International Journal of Statistics and Applied Mathematics 2020; 5(6): 38-42

International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
Maths 2020; 5(6): 38-42

www.mathsjournal.com
Received: 14-09-2020
Accepted: 16-10-2020

MA Alargt
Department of Statistics,
Faculty of Science, Al-Asmaria
Islamic University, Libya

AM Ben Aros
Department of Statistics,
Faculty of Science,
Elmergib University, Libya

Corresponding Author:
MA Alargt
Department of Statistics,
Faculty of Science, Al-Asmaria
Islamic University, Libya

Asymptotic properties of the finite fourier transform for a filtered series

MA Alargt and AM Ben Aros

Abstract
This paper presents a new approach to construct the finite Fourier transform under the assumption the Series a strictly stationary and has been filtered. The modified Series is defined and is used to study finite Fourier transform and its statistical properties.

Keywords: Finite fourier transform, continuous time series, cumulants, asymptotic normality, filtered and data window

1. Introduction

In the analysis of time series there is often occasion to apply some manipulatory operation. Specifically, consider an operation whose domain consists of r vector- valued series $Y(t), t \in R$ and whose range consists of s vector-valued series $Z(t), t \in R$ then $s \times r$ linear filters as takes the form

$$Z(t) = \mu + \sum_{u=-\infty}^{\infty} a(t-u)Y(t)$$
$$= \mu + \sum_{u=-\infty}^{\infty} a(u)Y(t-u) \qquad 1.1$$

Where $a(u), u \in R$ is a sequence of $s \times r$ matrices satisfying

$$\sum_{u=-\infty}^{\infty} |a(u)| < \infty \qquad 1.2$$

Such a filter an $s \times r$ summable filter and denoted it by $\{a(u)\}$. The transfer function of the filter 1.1 is seen to by given by

$$A(\lambda) = \sum_{u=-\infty}^{\infty} a(u) \exp\{-i\lambda t\} \qquad for \quad -\infty < \lambda < \infty. \qquad 1.3$$

And if $f_{YY}(\lambda), f_{ZZ}(\lambda)$ signify the $r \times r$ and $s \times s$ matrices of second-order spectra of $Y(t)$ and $Z(t)$, respectively. Then

$$f_{ZZ}(\lambda) = A(\lambda) f_{YY}(\lambda) \overline{A(\lambda)'} \qquad 1.4$$

Now we consider the problem of determining an s vector μ, and an $s \times s$ filter $\{a(u)\}$ such that the value

$$\mu + \sum_{u=-\infty}^{\infty} a(t-u)Y(u) \qquad 1.5$$

is near the value $Z(t)$. Suppose we measure closeness by the $s \times s$ Hermitian matrix

$$E\left\{[Z(t) - \underline{\mu} - \sum_{u=-\infty}^{\infty} a(t-u)Y(u)][Z(t) - \underline{\mu} - \sum_{u=-\infty}^{\infty} a(t-u)Y(u)]^T\right\} \qquad 1.6$$

Then the μ and $a(u)$ minimizing 1.6 are given by

$$\underline{\mu} = C_Z - \left(\sum_{u=-\infty}^{\infty} a(u)\right) C_Y = C_Z - A(0)C_Y, \qquad 1.7$$

$$a(u) = (2\pi)^{-1}\int_0^{2\pi} A(\alpha)\, Exp\{iu\alpha\}\, d\alpha, \qquad 1.8$$

where,

$$A(\lambda) = f_{ZY}(\lambda)\, f_{YY}(\lambda)^{-1}, \qquad 1.9$$

and $A(\lambda)$ is the transfer function of the $s \times r$ filter achieving the indicated minimum $A(\lambda)$ called the complex regression coefficient of $Z(t)$ on $Y(t)$ at frequency λ. Now we study the asymptotic properties of expanded finite Fourier transform under the relation $Z(t) = \mu + \sum_{u=-\infty}^{\infty} a(t-u)Y(u)$, the expanded finite Fourier transform discussed in Brillinger (1981), Ghazal *et al.* (1997), Farag (2000), and Ghazal, El-hassanien (2007), studied the asymptotic properties of the finite Fourier transform in the case where all observations are available, Ghazal, Mokaddis and El-Desokey (2010), studied the asymptotic properties of the finite Fourier transform in the case where there are some randomly missing observations for one vector valued, We will study the continuous expanded finite Fourier transform under the assumption the Series has been Filtered. The paper is organized as follows : In Section(1) Introduction, Section (2) we will study the Asymptotic properties of the modified series, Section (3) will be considered the expanded finite Fourier transform under the assumption the Series has been Filtered.

2. The modified Series

Let $Y(t), r$ vector-valued strictly stationary. We consider there are linear relation between $Y(t)$ and $Z(t)$, then we may define the modified series

$$Z(t) = \mu + \sum_{u=-\infty}^{\infty} a(t-u)Y(t) \qquad 2.1$$

$$Z(t) = \mu + A(\lambda)Y(t) \qquad 2.2$$

With components

$$Z_a(t) = \mu + A(\lambda)Y(t) \qquad 2.3$$

Where $A(\lambda)$ given by 1.9.
Suppose that

$$E\left\{ [Y(t+u) - C_Y][Z(t) - C_Z]' \right\} = C_{YZ}(u) = \int_{-\infty}^{\infty} f_{YZ}(\lambda) \exp(i\lambda u) du \qquad 2.4$$

And we defined the second spectral density by

$$f_{YZ}(\lambda) = (2\pi)^{-1} \int_{-\infty}^{\infty} C_{YZ}(u) \exp\{-i\lambda u\} du \quad , for \lambda \in R \qquad 2.5$$

3. Asymptotic Properties for Expanded Finite Fourier Transform

Assumption I.

Let $Y(t)$ is a strictly stationary continuous time series all of whose moments exist. For each $j=1, \ldots, k-1$ and k - tuple $a_1, \ldots, a_k$ we have

$$\int_{R^{k-1}} |t_j| \left| C_{a_1}, \ldots, {}_{a_{k-1}}(t_1, \ldots, t_{k-1}) dt_1, \ldots, dt_{k-1} \right| < \infty, \quad k = 2, 3, \ldots \tag{3.1}$$

We may define its cumulant spectral density by

$$f_{a_1, ,a_k}(\lambda_1, \ldots, \lambda_k) = (2\pi)^{k+1} \int_{R_{k-1}}^{T} C_{a_1, ,a_k}(t_1, \ldots, t_{k-1}) \times$$
$$\times \exp\left[-i \sum_{j=1}^{k-1} \lambda_j t_j \right] dt_1, \ , dt_{k-1,} \quad \lambda \in R, a_j = \overline{1, r}, k = 2, \ ,3, \ \ldots \tag{3.2}$$

Assumption II

Let $d_a^{(T)}(t) = d_a\left(\frac{t}{T}\right), t \in [0, T]$ is bounded, is of bounded variation and vanishes for all t outside the interval $[0, T]$, that is called data window.

We define the continuous expanded finite Fourier transform as

$$h_a^{(T)}(\lambda) = \frac{1}{\sqrt{2\pi \int_0^T \left[d_a^{(T)}(t) \right]^2}} \int_0^T d_a^{(T)}(\lambda) Z(t) \exp(-i\lambda t) dt \tag{3.3}$$

Where

$$Z(t) = A(\lambda) Y(t) \tag{3.4}$$

Theorem 3.1

Let $Z(t)$ be defined as 2.3. Then

$$E[Z(t)] = 0 \tag{3.5}$$
$$Cov[Z_{a_1}(t_1), Z_{a_2}(t_2)] = A(\lambda) C_{a_1 a_2}(u) A(\lambda)' \tag{3.6}$$
$$Var(Z_a(t)) = C_{aa}(0) \tag{3.7}$$

Proof

Since $Y(t)$ and $Z(t)$ are strictly stationary then 3.5 is obtained.

$$Cov\{Z_{a_1}(t_1), Z_{a_2}(t_2)\} =$$

$$= Cov\{\mu + A(\lambda) Y_{a_1}(t_1), \mu + A(\lambda) Y_{a_2}(t_2)\}$$
$$= A(\lambda) Cov\{Y_{a_1}(t_1), Y_{a_2}(t_2)\} A(\lambda)'$$
$$= A(\lambda) C_{a_1 a_2}(t_1 - t_2) A(\lambda)'$$

put $t_1 - t_2 = u$

$$Cov\{Z_{a_1}(t_1), Z_{a_2}(t_2)\} = A(\lambda) C_{a_1 a_2}(u) A(\lambda)'$$

Now,

$$Var\left[Z_a\right]=Cov\left\{Z_a(t),Z_a(t)\right\}$$
$$=Cov\left\{\mu+A(\lambda)Y_a(t),\mu+A(\lambda)Y_a(t)\right\}$$
$$=A(\lambda)Cov\left[y_a(t),Y_a(t)\right]A(\lambda)'$$
$$=A(\lambda)Cov\left[t-t\right]A(\lambda)'$$
$$=A(\lambda)C_{aa}(0)A(\lambda)'$$

which complete the proof.

Theorem 3.2

Let $Y(t)(t=0,\ \pm1,\ \ldots)$ be a strictly stationary r - vector valued time series with mean Zero, and satisfy Assumption I. Let $Z(t)$ be defined as 2.2 and $h_a^{(t)}(\lambda)$ be defined as 3.3, then

$$E\left[h_a^{(t)}(\lambda)\right]=0 \tag{3.8}$$

$$Cov\left[h_{a_1}(\lambda_1),h_{a_2}^{(t)}(\lambda_2)\right]=$$
$$=\int_{-T}^{T}A(\lambda)C_{a_1a_2}(u)A(\lambda)'\exp\left(-i\lambda_1u\right)U_{a_1a_2}^{(T)}(u,\lambda_1-\lambda_2)\,du_1 \tag{3.9}$$

$$=\int_R A(\lambda)\,f_{a_1a_2}(v)\,A(\lambda)'\,\Phi_{a_1a_2}^{(T)}(\lambda_1-v,\lambda_2-v)\,dv \tag{3.10}$$

where

$$U_{a_1a_2}^{(T)}(x)=\int_0^T h_a^{(T)}(t)\exp(-i\lambda t)\,dt \tag{3.11}$$

$$\Phi_{a_1a_2}^{(T)}(x,y)=(2\pi)^{-1}\left[\int_0^T\int_0^T(h_{a_1}^{(T)}(t_1)^2(h_{a_2}^{(T)}(t_2)^2\right]^{-\frac{1}{2}}U_{a_1}^{(T)}(x)U_{a_2}^{(T)}(y) \tag{3.12}$$

Proof

Since $Y(t)$ and $Z(t)$ are strictly stationary then 3.8 is obtained.

$$Cov\left\{h_{a_1}^{(T)}(\lambda_1),h_{a_2}^{(T)}(\lambda_2)\right\}=(2\pi)^{-1}\left[U_{a_1a_2}^{(T)}(0)\right]^{-1}\times$$
$$\times\int_0^T\int_0^T\exp\left(-i(\lambda_1t_1-\lambda_2t_2\right)d_{a_1}^{(T)}(t_1)\,d_{a_2}^{(T)}(t_2)\,Cov\left[Z_{a_1}(t_1),Z_{a_2}(t_2)\right] \tag{3.13}$$

Setting $t_1-t_2=u,t_2=t$ into 3.13 we get,

$$Cov\left\{h_{a_1}^{(T)}(\lambda_1),h_{a_2}^{(T)}(\lambda_2)\right\}=(2\pi)^{-1}\left[U_{a_1a_2}^{(T)}(0)\right]^{-1}\times$$
$$\times\int_{-T}^{T}A(\lambda)C_{a_1a_2}(u)A(\lambda)'\exp\left(-i\lambda u\right)du\times$$
$$\times\int_{-T}^{T}d_{a_1}^{(T)}(t_1+u)\,d_{a_2}^{(T)}(t)\exp\left(-it\lambda(\lambda_1-\lambda_2)\right)dt_1dt_2$$

which satisfy equation 3.6. Substituting 2.4 into 3.9 we get,

$$Cov\left[h_{a_1}^{(T)}(\lambda_1), h_{a_2}^{(T)}(\lambda_2)\right] = \int_R A(\lambda)\, f_{a_1a_2}(v)\,\Phi_{a_1a_2}^{(T)}(\lambda_1 - v, \lambda_2 - v)\,dv$$

where $\Phi_{a_1a_2}^{(T)}$ is given by 3.12.
Hence the proof is complete.
For $\lambda_1 = \lambda_2$ and $\lambda - v = \gamma$. Then

$$Var\left[h_{a_1}^{(T)}(\lambda)\right] = \int_{-\infty}^{\infty} f_{aa}(\lambda - \gamma)\,\Phi_{aa}^{(T)}(\gamma)\,d\gamma$$
.

Theorem 3.3

If the spectral density function $f_{aa}(X), a = \overline{1,r}, X \in R$ is bounded and continuous at a point $X = \lambda, \lambda \in R$ and the function $\Phi_{aa}^{(T)}(x)$ be defined as 3.12, then

$$\lim_{T\to\infty} Cov\left[h_{a_1}^{(T)}(\lambda_1), h_{a_2}^{(T)}(\lambda_2)\right] = 0,\ for\,all\ \{a_1, a_2 = \overline{1,r}\} \quad 3.14$$

$$\lim_{T\to\infty} Dh_{a_1}^{(T)}(\lambda) = A(\lambda)\, f_{aa}(\lambda)\, A(\lambda)',\ a = \overline{1,r} \quad 3.15$$

Conclusion

The continuous expanded finite Fourier transform are considered under the Assumption the Series has been filtered, the asymptotic properties are studied.

References

1. Brillinger DR. Asymptotic properties of spectral estimate of second order, Biometrika 1969;56(2):375-390.
2. Brillinger DR. Time series data analysis and theory; SIAM 2001.
3. Ghazal MA. Statistical Analysis of broanded periodogram for continuous time stationary processes; Journal of applied Mathematics and Computation 2001;24:343-349.
4. Ghazal MA. On the average spectral of continuous time process construction and estimation; Journal of Combinatorics. Information& Statistics 2005;29(1-4):81-91.
5. Ghazal MA, Elhassanien A. Spectral analysis with randomly missed observations; The 40th Annual Conference on Statistics, Computer Sciences and Operation Research 2005, 149-165.
6. Ghazal MA, Elhassanien A. Periodogram analysis with missing observations; journal of Applied Mathematics& Computing 2006;22(1):209-222.
7. Ghazal MA, Elhassanien A. Nonparametric spectral analysis of continuous time series; Bulletin of Statistics& Economics 2007;1(S07):41-53.
8. Ghazal MA, Farag EA. Estimating the spectral density autocovariance function and spectral measure of continuous stationary process; Journal of the Egyption Mathematical Society 2002;10(1):41-58.
9. Ghazal MA, Farag EA, El-desoky EA. Some properties of the Discrete expanded finite Fourier transform with missed observations; Ain Shames University, Faculty of Engerineering 2005;40(3).
10. Ghazal MA, Mokaddis GS, Farag EA, El-desoky EA. Asymptotic properties of spectral estimates of second –order with missed observations; International Journal of Applied Mathematics.
11. Ghazal MA, El-desoky EA, Alargt MA. Asymptotic Properties of the discrete stability time series with missed observations between tow-vector valued stochastic process, Int. R. J. of Engineering and Technology (IRJET) 2017;4(4).
12. Ghazal MA, El-desoky EA, Ben Aros AM. Statistical Analysis of Linear Stability Continuous Time Series Between Two - Vector Valued Stochastic Process, Int. J of Scientific Engineering Research 2017;8(4):2229-5518.
13. Ghazal MA, El-desoky EA, Ben Aros AM. Periodogram Analysis with Missed Observation between Two – Vectors valued Stochastic Process, Int. J. Of Advanced Research 2017;5(11):2320-5407.
14. Ghazal MA, El-desoky EA, Alargt MA. Statistical Properties of the Periodogram for tow-vector valued Stability Series with Missed Observations, Int. J. of Statistics and Applied Mathematics 2017;4(4).