces. This new class of sets lies between group acting on prw-closed sets and prw - open sets. We also obtain some results of group acting on $prwlc$-sets in topological spaces.

Keywords: Locally closed set, pre-closed set, group acting on prw - closed sets, prw - open sets

1. Introduction
Generalized Closed sets have been initiated and studied by N. Levine [5]. The introduction of locally closed sets in general topology was investigated by Bourbaki [3]. Ganster and Reilly [4] further developed the properties of locally closed sets by defining LC-continuity and LC-irresoluteness. In 1996, Balachandran *et al.* [2] introduced and investigated the concepts of generalized locally closed sets and obtained different notions of continuity called GLC-continuity and GLC-irresolute maps. Mikhail Tkačcnko [8] introduced the concepts of topological groups. In this paper, we introduce and studies on group acting on $prwlc$- sets in topological spaces.

The whole paper, Group (G, τ) (or simply G) always means a topological group on which no separation axioms are assumed unless explicitly stated. For a subset M of a group G, $cl(M), pcl(M)$ and $int(M)$ denote the closure of M, pre-closure of M, the interior of M respectively.

2. Preliminaries
We require the following definitions.

Definition 2.1
A topological group is a set G with two structures: (i). G is a group, and (ii) G is a topological space, Such that the two structures are compatible. (i.e)., the multiplication map $\mu: G \times G \longrightarrow G$ and the inversion map $v: G \longrightarrow G$ are both continuous.

Definition 2.2
A group G acting on a subset M is called regular open (briefly r-open) [6] set if $M = int(cl(M))$ and regular closed (briefly r-closed) [6] set if $M = cl(int(M))$.

Definition 2.3
A group G acting on a subset M is called pre-open [16] set if $M \subseteq int(cl(M))$ and pre-closed [16] set if $cl(int(M)) \subseteq M$.

Definition 2.4
A topological group (G, τ) acting on a subset M is said to be generalized pre-regular closed (briefly gpr-closed) [7] set if $pcl(M) \subseteq U$ whenever $M \subseteq U$ and U is regular open in G.

Definition 2.5
A topological group (G, τ) acting on a subset M is said to be regular semi-open (briefly rs-open) [11] set if there is a regular open set U such that $U \subset M \subset Cl(U)$.

Definition 2.6
A topological group (G, τ) acting on a subset M is said to be pre-regular weakly closed (briefly prw-closed) [11] set if $pcl(M) \subseteq U$ whenever $M \subseteq U$ and U is rs-open in G.

Definition 2.7
A subset M acting on (G, τ) is said to be locally closed [4] if $M = I \cap R$, where I is open and R is closed in G.

Definition 2.8
A subset M acting on (G, τ) is said to be Pre-locally closed (pl-closed) [7] if $M = I \cap R$, where I is Pre-open and R is Pre-closed in G.

3. Group action on prw-locally closed sets

Definition 3.1
A subset M of a topological group (G, τ) is said to be Pre-regular weakly locally closed (briefly $prwlc$-set) set if $M = I \cap R$, where I is prw-open and R is prw-closed in G.

Definition 3.2
A topological group (G, τ) acting on a subset M is called $prwlc^*$-set if there exists prw-open set I and a closed set R such that $M = I \cap R$.

Definition 3.3
A topological group (G, τ) acting on a subset M is called $prwlc^{**}$-set if there exists open set I and a prw-closed set R such that $M = I \cap R$.

Theorem 3.4
A subset M of a group G is locally closed set then it is Pre-locally closed in G. The converse is not true.

Proof
Let M be locally closed. Then by the definition it is the intersection of open and closed set. Since all open set (closed set) is Pre-open (Pre-closed). Then M is Pre-locally closed.

Example 3.5
Let $G = \{1, 3, 5, 7\}$ be a group acting on a topology $\tau = \{G, \emptyset, \{1, 3\}\}$. Here $M = \{5, 7\}$ is Pre-locally closed in (G, τ). But it is not locally closed.

Theorem 3.6
A topological group acting on a subset M is $prwlc^*$-set iff $M = B \cap pcl(M)$ for some prw-open set B in G.

Proof
Necessity: Let a subset M is $prwlc^*$-set. Then $M = B \cap R$. That is, it is the intersection of prw-open set B and a closed set R. Since $B \supset M$ and $pcl(M) \supset M$. We have $B \cap pcl(M) \supset M$. Also, since $R \supset pcl(M)$. Then $M = B \cap R \supset B \cap pcl(M)$. Hence
$M = B \cap pcl(M)$.

Sufficiency: Let $M = B \cap pcl(M) = B \cap R$. Since B is prw-open and R is closed. Hence M is $prwlc^*$-set.

Theorem 3.7
A subset M of (G, τ) is gpr- locally closed. Then it is $prwlc$-set.

Proof.
Let $M = C \cap F$. Where C is gpr-open and F is gpr-closed. Since every gpr-closed (pr-open) set is prw-closed (prw-open) set. Hence M is $prwlc$-set.

Theorem 3.8
If a subset M of a topological group (G, τ) is locally closed then it is $prwlc(G, \tau)$, $prwlc^*(G, \tau)$ and $prwlc^{**}(G, \tau)$.

Proof
Let $M = C \cap S$. Where C is open and S is closed in (G, τ). Since all open set is prw-open and all closed set is prw-closed set. Then M is $prwlc(G, \tau)$, $prwlc^*(G, \tau)$ and $prwlc^{**}(G, \tau)$.
The Converse is not true as shown in the following example.

Example 3.9
Let $G = \{1, 3, 5, 7\}$ be a group acting on a topology $\tau = \{\emptyset, G, \{1, 3\}\}$. Then the lc- sets are $\emptyset, G, \{1, 3\}, \{5, 7\}$ and $prwlc(G, \tau) = prwlc^*(G, \tau) = prwlc^{**}(G, \tau) = P(G)$. Here $\{1, 5\}$ is $prwlc(G, \tau)$, $prwlc^*(G, \tau)$ and $prwlc^{**}(G, \tau)$. But not locally closed in (G, τ).

Theorem 3.10
If $M, N \in prwlc(G, \tau)$, then $M \cap N \in prwlc(G, \tau)$.

Proof
Assume that, there exists prw-open sets X and Y such that $M = X \cap pcl(M)$ and $N = Y \cap pcl(N)$. Therefore, $M \cap N = (X \cap Y) \cap (pcl(M) \cap pcl(N))$. Since $X \cap Y$ is prw-open set and $pcl(M) \cap pcl(N)$ is closed. Hence, $M \cap N \in prwlc(G, \tau)$.

4. Conclusion
In this paper, Group action on $prwlc$-sets in topological spaces are clearly defined. In future, the relation of group action on $prwlc$-set properties can be extended to various closed sets.

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