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Ratio-product estimator in stratified double sampling based on coefficient of skewness of the auxiliary variable

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Abstract
In this paper, a ratio-product estimator for estimating the population mean of the study variable based on the population coefficient of skewness of the auxiliary variable is suggested in stratified double sampling. Asymptotic optimum estimator and its approximate bias and variance expressions are derived. Properties of the suggested estimator are studied with some known existing estimators identified as special members of this class of estimators. Analytical and numerical investigations showed that the suggested estimator is more efficient than the conventional regression estimator of mean in stratified double sampling and existing estimators of its class in stratified double sampling. Analysis and evaluation are presented.

Keywords: Asymptotic optimum estimator, efficiency, large sample approximation, optimality conditions, percentage relative efficiency

1. Introduction
The ratio estimation is gaining increased relevance in Statistical Estimation Theory compared to the regression estimation because of its improved precision in estimating the population or subpopulation parameters. But the regression estimator, in spite of its lower practicability, seems to be holding a unique position due to its sound theoretical basis. The classical ratio and product estimators, even though considered to be more useful in many practical situations in fields like Agriculture, Forestry, Economics and population studies, have roughly equal efficiencies with those of the linear regression.

This limitation has prompted most survey Statisticians to carry out research towards the modification of the existing ratio, product or classes of ratio and product estimators of the population mean in survey sampling to provide better alternatives and improve efficiency. The authors who have proposed improved estimators include [1-20].

In this paper, based on [21], a new ratio-product estimator for estimating the population mean of the study variable using the population coefficient of skewness of the auxiliary variable is suggested in stratified double sampling. The choice is obvious; coefficient of skewness and its functions are unaffected by extreme values or the presence of outliers. Further, it has always strong correlation with other population parameters like the mean and variance.

2. Basic notations and definitions
Consider a finite population $U = (U_1, U_2, \ldots, U_N)$ of size (N). Let (X) and (Y) denote the auxiliary and study variables taking values X_i and Y_i respectively on the i-th unit $U_i (i = 1, 2, \ldots, N)$ of the population.

The theory of double sampling for stratification was first given by [22]. The population is stratified into H strata such that the h-th stratum consists of N_h units and $\sum_{h=1}^{H} N_h = N$, $\sum_{h=1}^{H} n_h = n$. From the N_h units a preliminary large sample of n'_h units is drawn using simple random sampling without replacement (*SRSWOR*) and the auxiliary variable x_{hi} is measured only. A subsample of n_h is then selected from the given preliminary large sample of n'_h units by *SRSWOR* and both the study variable y_{hi} and the auxiliary variable x_{hi} are measured.

In this study, it is assumed that the second sample is drawn independently of the first, so that the n_h does not depend on the n'_h except for the assumption $n_h \leq n'_h$.

Let $\bar{x}'_h = \frac{1}{n'_h}\sum_{i=1}^{n'_h} x_{hi}$, $S'^2_{hx} = \frac{1}{n'_h-1}\sum_{i=1}^{n'_h}(x_{hi} - \bar{x}'_h)^2$, denote the first phase sample mean and variance respectively for the auxiliary variable. Similarly, let, $S^2_{hx} = \frac{1}{n_h-1}\sum_{i=1}^{n_h}(x_{hi} - \bar{x}_h)^2$, $S^2_{hy} = \frac{1}{n_h-1}\sum_{i=1}^{n_h}(y_{hi} - \bar{y}_h)^2$, $\bar{x}_h = \frac{1}{n_h}\sum_{i=1}^{n_h} x_{hi}$, $\bar{y}_h = \frac{1}{n_h}\sum_{i=1}^{n_h} y_{hi}$ denote the second phase sample means and variances for the auxiliary variable and study variable respectively.

Let consider the following definitions; $E(e_{hx}) = E(e_{hy}) = E(e'_{h\beta}) = E(e_{h\beta}) = 0, E(e^2_{hy}) = \alpha_h C^2_{hy}, E(e^2_{hx}) = \alpha_h C^2_{hx}, E(e^2_{h\beta}) = \alpha_h C^2_{h\beta}, E(e'^2_{h\beta}) = \alpha'_h C^2_{h\beta}, E(e_{hy}e_{hx}) = \alpha_h \rho_{hyx} C_{hy} C_{hx}$, $E(e_{hy}e_{h\beta}) = \alpha_h \rho_{hy\beta} C_{hy} C_{h\beta}$, $E(e_{hy}e'_{h\beta}) = \alpha'_h \rho_{hy\beta} C_{hy} C_{h\beta}$, $E(e'_{h\beta}e_{h\beta}) = \alpha'_h C^2_{h\beta}$, $\alpha'_h = \left(\frac{1}{n'_h} - \frac{1}{N_h}\right)$, $\alpha_h = \left(\frac{1}{n_h} - \frac{1}{n'_h}\right)$ and $\gamma_h = \left(\frac{1}{n_h} - \frac{1}{N_h}\right)$; $\tau_h = \rho_{hy\beta} C_{hy}/C_{h\beta}$, $g' = (N - n')/(N - 1)$, $n' = \sum_{h=1}^{H} n'_h$, $n = \sum_{h=1}^{H} n_h$, where the parameters above are defined as:

$\bar{y}_h$ is the second phase sample stratum mean of the study variable
$\bar{Y}_h$ is the second phase population stratum mean of the study variable
$\bar{x}'_h$ is the first phase sample stratum mean of the auxiliary variable
$\bar{x}_h$ is the second phase sample stratum mean of the auxiliary variable
$\bar{X}_h$ is the second phase population stratum mean of the auxiliary variable
s'^2_{hx} is the first phase sample stratum variance of the auxiliary variable
s^2_{hx} is the second phase sample stratum variance of the auxiliary variable
S^2_{hx} is the second phase population stratum variance of the auxiliary variable
$\beta'_{1h}(x)$ is the first phase sample coefficient of skewness of the auxiliary variable
$\beta_{1h}(x)$ is the second phase sample coefficient of skewness of the auxiliary variable
$\mathrm{B}_{1h}(x)$ is the second phase population coefficient of skewness of the auxiliary variable
C^2_{hx} is the coefficient of variation of the auxiliary variable
C^2_{hy} is the coefficient of variation of the auxiliary variable
ρ_{hxy} is the correlation coefficient between the auxiliary variable and the study variable.
$\rho_{hy\beta}$ is the correlation coefficient between the mean of the study variable and coefficient of skewness of the auxiliary variable.

3. The suggested estimator

Proposed a separate ratio-product estimator of mean using known values of the population mean of the auxiliary variable in stratified random sampling as given by the following [21]:

$$\hat{\bar{\mathcal{Y}}}_{vs} = \sum_{h=1}^{H} w_h \bar{y}_h \left[\xi_h \frac{\bar{X}_h}{\bar{x}_h} + (1 - \xi_h)\frac{\bar{x}_h}{\bar{X}_h}\right] \quad \ldots (1)$$

Where ξ_h is a real constant to be determined in such away that minimizes $\hat{\bar{\mathcal{Y}}}_{vs}$. Motivated by [21], a new separate ratio-product estimator of mean using known values of the population coefficient of skewness of the auxiliary variable is suggested in stratified double sampling as follows:

$$\hat{\bar{\mathcal{Y}}}_{pr} = \sum_{h=1}^{H} w_h \bar{y}_h \left[\xi_h \frac{\beta'_{1h}(x)}{\beta_{1h}(x)} + (1 - \xi_h)\frac{\beta_{1h}(x)}{\beta'_{1h}(x)}\right] \quad \ldots (2)$$

3.1 Variance estimation for the suggested estimator

This section derives the estimator of variance for the suggested estimator using the large sample approximation (*LASAP*) method.

Let $e_{hy} = \left(\frac{\bar{y}_h - \bar{Y}_h}{\bar{Y}_h}\right)$ so that $\bar{y}_h = \bar{Y}_h(1 + e_{hy})$ … (3)

Let $e_{h\beta} = \left(\frac{\beta_{1h}(x) - \mathrm{B}_{1h}(x)}{\mathrm{B}_{1h}(x)}\right)$ so that $\beta_{1h}(x) = \mathrm{B}_{1h}(x)(1 + e_{h\beta})$ … (4)

Let $e'_{h\beta} = \left(\frac{\beta'_{1h}(x) - \mathrm{B}_{1h}(x)}{\mathrm{B}_{1h}(x)}\right)$ so that $\beta'_{1h}(x) = \mathrm{B}_{1h}(x)(1 + e'_{h\beta})$ … (5)

Let $e_{hx} = \left(\frac{\bar{x}_h - \bar{X}_h}{\bar{X}_h}\right)$ so that $\bar{x}_h = \bar{X}_h(1 + e_{hx})$ … (6)

Expressing (2) in terms of the e's [that is substituting (3-5) in (2)] gives:

$$\hat{\bar{\mathcal{Y}}}_{pr} = \sum_{h=1}^{H} w_h \bar{Y}_h(1 + e_{hy})\left[\xi_h \frac{(1+e'_{h\beta})}{(1+e_{h\beta})} + (1 - \xi_h)\frac{(1+e_{h\beta})}{(1+e'_{h\beta})}\right]$$

$$\hat{\bar{\mathcal{Y}}}_{pr} = \sum_{h=1}^{H} w_h \bar{Y}_h(1 + e_{hy})\left[\xi_h(1 + e'_{h\beta})(1 + e_{h\beta})^{-1} + (1 - \xi_h)(1 + e_{h\beta})(1 + e'_{h\beta})^{-1}\right] \quad \ldots (7)$$

Taking Taylor's series expansion of $(1 + e_{h\beta})^{-1}$ and $(1 + e'_{h\beta})^{-1}$ gives

$$(1 + e_{h\beta})^{-1} = (1 - e_{h\beta} + e^2_{h\beta} - e^3_{h\beta} + \cdots) \quad \ldots (8)$$

$$(1 + e'_{h\beta})^{-1} = (1 - e'_{h\beta} + e'^2_{h\beta} - e'^3_{h\beta} + \cdots) \quad \ldots (9)$$

Substituting (8) and (9) in (7) and retaining terms to the first order of approximation gives:

$$\hat{\bar{\mathcal{Y}}}_{pr} = \sum_{h=1}^{H} w_h \bar{Y}_h(1 + e_{hy})\left[(1 - e'_{h\beta} + e'^2_{h\beta} + e_{h\beta} - e_{h\beta}e'_{h\beta}) + \xi_h(e^2_{h\beta} - e'^2_{h\beta} + 2e'_{h\beta} - 2e_{h\beta})\right]$$

So that

$$(\hat{\bar{y}}_{pr} - \bar{Y}) = \sum_{h=1}^{H} w_h \bar{Y}_h \left[\left(e_{hy} + e_{h\beta} - e'_{h\beta} + e'^2_{h\beta} - e'_{h\beta} e_{h\beta} - e_{hy} e'_{h\beta} + e_{hy} e_{h\beta} \right) + \xi_h \left(e^2_{h\beta} - e'^2_{h\beta} + 2e'_{h\beta} - 2e_{h\beta} + 2e_{hy} e'_{h\beta} - 2e_{hy} e_{h\beta} \right) \right] \quad \ldots (10)$$

Taking expectation of both sides of (10) gives the biased of $\hat{\bar{y}}_{pr}$ as:

$$Biased\left(\hat{\bar{y}}_{pr}\right) = \sum_{h=1}^{H} w_h \bar{Y}_h (\alpha_h - \alpha'_h) C^2_{h\beta} [1 - (2\xi_h - 1)\tau_h]$$

Squaring both sides of (10) and retaining terms to the first order of approximation gives:

$$(\hat{\bar{y}}_{pr} - \bar{Y})^2 = \sum_{h=1}^{H} w_h^2 \bar{Y}_h^2 \left[\left(e^2_{hy} + e^2_{h\beta} + e'^2_{h\beta} - 2e'_{h\beta} e_{h\beta} - 2e_{hy} e'_{h\beta} + 2e_{hy} e_{h\beta} \right) + 4\xi_h \left(e_{hy} e'_{h\beta} - e_{hy} e_{h\beta} + 2e_{h\beta} e'_{h\beta} - e^2_{h\beta} - e'^2_{h\beta} \right) + 4\xi_h^2 \left(e^2_{h\beta} + e'^2_{h\beta} - 2e_{h\beta} e'_{h\beta} \right) \right] \quad \ldots (11)$$

Taking expectation of both sides of (11) gives the variance of $\hat{\bar{y}}_{pr}$ as:

$$\hat{V}(\hat{\bar{y}}_{pr}) = \sum_{h=1}^{H} w_h^2 \bar{Y}_h^2 \left[\gamma_h C^2_{hy} + (\alpha_h - \alpha'_h)(2\xi_h - 1)[(2\xi_h - 1) - 2\tau_h] C^2_{h\beta} \right] \quad \ldots (12)$$

The $\hat{V}(\hat{\bar{y}}_{pr})$in (12) is minimized when

$$\xi_h = \frac{(1+\tau_h)}{2} = \xi_{h,opt}\ (say) \quad \ldots (13)$$

Substituting the value of $\xi_{h,opt}$ in (13) for ξ_h in (2) gives the asymptotically optimum estimator (AOE) of mean ($\bar{Y}$) in stratified double sampling as:

$$\hat{\bar{y}}_{pr,opt} = \sum_{h=1}^{H} w_h \bar{y}_h \left[\left(\frac{1+\tau_h}{2} \right) \frac{\beta'_{1h}(x)}{\beta_{1h}(x)} + \left(\frac{(1-\tau_h)}{2} \right) \frac{\beta_{1h}(x)}{\beta'_{1h}(x)} \right] \quad \ldots (14)$$

Similarly, substituting the value of $\xi_{h,opt}$ in (13) for ξ_h in (12) gives the variance of an asymptotically optimum estimator (AOE) $\hat{\bar{y}}_{pr,opt}$ (or minimum variance of $\hat{\bar{y}}_{pr}$) as:

$$\hat{V}(\hat{\bar{y}}_{pr,opt}) = \sum_{h=1}^{H} w_h^2 \bar{Y}_h^2 C^2_{hy} \left[\alpha_h \left(1 - \rho^2_{hy\beta} \right) + \alpha'_h \rho_{hy\beta} \right] \quad \ldots (15)$$

3.2 Properties of the suggested estimator

This section studies the properties of the suggested estimator and identifies some special members of its family and derives their biased and variance expressions under certain prescribed conditions.

i) Property 1: When $\xi_h = 1$; then the suggested estimator in (2) reduces to a separate ratio estimator defined by:

$$\hat{\bar{y}}_{pr1} = \sum_{h=1}^{H} w_h \hat{R}_h \beta'_{1h}(x) \quad \ldots (16)$$

$$Bias\left(\hat{\bar{y}}_{pr1}\right) = \sum_{h=1}^{H} w_h \bar{Y}_h (\alpha_h - \alpha'_h) C^2_{h\beta} [1 - \tau_h] \quad \ldots (17)$$

$$\hat{V}(\hat{\bar{y}}_{pr1}) = \sum_{h=1}^{H} w_h^2 \bar{Y}_h^2 \left[\alpha_h C^2_{hy} + (\alpha_h - \alpha'_h)[1 - 2\tau_h] C^2_{h\beta} \right] \quad \ldots (18)$$

Where $\hat{R}_h = \bar{y}_h / \beta_{1h}(x)$; $\beta_{1h}(x) \neq 0$ is the estimate of the ratio $R_h = \bar{Y}_h / B_{1h}(x)$; $B_{1h}(x) \neq 0$ of the hth stratum in the population. This estimator is only efficient if the variables are strongly positively correlated [21].

ii) Property 2: When $\xi_h = 0$; then the suggested estimator in (2) reduces to a separate product estimator defined by:

$$\hat{\bar{y}}_{pr2} = \sum_{h=1}^{H} w_h \frac{\hat{P}_h}{\beta'_{1h}(x)} \quad \ldots (19)$$

$$Bias\left(\hat{\bar{y}}_{pr2}\right) = \sum_{h=1}^{H} w_h \bar{Y}_h (\alpha_h - \alpha'_h) C^2_{h\beta} [1 + \tau_h] \quad \ldots (20)$$

$$\hat{V}(\hat{\bar{y}}_{pr2}) = \sum_{h=1}^{H} w_h^2 \bar{Y}_h^2 \left[\alpha_h C^2_{hy} + (\alpha_h - \alpha'_h)[1 + 2\tau_h] C^2_{h\beta} \right] \quad \ldots (21)$$

Where $\hat{P}_h = \bar{y}_h \beta_{1h}(x)$ is the estimate of the product $P_h = \bar{Y}_h B_{1h}(x)$ of the hth stratum in the population. This estimator will often be used if the two variables are supposed to be strongly negatively correlated [21].

iii) Property 3: When $\xi_h = -1$; then the suggested estimator in (2) reduces to an estimator of the form:

$$\hat{\bar{y}}_{pr3} = \sum_{h=1}^{H} w_h \bar{y}_h \left[2 \frac{\beta_{1h}(x)}{\beta'_{1h}(x)} - \frac{\beta'_{1h}(x)}{\beta_{1h}(x)} \right] \quad \ldots (22)$$

$$Bias\left(\hat{\bar{y}}_{pr3}\right) = \sum_{h=1}^{H} w_h \bar{Y}_h (\alpha_h - \alpha'_h) C^2_{h\beta} [1 + 3\tau_h] \quad \ldots (23)$$

$$\hat{V}(\hat{\bar{y}}_{pr3}) = \sum_{h=1}^{H} w_h^2 \bar{Y}_h^2 \left[\alpha_h C^2_{hy} + 3(\alpha_h - \alpha'_h)[3 + 2\tau_h] C^2_{h\beta} \right] \quad \ldots (24)$$

Remarks

Following from properties (1-3); it is observed that at the same optimum condition (that is $\xi_{h,opt} = \{(1 + \tau_h)/2\}$), every identified member of the suggested class of estimators $\left(\hat{\bar{y}}_{pr1} - \hat{\bar{y}}_{pr3}\right)$ has equal optimal efficiency with variance estimator as given by:

$$\hat{V}\left(\hat{\bar{y}}_{pr*}\right) = \sum_{h=1}^{H} w_h^2 \bar{Y}_h^2 C^2_{hy} \left[\alpha_h - (\alpha_h - \alpha'_h) \rho^2_{hy\beta} \right] \quad \ldots (25)$$

4. Analytical study
4.1 Efficiency comparisons
This section compares the efficiency of the suggested estimator with the regression estimator of mean in stratified double sampling designated as the global estimator in this paper. The variance of the conventional regression estimator of mean in stratified double sampling $(\hat{\bar{y}}_{REG})$ is defined by [23] as:

$$\hat{V}(\hat{\bar{y}}_{REG}) = \sum_{h=1}^{H} w_h^2 S_{hy}^2 \left[\alpha_h \left(1-\rho_{hxy}^2\right) + \alpha_h'\right] \quad \ldots (26)$$

The suggested estimator $\left(\hat{\bar{y}}_{pr}\right)$ would be more efficient than the regression estimator $(\hat{\bar{y}}_{REG})$ if and only if:

$$\hat{V}\left(\hat{\bar{y}}_{pr}\right) < \hat{V}(\hat{\bar{y}}_{REG}) \quad \ldots (27)$$

So that the optimality condition is:

$$\alpha_h' > \frac{\alpha_h\left(\rho_{hxy}^2-\rho_{hy\beta}^2\right)}{(1-\rho_{hy\beta})} \quad \ldots (28)$$

When the optimality condition is satisfied, the suggested estimator is more efficient than the regression estimator.

4.2 The percent relative efficiency (PRE)
The percent relative efficiency (PRE) of an estimator ϕ with respect to the conventional unbiased estimator of mean in stratified double sampling $(\hat{\bar{y}}_{ds})$ is defined by:

$$PRE(\phi, \hat{\bar{y}}_{ds}) = \frac{V(\hat{\bar{y}}_{ds})}{V(\phi)} \times 100 \quad \ldots (29)$$

The variance of the conventional unbiased estimator of mean in stratified double sampling $(\hat{\bar{y}}_{ds})$ is defined by [23] as:

$$V(\hat{\bar{y}}_{ds}) = \sum_{h=1}^{H} \left(\frac{w_h^2 S_{hy}^2}{n_h}\right) + \frac{g'}{n'} \sum_{h=1}^{H} w_h (\bar{Y}_h - \bar{Y})^2 \quad \ldots (30)$$

5. Empirical study
To judge the relative performances of the suggested estimator over the regression estimator, data set from [24] given in table 1 was considered.
Numerical results for the efficiency comparisons, shows that:

i) $\alpha_1' = 0.0474 > \frac{\alpha_1\left(\rho_{1xy}^2-\rho_{1y\beta}^2\right)}{(1-\rho_{1y\beta})} = -0.3213$

ii) $\alpha_2' = 0.0368 > \frac{\alpha_2\left(\rho_{2xy}^2-\rho_{2y\beta}^2\right)}{(1-\rho_{2y\beta})} = -1.1032$

iii) $\alpha_3' = 0.0235 > \frac{\alpha_3\left(\rho_{3xy}^2-\rho_{3y\beta}^2\right)}{(1-\rho_{3y\beta})} = -0.0211$

Therefore since the optimality condition is satisfied in accordance with section 4.1, it is concluded that the new estimator is more efficient than the regression estimator in stratified double sampling.
Numerical results for the percent relative efficiency ($PREs$) in table 2 reveals that the suggested estimator $\hat{\bar{y}}_{pr}$ has 135 percent gains in efficiency while the conventional regression estimator in stratified double sampling $\hat{\bar{y}}_{REG}$ has 95 percent gains in efficiency; this shows that the suggested estimator $\hat{\bar{y}}_{ds}$ is 40 percent more efficient than the regression estimator in stratified double sampling $\hat{\bar{y}}_{REG}$. This means that in using the suggested estimator $\hat{\bar{y}}_{pr}$ one will have 40 percent efficiency gain over the regression estimator in stratified double sampling $\hat{\bar{y}}_{REG}$.

Table 1: Data set adapted from [24]

Parameter	Stratum 1	Stratum 2	Stratum 3
N_h	52	76	82
n_h'	15	20	28
n_h	4	5	7
$\bar{X}_h$	6.813	10.12	7.967
$\bar{Y}_h$	417.33	503.375	340.00
S_{hx}^2	15.9712	132.66	38.438
S_{hy}^2	74775.467	259113.70	65885.6
S_{hxy}	1007.6547	5709.1629	1404.71
α_h'	0.0474	0.0368	0.0235
γ_h	0.2308	0.1868	0.1307
ρ_{hyx}	0.703	0.738	0.805
$\rho_{hy\beta}$	0.86	0.764	0.826
$\rho_{hx\beta}$	0.82	0.803	0.782

Table 2: Performance of estimators

Estimator	Variance	PRE$[\phi, \hat{\bar{y}}_{ds}]$
$\hat{\bar{y}}_{ds}$	9425.2024	100
$\hat{\bar{y}}_{pr}$	4015.8967	234.6973
$\hat{\bar{y}}_{REG}$	4823.9761	195.3844

6. Conclusion

This study introduces a new ratio-product estimator for estimating the population mean of the study variable based on the population coefficient of skewness of the auxiliary variable in stratified double sampling. The bias and variance expressions for the suggested estimator have been derived under large sample approximation. Asymptotic optimum estimator and its approximate bias and variance expressions are equally derived. Properties of the suggested estimator are studied with some known existing estimators identified as special members of this class of estimators. Analytical and numerical results showed that the suggested estimator $\hat{\bar{y}}_{pr}$, is more efficient than the regression estimator $\hat{\bar{y}}_{REG}$ and by extension than the traditional ratio estimator in stratified double sampling (since the regression estimator $\hat{\bar{y}}_{REG}$ is always more efficient than the ratio estimator [23, 25]).

It is observed that the new estimator $\hat{\bar{y}}_{pr}$, is very attractive and should be preferred in practice as it provides consistent and more precise parameter estimates than existing estimators in stratified double sampling.

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