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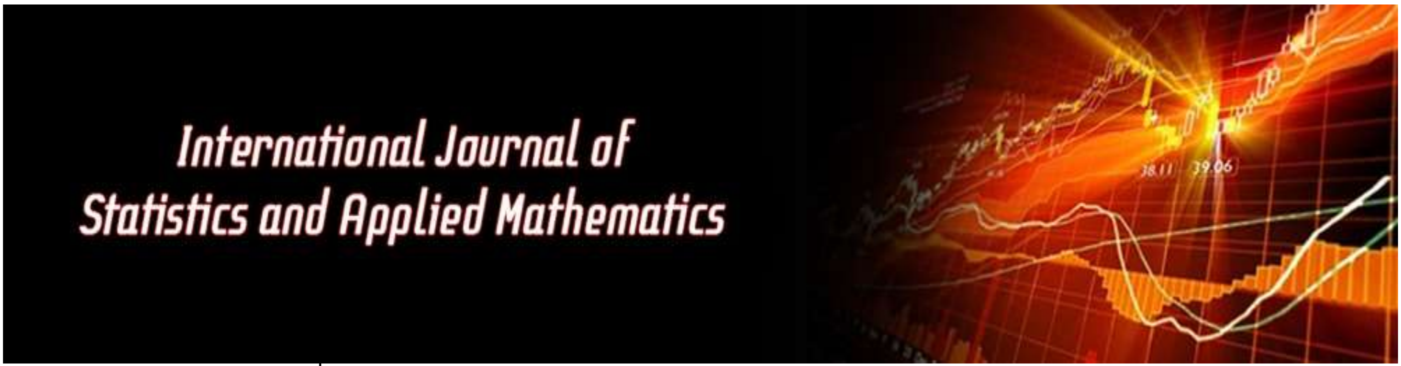

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Generalızatıon of mappıngs ın fuzzy topologıcal spaces

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Abstract
In this paper, we introduce fuzzy generalized regular-open and fuzzy generalized regular-closed maps in fuzzy topological spaces and obtain certain characterization of the generalized regular-closed and generalized regular open maps.

Keywords: Fuzzy gr-closed sets, fuzzy gr-open sets, gr-open maps, gr-closed maps

Introduction

The concept of a fuzzy subset was introduced and studied by LA Zadeh in the year 1965. The subsequent research activities in this area and related areas have found applications in many branches of science and engineering. In the year 1965, LA Zadeh [1] introduced the concept of fuzzy subset as a generalization of that of an ordinary subset. The introduction of fuzzy subsets paved the way for rapid research work in many areas of mathematical science. In the year 1968, CL Chang [2] introduced the concept of fuzzy topological spaces as an application of fuzzy sets to topological spaces. Subsequently several researchers contributed to the development of the theory and applications of fuzzy topology. The theory of fuzzy topological spaces can be regarded as a generalization theory of topological spaces. An ordinary subset A or a set X can be characterized by a function called characteristic function

$\mu_A : X \rightarrow [0,1]$ of A, defined by
$\mu_A(x) = 1$, if $x \in A$.
$= 0$, if $x \notin A$.

Thus an element $x \in X$ is in A if $\mu_A(x) = 1$ and is not in A if $\mu_A \longrightarrow (x) = 0$. In general if X is a set and A is a subset of X then A has the following representation. $A = \{ (x, \mu_A(x)): x \in X\}$, here $\mu_A(x)$ may be regarded as the degree of belongingness of x to A, which is either 0 or 1. Hence A is the class of objects with degree of belongingness either 0 or 1 only. Prof. L.A.Zadeh [1] introduced a class of objects with continuous grades of belongingness ranging between 0 and 1 ; he called such a class as fuzzy subset. A fuzzy subset A in X is characterized as a membership function μ_A : X [0,1], which associates with each point in x a real number $\mu_A(x)$ between 0 and 1 which represents the degree or grade membership of belongingness of x to A.

The purpose of this paper is to introduce a new class of fuzzy sets called fuzzy gr-closed sets in fuzzy topological spaces and investigate certain basic properties of these fuzzy sets. Among many other results it is observed that every fuzzy closed set is fuzzy gr-closed but not conversely. Also we introduce fuzzy gr-open sets in fuzzy topological spaces and study some of their properties.

1. Preliminaries

1.1Definition: [1] A fuzzy subset A in a set X is a function $A : X \rightarrow [0, 1]$. A fuzzy subset in X is empty iff its membership function is identically 0 on X and is denoted by 0 or μ_ϕ.The set X can be considered as a fuzzy subset of X whose membership function is identically 1 on X and is denoted by $\mu_x$or I_x. In fact every subset of X is a fuzzy subset of X but not conversely. Hence the concept of a fuzzy subset is a generalization of the concept of a subset.

1.2 Definition: [1] If A and B are any two fuzzy subsets of a set X, then A is said to be included in B or A is contained in B iff $A(x) \leq B(x)$ for all x in X. Equivalently, $A \leq B$ iff $A(x) \leq B(x)$ for all x in X.

1.3 Definition: [1] Two fuzzy subsets A and B are said to be equal if $A(x) = B(x)$ for every x in X.
Equivalently $A = B$ if $A(x) = B(x)$ for every x in X.

1.4 Definition: [1] The complement of a fuzzy subset A in a set X, denoted by A' or $1 - A$, is the fuzzy subset of X defined by $A'(x) = 1 - A(x)$ for all x in X. Note that $(A')' = A$.

1.5 Definition: [1] The union of two fuzzy subsets A and B in X, denoted by $A \vee B$, is a fuzzy subset in X defined by $(A \vee B)(x) = \mathrm{Max}\{\mu_A(x), \mu_B(X)\}$ for all x in X.

1.6 Definition: [1] The intersection of two fuzzy subsets A and B in X, denoted by $A \wedge B$, is a fuzzy subset in X defined by $(A \wedge B)(x) = \mathrm{Min}\{A(x), B(x)\}$ for all x in X.

1.7 Definition: [1] A fuzzy set on X is 'Crisp' if it takes only the values 0 and 1 on X.

1.8 Definition: [2] Let X be a set and be a family of fuzzy subsets of X. τ is called a fuzzy topology on X iff τ satisfies the following conditions.

1. μ_ϕ ; $\mu_X \in \tau$: That is 0 and 1 $\in \tau$
2. If $G_i \in \tau$ for $i \in I$ then $\bigvee_{i \in I} G_i \in \tau$
3. If $G,H \in \tau$ then $G \wedge H \in \tau$

The pair (X, τ) is called a fuzzy topological space (abbreviated as fts). The membersof $\tau$are called fuzzy open sets and a fuzzy set A in X is said to be closed iff $1 - A$ is an fuzzy open set in X.

1.9 Remark: [2] Every topological space is a fuzzy topological space but not conversely.

1.10 Definition: [2] Let X be a fts and A be a fuzzy subset in X. Then $\wedge$ {B: B is a closed fuzzy set in Xand $B \geq A$} is called the closure of A and is denoted by A or cl (A).

1.11 Definition: [2] Let A and B be two fuzzy sets in a fuzzy topological space (X,τ) and let $A \geq B$. Then B is called an interior fuzzy set of A if there exists $G \in \tau$ suchthat $A \geq G \geq B$, the least upper bound of all interior fuzzy sets of A is called theinterior of A and is denoted by A^0.

1.12 Definition: [3] A fuzzy set A in a fts X is said to be fuzzy semiopen if and only if there exists a fuzzy open set V in X such that $V \leq A \leq cl(V)$.

1.13 Definition: [3] A fuzzy set A in a fts X is said to be fuzzy semi-closed if and only if there exists a fuzzy closed set V in X such that $int(V) \leq A \leq V$. It is seen that a fuzzy set A is fuzzy semiopen if and only if 1-A is a fuzzy semi-closed.

1.14 Theorem: [3] The following are equivalent:

(a) $\mu$is a fuzzy semiclosed set,
(b) μ^C is a fuzzy semiopen set,
(c) $int(cl(\mu)) \leq \mu$.
(d) $int(cl(\mu)) \geq \mu^c$

1.15 Theorem: [3] Any union of fuzzy semiopen sets is a fuzzy semiopen set and (b) any intersection of fuzzy semi closed sets is a fuzzy semi closed.

1.16 Remark: [3]

1. Every fuzzy open set is a fuzzy semiopen but not conversely.
2. Every fuzzy closed set is a fuzzy semi-closed set but not conversely.
3. The closure of a fuzzy open set is fuzzy semiopen set
4. The interior of a fuzzy closed set is fuzzy semi-closed set

1.17Definition: [3] A fuzzy set $\mu$of a fts X is called a fuzzy regular open set of X if int (cl(μ).= μ.

1.18 Definition: [3] A fuzzy set $\mu$of fts X is called a fuzzy regular closed set of X if $cl(int(\mu)) = \mu$.

1.19 Theorem: [3] A fuzzy set $\mu$of a fts X is a fuzzy regular open if and only if μ^C fuzzy regular closed set.

1.20 Remark: [3]

1. Every fuzzy regular open set is a fuzzy open set but not conversely.
2. Every fuzzy regular closed set is a fuzzy closed set but not conversely.

1.21 Theorem: [3]

1. The closure of a fuzzy open set is a fuzzy regular closed.
2. The interior of a fuzzy closed set is a fuzzy regular open set.

1.22 Definition: [4] A fuzzy set αin fts X is called fuzzy rw-closed if $cl(\alpha) \leq \mu$whenever $\alpha \leq \mu$and $\mu$is regular semi-open in X.

1.23 Definition: [5] A fuzzy set α in fts X is called fuzzy pgprw closed if $p\text{-}cl(\alpha) \leq \mu$whenever $\alpha \leq \mu$ and μ is rgα −open set in X.

1.24 Definition: [5] A fuzzy set α of a fts X is fuzzy pgprw-open set, if it's complement α^c is a fuzzy pgprw-closed in fts X.

1.25 Definition: [2] Let X and Y be fts. A map f: X $\rightarrow$ Y is said to be a fuzzy continuous mapping if $f^{-1}(\mu)$ is fuzzy open in X for each fuzzy open set μ in Y.

1.26 Definition: [6] A mapping f: X $\rightarrow$ Y is said to be fuzzy irresolute iff f^{-1} (B) is fuzzy semi-open in X is fuzzy open in Y.

1.27 Definition: [7] A mapping f: X$\longrightarrow$ Y is said to be fuzzy pgprw irresolute if the inverse image of every fuzzy pgprw-open in Y is a fuzzy pgprw-open set in X.

2. Fuzzy gr-open maps and fuzzy gr-closed maps in fuzzy topological spaces

Definition 2.1: Let X and Y be two fts. A map f: (X,T_1) (Y,T_2) is called fuzzy genealized regular-open map (fuzzy gr-open map) if the image of every fuzzy open set in X is fuzzy gr-open in Y.

Theorem 2.2: Every fuzzy open map is a fuzzy gr-open map.

Proof: Let f :(X,T_1)$\longrightarrow$(Y,T_2) be a fuzzy open map and and μ be a fuzzy open set in fts X. Then f(μ) is a fuzzy open set in fts Y. Since every fuzzy open set is fuzzy gr-open,f(μ) is a fuzzy gr-open set in fts Y. Hence f is a fuzzy gr-open map.
The converse of the above theorem need not be true in general as seen from the following example.

Example 2.3: Let X= Y={a, b, c} and the functions α, β: X [0, 1] be defined as

α (x) = 1 if x = a,c
0 otherwise
β(x) = 1 if x = a
0 Otherwise

Consider T_1= {1,0,α}, T_2 = {1,0,β}then$\longrightarrow$(X,T_1) (Y,T_2)are fts.Let map f: X$\rightarrow$Y be the identity map. Then this function is gr-open map but it is not fuzzy open. Since the image of the fuzzy open set α (x) in X is fuzzy set β in Y which is not fuzzy open.

Remark 2.4: If f: (X, T_1) $\rightarrow$ (Y, T_2) and g:(Y,T_2) $\rightarrow$ (Z,T_3) be two fuzzy gr—open map then composition g of: (X,T_1) $\rightarrow$ (Z,T_3) need not be fuzzy gr-open as seen from the following example

Example 2.5: Let X=Y=Z= {a, b, c} and the function$\alpha, \beta, \gamma, \delta$: X [0,1] be defined as
α (x) = 1 if x = a
0 otherwise
β(x) = 1 if x = c
0 otherwise
γ(x) = 1 if x = a,c
0 otherwise
δ(x) = 1 if x = a,b
0 otherwise

Consider T_1 = {1, 0, α, δ}, T_2 ={1,0,α} and T_3 = {0,1, α, β, γ}. Let map f: (X T_1) $\rightarrow$ (Y, T_2) and g: (Y,T_2) $\rightarrow$ (Z,T_3) be the identity maps then f and g are fuzzy gr-open map but their composition gof:(X,T_1) $\rightarrow$ (Z,T_3) is not fuzzy gr-open as β: X $\rightarrow$ [0,1] is fuzzy open in X But (gof)(β)=β is not fuzzy gr-open in Z.

Theorem 2.6: If f: (X,T_1) $\longrightarrow$(Y,T_2) is fuzzy open map and g:(Y,T_2) $\longrightarrow$(Z,T_3) is fuzzy gr—open map then their composition gof: (X,T_1) $\rightarrow$ (Z,T_3) is fuzzy gr-open map.

Proof: Let α be fuzzy open set in (X, T_1). Since f is fuzzy open map,f(α) is a fuzzy open set in (Y,T_2). Since g is a fuzzy gr-open map g(f(α))= (gof)(α). Thus gof is a fuzzy gr-open map.

Definition 2.7: Let X and Y be two fuzzy topological spaces. A map f:(X,T_1) (Y,T_2) is called fuzzy generalized regular closed map (fuzzy gr-closed map)if the image of every fuzzy closed set in X is a fuzzy closed set in Y.

Theorem 2.8: Let f: (X, T_1)$\rightarrow$(Y, T_2) be fuzzy closed map then f is a fuzzy gr closed map.

Proof: Let α be a fuzzy closed set in (X, T_1). Since f is fuzzy closed map,f(α) is a fuzzy closed map, f(α) is a fuzzy closed set in (Y,T_2). since every fuzzy closed set is fuzzy gr-closed, f(α) is a fuzzy gr-closed set in (Y,T_2).Hence f is fuzzy gr-closed map.
The converse of the theorem need not be true in general as seen from the following example

Example 2.9: Let X= Y={a, b, c} and the functions α, β: X $\rightarrow$ [0, 1] be defined as
α (x) = 1 if x = a,c
0 otherwise
β(x) = 1 if x = a
0 Otherwise

Consider T_1= {1,0,α} and T_2= {1,0,β}; then (X,T_1) and (Y,T_2) are fts. Let f: (X,T_1)$\rightarrow$(Y,T_2) be the identity map. Then f is a fuzzy gr-closed map but it is not a fuzzy closed map, since the image of the closed set α in X is not a fuzzy closed set in Y.

Remark 2.10: The composition of two fuzzy gr-closed maps need not be a fuzzy gr-closed map

Example 2.11: Consider the fts (X, T_1),(Y,T_2)(Z,T_3) and mappingr defined in 2.5. The maps f and g are fuzzy gr-closed but their composition is not fuzzy gr-closed as β:X$\rightarrow$[0, 1] is a fuzzy closed set in X but (gof)(β)=β is not fuzzy gr-closed in Z.

Theorem 2.12: If a map f: (X,T_1) $\longrightarrow$ (Y,T_2) and g : (Y,T_2) (Z,T_3) be two maps then composition of two gr-closed maps isfuzzy gr-closed map.

Proof: Let α be a fuzzy closed set in (X, T_1). Since f is a fuzzy closed map, f(α) is a fuzzy closed set in (Y,T_2). Since g is a fuzzy gr-closed map but g(f(α)) is a fuzzy gr-closed set in (Z,T_3) but g(f(α))=(gof)(α). Thus gof is fuzzy gr-closed map.

Theorem 2.13: A map f: X $\longrightarrow$Y is fuzzy gr-closed map if for each fuzzy set δ of Y and for each fuzzy open set μ of X s.t μ ≥$f^{-1}(\delta)$, there is a fuzzy gr-open map set α of Y s.t $\delta \leq \alpha$ and $f^{-1}(\alpha) \leq \mu$.

Proof: Suppose that f is fuzzy gr-closed map.Let δ be a fuzzy subset of Y and μ be a fuzzy open set of X s.t $f^{-1}(\delta) \leq \mu$. Let α = 1 – f(1- μ) is fuzzy gr-open set in fts Y. note that $f^{-1}(\delta) \leq \mu$.Which implies $\delta \leq \alpha$ and $f^{-1}(\alpha) \leq \mu$. For the converse, Suppose that μ is a fuzzy closed set in X, Then f^{-1}(1-f (μ)≤1- μAnd 1- μ is fuzzy open by hypothesis, there is a fuzzy gr-open set α of Y s.t (1-f (μ)≤ α. And $f^{-1}(\alpha) \leq 1-\mu$. Therefore $\mu \leq 1- f^{-1}(\alpha)$. Hence 1- α ≤ μ, f(1-$f^{-1}(\alpha)$) ≤ 1- α.Which implies f (μ) = 1- α. Since 1- α is fuzzy gr-closed f (μ) is fuzzy gr-closed and thus f is fuzzy gr-closed.

Lemma 2.14: Let f :(X, T_1)$\rightarrow$ (Y,T_2) be fuzzy irresolute map and α be fuzzy rgα open in Y then $f^{-1}(\alpha)$ is fuzzy rgα open in X.

Proof: Let α be fuzzy rgα-open in Y. To prove $f^{-1}(\alpha)$ is fuzzy rgα open in X that is to prove $f^{-1}(\alpha)$ is both fuzzy semi-open and fuzzy semi-closed in X.Now α is fuzzy semi-open in Y. Since f is fuzzy irresolute map,$f^{-1}(\alpha)$ is fuzzy semi-open in X.
Now α is fuzzy semi-closed in Y as fuzzy rgα open set is fuzzy semi-closed then 1- α is Fuzzy semi-open in Y. Since f is fuzzy irresolute map, f^{-1}(1- α) is a fuzzy semi open in X.
But f^{-1}(1- α) = 1 - $f^{-1}(\alpha)$ is fuzzy semi-open in X and so $f^{-1}(\alpha)$ is semi closed in X. Thus $f^{-1}(\alpha)$ is both fuzzy semi-open

and fuzzy semi-closed in X and hence $f^{-1}(\alpha)$ is fuzzy rgα open in X.

Thorem 2.15: If a map f: $(X, T_1) \longrightarrow (Y, T_2)$ is fuzzy irresolute map and fuzzy gr-closed and α is fuzzy gr-closed set of X,then $f(\alpha)$ is a fuzzy gr-closed set in Y.

Proof: Let α be a fuzzy closed set of X.Let $f(\alpha) \leq \mu$. Where μ is fuzzy rgα open in Y.Since Since f is fuzzy irresolute map, $f^{-1}(\mu)$ is a fuzzy rgα open in X, by lemma 2.14 and $\alpha \leq f^{-1}(\mu)$. Since α is a fuzzy gr-closed set in X, P-cl$(\alpha) \leq f^{-1}(\mu)$. Since f is a fuzzy gr-closed set contained in the fuzzy rgα open setμ, which implies cl (f(p-cl$(\alpha)) \leq \mu$ and hence cl$(f(\alpha)) \leq \mu$. Therefore $f(\alpha)$ is a fuzzy gr-closed set in Y.

Corollary 2.16: If a map f: $(X, T_1) \rightarrow (Y, T_2)$ is fuzzy irresolute map and fuzzy closed and α is a fuzzy gr-closed set in fts.

Proof: The proof follows from the theorem 2.15 and the fact that every fuzzy closed map is a fuzzy gr-closed map.

Theorem 2.17: Let f: $X \rightarrow Y$ and g: $Y \rightarrow Z$ be two mappingr s.t

i. If f is fuzzy continuous map and subjective, then g is fuzzy gr-closed map.
ii. If g is fuzzy gr-irresolute map and injective then f is fuzzy gr-closed map.

Proof:
1. Let μ be a fuzzy closed set in Y, since f is fuzzy continuous map,$f^{-1}(\mu)$ is a fuzzy closed set in X. Since gof is a fuzzy gr closed map,(gof)($f^{-1}(\mu)$) is a fuzzy gr-closed set in Z. but (gof) ($f^{-1}(\mu)$) = $g(\mu)$ as f is surjective thus g is fuzzy gr closed map.
2. Let β be a fuzzy closed set of X then (gof)(β) is a fuzzy gr closed set in Z, Since (gof) is a fuzzy gr-closed map. Since g is fuzzy gr-irresolute map g^{-1} (gof)(β) is fuzzy gr-closed in Y. But g^{-1} (gof)$(\beta) = f(\beta)$ as g is injective. Thus f is fuzzy gr-closed map.

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