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Infinite triple integral representation for the polynomial set $S_n(x, y)$

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Abstract
In the present paper an attempt has been made to express an Infinite Triple Integral representation for the polynomial set $S_n(x, y)$. Many interesting new results may be obtained as particular cases on separating the parameter. This polynomial Set covers as many as forty-one orthogonal and non-orthogonal polynomials and have been deduced as particular as cases. The newly defined generalized Hypergeometric polynomial Set Sn(X,Y) may be immense use in new phase of Mathematics relvent to Physics, Chemistry, Engineering and Social sciences. These integral representations have been given in the form of theorems together with a number of new and interesting particular cases, which may be useful for scientists and engineers.

Keywords: Appell function generalized hypergeometric polynomial integral representation, orthogonal polynomial

Introduction

We define the generalized hypergeometric polynomial set $S_n(x, y)$ by means of the generating functions,

$$e^{\lambda y t}\, F\left[\begin{matrix}(G_r); & \\ & \lambda_1 y^{e_1} t^{e_1} \\ (H_s); & \end{matrix}\right] \times F\left[\begin{matrix}(a_p);(A_h);(C_u) & \\ & \lambda_3 x^{e_3} t, \lambda_2 x^{e_2} y^{-e_2} t^{e_2} \\ (b_q);(B_k);(D_v) & \end{matrix}\right]$$

$$= \sum_{n=0}^{\infty} S_{n_1;e_1;e_2;e_3;(H_s);(b_q);(B_k);(D_v)}^{\lambda;\lambda_1;\lambda_2;\lambda_3;(G_r);(a_p);(A_h);(C_u)}(x, y)\, t^n \quad \ldots \quad (1.1)$$

Where $\lambda, \lambda_1, \lambda_2, \lambda_3$ are real and e_1, e_2, e_3 are positive integers.
The left hand side of (1.1) contains Appell function of two variables in the notation of Burchnall and Chaundy [1]. The polynomial set contains a number of parameters, for simphilifys, we shall denote.

$$S_{n;e_1;e_2;e_3;(H_s)(b_q);(B_k);(D_v)}^{\lambda;\lambda_1;\lambda_2;\lambda_3;(G_r);(a_p);(A_h);(C_u)}(x, y)$$ by $Sn(x, y)$.

Where n denote the order of the polynomial set.
After little simplification (1.1) gives

$$S_n(x,y)=\sum_{\substack{m,m_1,m_2>0\\ m+e_1m_1+e_2m_2\le 0}}\frac{\Delta(m_1,m_2)}{(n-m-e_1m_1-e_2m_2)!}\quad\ldots \tag{1.2}$$

Where

$$\Delta(m_1,m_2)=\frac{[(a_p)]_{n-m-e_1m_1-(e_2-1)m_2}}{[(b_q)]_{n-m-e_1m_1-(e_2-1)m_2}}\times\frac{[(A_h)]_{n-m-e_1m_1-e_2m_2}[(G_r)]_{m_1}[(C_u)]_{m_2}}{[(B_k)]_{n-m-e_1m_1-e_2m_2}[(H_s)]_{m_1}[(D_v)]_{m_2}}$$

$$\times\frac{x^{m_2e_2}\lambda^m\lambda_1^{m_1}\lambda_2^{m_2}\left(\lambda_3x^{e_3}\right)^{n-m-e_1m_1-e_2m_2}\;y^{m+e_1m_1+e_2m_2}}{m!\quad m_1!\quad m_2!\;\;(n-m-e_1m_1-e_2m_2)!}$$

The polynomial set $S_n(x, y)$ happens to the generalization of as many as forty one orthogonal and non-orthogonal polynomials.

Notations

1. $(m) = 1, 2, 3, \ldots\ldots, m.$
2. $(Ap) = A1, A2, A3, \ldots\ldots Ap.$
3. $[(Ap)] = A1, A2, A3\ldots\ldots Ap.$
4. $[(Ap)]n = (A1)n\,(A2)n, (A3)n\ldots\ldots (Ap)n.$
5. $\Delta(a,b)=\frac{b}{a},\frac{b+1}{a},\ldots\ldots\frac{b+a-1}{a}.$
6. $\Gamma(a\pm b)=\Gamma(a+b)\Gamma(a-b).$
7. $\Gamma_*\Gamma_*(a+b)=\Gamma(a+b)\Gamma(a+b).$
8. $K=\frac{[(a_p)]_n[(A_h)]_n\left(\lambda_3x^{e_3}\right)^n}{[(bq)]_n[(B_k)]_n\,n!}$

Theorem

For $e2>1$

$$S_n(x,y)=\frac{8\Gamma\left(1+\frac{\nu}{2}\pm\mu\right)}{i^n\sqrt{\pi}\,\Gamma\left(\frac{1}{2}\pm\mu\right)\Gamma\left(\frac{1}{2}+\frac{\nu}{2}\right)\Gamma\left(1+\frac{\nu}{2}\right)\int_0^\infty f(t)J_n(t)dt}$$

$$\times\int_0^\infty\int_0^\infty\int_0^\infty\left(x^2+y^2\right)^{\frac{\nu-1}{2}}\left(x^2+y^2+z^2\right)^{\frac{-\nu}{2}}\exp\left[i\left(x^2+y^2+z^2\right)\frac{x^2-y^2}{x^2+y^2}\right]$$

$$\times\cos\left[2\mu\left\{\tan^{-1}\left(\frac{x^2+y^2}{2}\right)\right\}f\left(x^2+y^2+z^2\right)\right]$$

$$\times\sum_{m=0}^{n}\sum_{m_1=0}^{\left[\frac{n-m}{e_1}\right]}\times\frac{[(a_p)]_{n-m-e_1m_1}[(A_h)]_{n-m-e_1m_1}[(G_r)]_{m_1}\lambda^m\lambda_1^{m_1}}{[(b_q)]_{n-m-e_1m_1}[(B_k)]_{n-m-e_1m_1}[(H_s)]_{m_1}\,m!\,m_1!}\times\frac{\left(\lambda_2x^{e_2}\right)^{n-m-e_1m_1}\;y^{m+e_1m_1}}{(n-m-e_1m_1)}$$

$$\times F\left[\begin{matrix}\Delta\left(e_2;-n+m+e_1m_1\right),\Delta\left(e_2-1;1-\left(b_q\right)-n+m+e_1m_1\right),\\ \Delta\left(e_2;1-\left(B_k\right)-n+m+e_1m_1\right),\left(C_u\right),\left(1+\frac{\nu}{2}\pm\mu\right),\sigma;\\ \dfrac{\lambda_2x^{e_2}\left\{-e_2\right\}^{e_2(q-p+h-k+1)}\left\{-\left(e_2-1\right)\right\}^{(e_2-1)(q-p)}}{\left(\lambda_2x^{e_3}y\right)^{e_2}}\\ \Delta\left(e_2-1;1-\left(a_p\right)-n+m+e_1m_1\right),\\ \Delta\left(e_2;1-\left(A_h\right)-n+m+e_1m_1\right),\left(D_v\right),\frac{1}{2}+\frac{\nu}{2},1+\frac{\nu}{2};\end{matrix}\right]dxdydz \quad \ldots \quad (3.1)$$

Proof:

$$I_6=\int_0^\infty\int_0^\infty\int_0^\infty\left(x^2+y^2\right)^{\frac{\nu-1}{2}}\left(x^2+y^2+z^2\right)^{-\frac{\nu}{2}}\exp\left[i\left(x^2+y^2+z^2\right)\frac{x^2-y^2}{x^2+y^2}\right]$$

$$\times\cos\left[2\mu\left\{\tan^{-1}\left(\frac{x^2+y^2}{2}\right)\right\}f\left(x^2+y^2+z^2\right)\right]$$

$$\times\sum_{m=0}^{n}\sum_{m_1=0}^{\left[\frac{n-m}{e_1}\right]}\times\frac{\left[\left(a_p\right)\right]_{n-m-e_1m_1}\left[\left(A_h\right)\right]_{n-m-e_1m_1}\left[\left(G_r\right)\right]_{m_1}\lambda^m\lambda_1^{m_1}}{\left[\left(b_q\right)\right]_{n-m-e_1m_1}\left[\left(B_k\right)\right]_{n-m-e_1m_1}\left[\left(H_s\right)\right]_{m_1}m!\,m_1!}\times\frac{\left(\lambda_2x^{e_2}\right)^{n-m-e_1m_1}y^{m+e_1m_1}}{\left(n-m-e_1m_1\right)!}$$

$$\times F\left[\begin{matrix}\Delta\left(e_2;-n+m+e_1m_1\right),\Delta\left(e_2-1;1-\left(b_q\right)-n+m+e_1m_1\right),\\ \Delta\left(e_2;1-\left(B_k\right)-n+m+e_1m_1\right),\left(C_u\right),\left(1+\frac{\nu}{2}\pm\mu\right),\sigma;\\ \dfrac{\lambda_2x^{e_2}\left\{-e_2\right\}^{e_2(q-p+k-h+1)}\left\{-\left(e_2-1\right)\right\}^{(e_2-1)(q-p)}}{\left(\lambda_2x^{e_3}y\right)^{e_2}}\\ \Delta\left(e_2-1;1-\left(a_p\right)-n+m+e_1m_1\right),\\ \Delta\left(e_2;1-\left(A_h\right)-n+m+e_1m_1\right),\left(D_v\right),\frac{1}{2}+\frac{\nu}{2},1+\frac{\nu}{2};\end{matrix}\right]dxdydz$$

$$=\sum_{m=0}^{n}\sum_{m_1=0}^{\left[\frac{n-m}{e_1}\right]}\sum_{m_2=0}^{\left[\frac{n-m-e_1m_1}{e_2}\right]}\frac{\left[\left(a_p\right)\right]_{n-m-e_1m_1}\left[\left(A_h\right)\right]_{n-m-e_1m_1}}{\left[\left(b_q\right)\right]_{n-m-e_1m_1}\left[\left(B_k\right)\right]_{n-m-e_1m_1}}$$

$$\times\frac{\left[\left(G_r\right)\right]_{m_1}\left(\lambda_2x^{e_2}\right)^{n-m-e_1m_1}y^{m+e_1m_1}\Delta_{m_2}\left[e_2;-n+m+e_1m_1\right]}{\left[\left(H_s\right)\right]_{m_1}\quad m!\quad m_1!\quad\left(n-m-e_1m_1\right)!}$$

$$\times\frac{\Delta_{m_2}\left[e_2-1;1-\left(b_q\right)-n+m+e_1m_1\right]\Delta_{m_2}\left[e_2;1-\left(B_k\right)-n+m+e_1m_1\right]}{\Delta_{m_2}\left[e_2-1;1-\left(a_p\right)-n+m+e_1m_1\right]\Delta_{m_2}\left[e_2;1-\left(A_p\right)-n+m+e_1m_1\right]}$$

$$\times\frac{\left[(C_u)\right]_{m_2}\left(1+\frac{\nu}{2}\pm\mu\right)_{m_2}(\sigma)_{m_2}\left(\lambda_2 x^{e_2}\right)^{m_2}(-e_2)^{e_2(q-p+k-h+1)m_2}\left\{-(e_2-1)\right\}^{(e_2-1)(q-p)m_2}}{\left[(D_v)\right]_{m_2}\left(\frac{1}{2}+\frac{\nu}{2}\right)_{m_2}\left(1+\frac{\nu}{2}\right)_{m_2}\left(\lambda_2 x^{e_3}y\right)^{e_2 m_2}}$$

$$\times\int_0^\infty\int_0^\infty\int_0^\infty\left(x^2+y^2\right)^{\frac{\nu-1+2m_2}{2}}\left(x^2+y^2+z^2\right)^{-\frac{\nu+2m_2}{2}}$$

$$\times\exp\left[i\left(x^2+y^2+z^2\right)\frac{x^2-y^2}{x^2+y^2}\right]\cos\left[2\mu\left\{\tan^{-1}\left(\frac{x^2+y^2}{2}\right)\right\}f\left(x^2+y^2+z^2\right)\right]dxdydz \quad \ldots \quad (3.2)$$

$$=\frac{i^n\sqrt{\pi}\,\Gamma\left(\frac{1}{2}\pm\mu\right)\Gamma\left(\frac{1}{2}+\frac{\nu}{2}\right)\Gamma\left(1+\frac{\nu}{2}\right)}{8\,\Gamma\left(1+\frac{\nu}{2}\pm\mu\right)\int_0^\infty f(t)\,J_n(t)\,dt}S_n(x,y)$$

On using [172] *viz*

$$\int_0^\infty\int_0^\infty\int_0^\infty\left(x^2+y^2\right)^{\frac{\nu-1}{2}}\left(x^2+y^2+z^2\right)^{\frac{\nu}{2}}\exp\left[i\left(x^2+y^2+z^2\right)\frac{x^2-y^2}{x^2+y^2}\right]$$

$$\times\cos\left[2\mu\left\{\tan^{-1}\left(\frac{x^2+y^2}{2}\right)\right\}\right]f\left(x^2+y^2+z^2\right)dxdydz$$

$$=\frac{i^n\sqrt{\pi}\,\Gamma\left(\frac{1}{2}\pm\mu\right)\Gamma\left(\frac{1+\nu}{2}\right)\Gamma\left(1+\frac{\nu}{2}\right)}{8\,\Gamma\left(1+\frac{\nu}{2}\pm\mu\right)}\int_0^\infty f(t)\,J_n(t)\,dt$$

Particular Cases of (3.1)

Separating the term corresponding to $\lambda = 0$ and putting $r = 0 = s = \lambda 1$ in (3.1), we obtaina a number of results on specializing the remaining parameters:

i) On making the substitution p = 0 = $q = h = k = u =$ v; $y = x$; $\lambda 3 = 1 = e3 = e1$; $e2 = 2$, $\lambda 2 = -1$; $x = 3y$ we achieve

$$g_n(x)=\frac{8\Gamma\left(1+\frac{\nu}{2}\pm\mu\right)(2x)^n}{n!\,\sqrt{\pi}\,i^n\,\Gamma\left(\frac{1}{2}\pm\mu\right)\Gamma\left(\frac{1}{2}+\frac{\nu}{2}\right)\Gamma\left(1+\frac{\nu}{2}\right)\int_0^\infty f(t)J_n(t)dt}$$

$$\times\int_0^\infty\int_0^\infty\int_0^\infty\left(x^2+y^2\right)^{\frac{\nu-1}{2}}\left(x^2+y^2+z^2\right)^{\frac{\nu}{2}}\exp\left[i\left(x^2+y^2+z^2\right)\frac{x^2-y^2}{x^2+y^2}\right]$$

$$\times\cos\left[2\mu\left\{\tan^{-1}\left(\frac{x^2+y^2}{2}\right)\right\}f\left(x^2+y^2+z^2\right)\right]$$

$$\times F\left[\begin{matrix}\Delta(2,-n),1+\alpha,\left(1+\frac{\nu}{2}\pm\mu\right),\sigma; \\ \qquad\qquad\qquad\qquad\qquad\qquad \frac{-1}{x^2} \\ 1+\beta,\frac{1}{2}+\frac{\nu}{2},1+\frac{\nu}{2};\end{matrix}\right]dxdydz$$

Where

$h_n^*(y)$ are the Humbert polynomials [2].

ii) On making the substitution $p = 0 = q = h = k = r = s$; $e2 = m$, $e3 = 1 = e1 = e$; $\lambda3 = \delta$; $\lambda2 = \lambda$ and replacing x and y by $x^{\frac{1}{c};\delta}$ we arrive at

$$B_n(x,y)=\frac{8\Gamma\left(1-\frac{\nu}{2}\pm\mu\right)\left[\left(\alpha_p\right)\right]_n(\nu x)^n}{\left[\left(\beta_q\right)\right]_n\, n!\,\sqrt{\pi}\, i^n\,\Gamma\left(\frac{1}{2}\pm\mu\right)\Gamma\left(\frac{1}{2}+\frac{\nu}{2}\right)}$$

$$\times\frac{1}{\Gamma\left(1+\frac{\nu}{2}\right)\int_0^\infty f(t)J_n(t)dt}\int_0^\infty\int_0^\infty\int_0^\infty\left(x^2+y^2\right)^{\frac{\nu-1}{2}}\left(x^2+y^2+z^2\right)^{-\frac{\nu}{2}}$$

$$\times\exp\left[i\left(x^2+y^2+z^2\right)\frac{x^2-y^2}{x^2+y^2}\right]\cos\left[2\mu\left\{\tan^{-1}\left(\frac{x^2+y^2}{2}\right)\right\}f\left(x^2+y^2+z^2\right)\right]$$

$$\times F\left[\begin{matrix}\Delta(m;-n)\Delta\left(m;1-\left(\beta_q\right)-n\right)\left(a_r\right),1+\frac{\nu}{2}\pm\mu,\delta; \\ \qquad\qquad\qquad \frac{\mu(-m)^{m(q-p+1)}}{(\nu xy)^m} \\ \Delta\left(m;1-\left(\alpha_p\right)-n\right),\left(b_s\right),\frac{1}{2}+\frac{\nu}{2},1+\frac{\nu}{2};\end{matrix}\right]dxdydz$$

Where

$Fn(x)$ are the polynomials defined by Shah [3].

iii) On putting $p = 0 = q = h = k = r = s$; $e2 = m$; $e3 = 1 = \lambda3$; $\lambda2 = \beta$; α for x, y we achieve

$$H_{n,m,\nu}(x)=\frac{8\Gamma\left(1-\frac{\nu}{2}\pm\mu\right)(\nu x)^n}{n!\,\sqrt{\pi}\, i^n\,\Gamma\left(\frac{1}{2}\pm\mu\right)\Gamma\left(\frac{1}{2}+\frac{\nu}{2}\right)\Gamma\left(1+\frac{\nu}{2}\right)\int_0^\infty f(t)J_n(t)dt}$$

$$\times\int_0^\infty\int_0^\infty\int_0^\infty\left(x^2+y^2\right)^{\frac{\nu-1}{2}}\left(x^2+y^2+z^2\right)^{-\frac{\nu}{2}}\exp\left[i\left(x^2+y^2+z^2\right)\frac{x^2-y^2}{x^2+y^2}\right]\times\cos\left[2\mu\left\{\tan^{-1}\left(\frac{x^2+y^2}{2}\right)\right\}f\left(x^2+y^2+z^2\right)\right]$$

$$\times F\left[\begin{matrix}\Delta(m;-n),1+\frac{\nu}{2}\pm\mu,\sigma; \\ -\left(\frac{-m}{\nu x}\right)^{m} \\ \frac{1}{2}+\frac{\nu}{2},1+\frac{\nu}{2};\end{matrix}\right]dxdydz$$

Where
Hn, m (α, α) are the generalized polynomials defined by Gupta and Jain [4].

iv) Putting $p = 0 = q = h = k = u = v = r = s$; $e3 = 1 = e1$; $\lambda 3 = \nu$, $\lambda 2 = -1$, we get

$$B_{n,\nu;(\alpha_p)}^{c;\mu;m;\beta_q}(x)=\frac{8\Gamma\left(1+\frac{\nu}{2}\pm\mu\right)\left[(\alpha_p)\right]_n(\nu x)^n}{\left[(\beta_q)\right]_n\ n!\ \sqrt{\pi}\ i^n\ \Gamma\left(\frac{1}{2}\pm\mu\right)\Gamma\left(\frac{1}{2}+\frac{\nu}{2}\right)\Gamma\left(1+\frac{\nu}{2}\right)}$$

$$\times\frac{1}{\int_0^\infty f(t)J_n(t)dt}\int_0^\infty\int_0^\infty\int_0^\infty\left(x^2+y^2\right)^{\frac{\nu-1}{2}}\left(x^2+y^2+z^2\right)^{-\frac{\nu}{2}}\exp\left[i\left(x^2+y^2+z^2\right)\frac{x^2-y^2}{x^2+y^2}\right]$$

$$\times\cos\left[2\mu\left\{\tan^{-1}\left(\frac{x^2+y^2}{2}\right)\right\}\right]f\left(x^2+y^2+z^2\right)$$

$$\times F\left[\begin{matrix}\Delta(m;-n),\Delta\left(m;1-(\beta_q)-n\right),1+\frac{\nu}{2}\pm\mu,\sigma; \\ \frac{\mu(-m)^{m(q-p+1)}}{(\nu x)^m} \\ \Delta\left(n;1-(\alpha_p)-n\right),\frac{1}{2}+\frac{\nu}{2},1+\frac{\nu}{2};\end{matrix}\right]dxdydz$$

Where *Hn, m, v* (*x*) are the generalized polynomials defined by Lahiri [5].

v) If we set $p = 0 = q = u = v = r = s$; $e2 = m$, $e3 = 1 = e1$; $y = x$, $\lambda 2 = \mu$, $\lambda 3 = v$, (βq) instead of Bk and (αp) instead of (Ap) then we achieve

$$G_n(\alpha,\beta;x)=\frac{8\Gamma\left(1+\frac{\nu}{2}\pm\mu\right)(\alpha)_n(\beta)_n(\nu x)^n}{(a+\beta)_n\ n!\ \sqrt{\pi}\ i^n\ \Gamma\left(\frac{1}{2}\pm\mu\right)\Gamma\left(\frac{1}{2}+\frac{\nu}{2}\right)\Gamma\left(1+\frac{\nu}{2}\right)\int_0^\infty f(t)J_n(t)dt}\times\int_0^\infty\int_0^\infty\int_0^\infty\left(x^2+y^2\right)^{\frac{\nu-1}{2}}\left(x^2+y^2+z^2\right)^{-\frac{\nu}{2}}$$

$$\times F\left[\begin{matrix}\Delta(2;-n),1-\alpha-\beta-n,1+\frac{\nu}{2}\pm\mu,\sigma; \\ \qquad\qquad\qquad\qquad\qquad\qquad\frac{1}{x^2} \\ 1-\alpha-n,1-\beta-n,\frac{1}{2}+\frac{\nu}{2},1+\frac{\nu}{2};\end{matrix}\right]dxdydz$$

$$\times\exp\left[i\left(x^2+y^2+z^2\right)\frac{x^2-y^2}{x^2+y^2}\right]\cos\left[2\mu\left\{\tan^{-1}\left(\frac{x^2+y^2}{2}\right)\right\}\right]f\left(x^2+y^2+z^2\right)$$

Where

$$B_{n,v;(\alpha_p)}^{c,\mu;m;(\beta_q)}(x)$$

Are the generalized polynomials defined by Bhargava [6].

vi) On making the substitution $q = 0 = r = s = u = \text{v}$; $e3 = 1 = e1 = p = h = k$; $e2 = 2 = \lambda 3$; $\lambda 2 = -1$, $a1 = \alpha$, $A1 = \beta$, $B1 = \alpha + \beta$, we obtain

$$R_{n,\nu}\left(\frac{1}{z}\right)=\frac{8\Gamma\left(1+\frac{\nu}{2}\pm\mu\right)(\nu)_n(2z)^n}{n!\,\sqrt{\pi}\,i^n\,\Gamma\left(\frac{1}{2}\pm\mu\right)\Gamma\left(\frac{1}{2}+\frac{\nu}{2}\right)\Gamma\left(1+\frac{\nu}{2}\right)\int_0^\infty f(t)J_n(t)dt}\times\int_0^\infty\int_0^\infty\int_0^\infty\left(x^2+y^2\right)^{\frac{\nu-1}{2}}\left(x^2+y^2+z^2\right)^{-\frac{\nu}{2}}\exp\left[i\left(x^2+y^2+z^2\right)\frac{x^2-y^2}{x^2+y^2}\right]$$

$$\times\cos\left[2\mu\left\{\tan^{-1}\left(\frac{x^2+y^2}{2}\right)\right\}\right]f\left(x^2+y^2+z^2\right)$$

$$\times F\left[\begin{matrix}\Delta(2;-n),1+\frac{\nu}{2}\pm\mu,\sigma; \\ \qquad\qquad\qquad\qquad-\frac{1}{z^2} \\ -n,1-\nu-n,\nu,\frac{1}{2}+\frac{\nu}{2},1+\frac{\nu}{2};\end{matrix}\right]dxdydz$$

Where $Gn(\alpha, \beta; x)$ are the generalized Badient polynomials [7].

(vii) If we take $q = 0 = k = u = r = s$; $p = 1 = h = \text{v} = e3$; $e2 = 2 = \lambda 3$; $\lambda 2= -1$; $a1 = 1$, $A1 = v$, $D1 = \nu$ and z for x and y we achieve

$$h_n(x)=\frac{8\Gamma\left(1-\frac{\nu}{2}\pm\mu\right)(\nu)_n(3\nu)^n}{n!\,\sqrt{\pi}\,i^n\,\Gamma\left(\frac{1}{2}\pm\mu\right)\Gamma\left(\frac{1}{2}+\frac{\nu}{2}\right)\Gamma\left(1+\frac{\nu}{2}\right)\int_0^\infty f(t)J_n(t)dt}\times\int_0^\infty\int_0^\infty\int_0^\infty\left(x^2+y^2\right)^{\frac{\nu-1}{2}}\left(x^2+y^2+z^2\right)^{-\frac{\nu}{2}}\exp\left[i\left(x^2+y^2+z^2\right)\frac{x^2-y^2}{x^2+y^2}\right]$$

$$\times\cos\left[2\mu\left\{\tan^{-1}\left(\frac{x^2+y^2}{2}\right)\right\}\right]f\left(x^2+y^2+z^2\right)$$

$$\times F\left[\begin{matrix}\Delta(3;-n),1+\frac{\nu}{2}\pm\mu,\sigma; \\ \qquad\qquad \frac{1}{4x^3} \\ \Delta(2;1-\nu-n),\frac{1}{2}+\frac{\nu}{2},1+\frac{\nu}{2};\end{matrix}\right] dxdydz$$

Where

$$R_{n,\nu}\left(\frac{1}{z}\right)$$

Are the Lommel polynomials [8; P. 112 (5)].

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