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Combined multiple forecasting model using regression

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Abstract
Accuracy in the forecasting techniques is most challenging in many areas of research. In order to encounter the inaccuracy of the forecast of a single model, weight combination forecasting models are increased accuracies in forecasting models. In the present paper, an attempt has been made by proposing some forecasting models with the estimation of their parameters besides the choice of weights in the combination of forecasts in the forecasting model. A method of multiple combining forecasts has been proposed by choosing optimum values for the weights for the forecasts and combined forecasting technique based on regression method has been developed.

Keywords: Forecast, weights, combination

1. Introduction

The conventional approach to forecasting involves choosing the forecasting method evaluated most suitable of the available methods and applying it to some precise situations. The preferences of a method depend upon the attributes of the time series and the type of application. The selection to the conventional approach is to collective information from diverse forecasting techniques by collective forecasts. This avoids the problem of having to choose a single method and rely completely on its forecasts. There are lot of studies have been proposed on combination of forecasts has with considerable success. Different techniques for combining forecasts with respect to weighted averages were proposed by Bates and Granger (1969) [1], Nelson (1972), New bold and Granger (1974 and 1977) [5], Bhattacharyya (1980) [2], Winkler (1982) [15], Urich (1983), Makridakis and Figlewski (1983) [8], Mahmoud (1984) [5], Zarnowitz (1984) [16]. In the combination it is assumed that the weights are stable over time. The weights are unstable in practice.

The fundamental techniques for different forecasts are known, we can get either combined forecast or a composite forecast. According to the classification, outcomes of wide range of forecasts are associated to arrive at a combined forecast, and the techniques itself are combined to come up with a composite model from which a composite forecast is acquired. The potentials of the composite forecasts are suggested by some discussants of Bates and Granger's article (1969) [1].

2. Method of combining two forecasts

Suppose there are two models A & B and $F_t^{(A)}, F_t^{(B)}$ be the Forecast for the period 't' from the two forecasting models, then the combined forecast for the time period 't' can be expressed as

$$F_t^{(c)} = w_A F_t^{(A)} + w_B F_t^{(B)} \qquad \rightarrow 2.1$$

Where 'w' is the combing parameter (or) weights that lies between 0 and 1 and varied at each time period.

Let the variance of $F_t^{(A)} \,\&\, F_t^{(B)}$ be $\sigma_A^2 \,\&\, \sigma_B^2$ which are known the combined variance σ_C^2 can be expressed as

$$\sigma_C^2 = w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 \qquad \rightarrow 2.2$$

$$\left(Since\ w_B^2 = 1 - w_A^2 \right)$$

If we assume $F_t^{(A)}$ and $F_t^{(B)}$ are independent we obtain the value of 'w' by minimizing σ_c^2 with respect to w.

The first order condition for minimization gives

$$\frac{\partial^2\left(\sigma_c^2\right)}{\partial w} = 0 \rightarrow 2w_A\sigma_A^2 - 2w_B\sigma_B^2 = 0$$

$\rightarrow 2.3$

$$\therefore\ w^* = \left[\frac{\sigma_B^2}{\sigma_A^2 + \sigma_B^2}\right] \rightarrow 2.4$$

By subtitling w* in 2.1 we get

$$F_t^{(c)} = \left[\frac{\sigma_B^2}{\sigma_A^2 + \sigma_B^2}\right]F_t^{(A)} + \left[1 - \left(\frac{\sigma_B^2}{\sigma_A^2 + \sigma_B^2}\right)\right]F_t^{(B)}$$

$$\therefore F_t^{(c)} = \frac{\sigma_B^2 F_t^{(A)} + \sigma_A^2 F_t^{(B)}}{\sigma_A^2 + \sigma_B^2} - 2.5$$

2.1 Observations

1. If the variance are same the w = ½ then $\sigma_c^2 = \frac{\sigma_A^2 + \sigma_B^2}{4}$
2. If $\sigma_A^2 = \sigma_B^2 = \sigma_C^2$ (say), then $\sigma^2 = \frac{\sigma^2}{2}$

3. Combined multiple forecast

Consider $F_t^{(1)}, F_t^{(2)}.......F_t^{(m)}$ be the forecast for the time period 't' from the 'm' forecasting methods respectively. Then the combined multiple forecast for the time period 't' which is a linear combination can be defined as

$$F_t^{(c)} = w_1F_t^{(1)} + w_2F_t^{(2)} + ... + w_mF_t^{(m)}$$

$$= \sum_{i=1}^{m} wi\ F_t^{(i)} \rightarrow 3.1$$

Where the weights are non-negative and $w_1 + w_2 + ... + w_m = 1$

$$\sum_{i=1}^{m} wi = 1 \rightarrow 3.2$$

Let X_t be the actual data from the time period 't' the forecast error or residual for the time period 't' is defined as

$$e_t = \left[x_t - F_t\right] \rightarrow 3.3$$

For the i^{th} forecasting method, the mean percentage forecast error (MPFE) is

$$\text{MPFE} = e_t^{-(i)} = \left[\sum_{i=1}^{m}\frac{X_t^{(i)} - F_t^{i}}{X_t^{(i)}} / n\right] \times 100 \rightarrow 3.4$$

Similarly the covariance matrix of the MPFE from the 'm' forecasting method is

$$\Phi = \left(\phi_{ij}\right) \rightarrow 3.5$$

$$\phi_i = Cov\left(e_t^{-(i)}, e_t^{-(j)}\right) \forall i \neq j = 1, 2, ...m$$

Let $\Phi^{-1} = \left(\left(a_{ij}\right)\right)_{m-m}$

If the elements of covariance matrix Φ are known then the optimal choice weights are given by

$$w_t = \frac{\sum_{p=1}^{m} a_{ip}}{\sum_{h=1}^{m}\sum_{p=1}^{m} a_{hp}} \rightarrow 3.6$$

When the elements of covariance matrix Φ are unknown, and if the forecast from 1 to m are unbiased, then the properties of MPFE's are same to the OLS residuals in the regression analysis. By using maximum likelihood method, the elements of Φ are given by

$$\hat{\phi}_j = \frac{\sum_{t=1}^{n} e_t^{-(i)} e_t^{(i)}}{n}, \forall\ i \neq j = 1, 2, ...m \rightarrow 3.7$$

$$\phi_j = \frac{\sum_{t=1}^{n} e_t^{(i)^2}}{n}, \forall\ i \neq j = 1, 2, ...m \rightarrow 3.8$$

Now the estimates of weights are given by

$$\hat{w}t = \frac{\sum_{p=1}^{m} \hat{a}ip}{\sum_{n=1}^{m}\sum_{p=1}^{m} \hat{a}_{np}} \rightarrow 3.9$$

Here $\Phi^{-1} = \left(\left(\hat{a}ij\right)\right)_{m,n}$ $\rightarrow 3.10$

4. Combined multiple forecasting techniques by using regression method

Consider $F_t^{(1)}, F_t^{(2)}......F_t^{(m)}$ be the forecast for the time period 't' from the 'm' different forecasting techniques respectively. Let $w_1, w_2..........w_n$ be the corresponding weights to be assigned forecasts

$\therefore$ The combined multiple forecasting techniques

$$F_t^{(c)} = w_0 + w_1\ F_t^{(1)} + w_2\ F_t^{(2)} + ... + w_m\ F_t^{(m)} \rightarrow 4.1$$

Minimization of the combined multiple forecasts MSFE is equivalent to the minimization of $E\left\{\in t^2\right\}$ in the following multiple linear regression models:

$$X_t = w_0 + w_1\ F_t^{(1)} + w_2 F_t^{(2)} + ... + w_m F_t^{(m)} + \in t\ ;\ t = 1, 2\ ....n \rightarrow 4.2$$

Here X_t is the actual data for the time period 't'. According to deviation from, the model (4.2) becomes

$$x_t = w_1 f_t^{(1)} + w_2 f_t^{(2)} + ... + w_m f_t^{(m)} + \in t\ ; t = 1, 2.....n \rightarrow 4.3$$

Here $x_t = \left[X_t - \overline{X}\right],\ f_t^{(i)} = \left[F_t^{(i)} - \overline{F}_t^{(i)}\right] \forall\ i = 1, 2, ...m$

In matrix notation (4.3) can be written as

$$X_{n\times1} = F_{n\times m}\ w_{m\times1} + \in_{n\times1} \rightarrow 4.4$$

Here $X = \begin{bmatrix} x_1 \\ x_2 \\ : \\ x_n \end{bmatrix}_{n\times 1}$, $F = \begin{bmatrix} f_1^{(1)}, & f_1^{(2)}\ldots\ldots & f_1^{(m)} \\ f_2^{(1)} & f_2^{(2)}\ldots\ldots & f_2^{(m)} \\ : & & \\ f_n^{(1)} & f_n^{(2)}\ldots\ldots & f_n^{(m)} \end{bmatrix}_{n\times m}$

$w = \begin{bmatrix} w_1 \\ w_2 \\ : \\ w_m \end{bmatrix}_{m\times 1}$, $\in = \begin{bmatrix} \in_1 \\ \in_2 \\ : \\ \in_n \end{bmatrix}_{n\times 1}$

The OLS estimator of 'w' is given by

$$\hat{w} = \left(F^1 F\right)^{-1} F^1 X \qquad \rightarrow 4.5$$

Also the OLS estimator of w_0 is given by

$$\hat{w}_0 = \overline{X} - \hat{w}_1 \overline{F}^{(1)} - \hat{w}_2 \overline{F}^{(2)} - \ldots\ldots\ldots\ldots\ldots - \hat{w}_m \overline{F}^{(m)} \qquad \rightarrow 4.6$$

It is clear that with respect to the combined multiple forecast given in 4.1, the sum of weights are not restricted one and it allows for the possibility of biased forecasts.

5. Observations

1. By applying the restricted least square method, we can obtain the estimates for the weights.
2. If we define weights by OLS residuals, the other different residuals like BLUES, studentized and recursive residuals can be used in the place of OLS residuals.

6. Conclusion

In this paper we have investigated the accuracy of combined forecasts consisting of weighted averages of forecasts from individual methods. These procedures relate the weights to reciprocals of sums of squared errors as opposed to basing the weights directly on an estimated covariance matrix of forecast errors. In the combination of forecasts the weights in the weighted averaging should remain stable over time.

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