International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
Maths 2021; 6(1): 39-44

www.mathsjournal.com
Received: 09-11-2020
Accepted: 22-12-2020

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A novel algorithm with a blend of single and sequential sampling (SSS) to reduce producer and consumer menace

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Abstract
In this article, the two sampling techniques namely single and sequential sampling are blended together so as to reduce the producer and consumer menace. When the inspection or testing costs per product are high or products are destructive during testing then screening of the rejected lots may leads to high risk and hence a heavy loss may occur for the producer. If the lot is not accepted basis on the first phase of inspection then the faith of consumer on the sampling protocol may be reduced. Therefore to eradicate these shortcomings, Single and Sequential Sampling (SSS) algorithms are blended to obtain a new sampling protocol. The producer is given another chance of acceptance but at the same time item by item is recommended if the lot is not accepted during first phase of inspection. Hence producers as well as consumers are loaded with smaller menace (risks). A new algorithm to sentence the lot is given and the related measures are derived. The OC function shows slower decrease in the probability of acceptance for known quality levels and it is found that the producer risk is lower when compared to other sampling plans. Tables are constructed to select the parameters of the sampling plans.

Keywords: Single sampling, sequential, operating characteristic function, SSS

Introduction

In single sampling plan the decision about accepting or rejecting lot is taken on basis of single sample result only and hence it is the most economical plan. But at the same time based on single sample, sometimes rejection of lots may be high even for a small deviation in the quality. Hence single sampling plans faces some shortcomings and may affect the goodwill of the producer. In the case of sequential sampling plan, sample items are examined one at time and after each time inspected one of three decisions is made. viz., to accept the lot, to reject lot or to continued sampling. But inspection of one or two items cannot lead to a good decision until sequences of m units are tested. In fact sequential process ultimately terminates with probability one. Thus sequential sampling plan also faces few drawbacks. Therefore to offset these shortcomings blending of Single and Sequential sampling (SSS) inspection is employed in the quality control inspection. This gives more protection to the Producer and at the same time if the first stage fails consumer is also satisfied with the procedure of unit by unit inspection in the second stage. Therefore the blend of single and sequential sampling plans is more advantages for producer and consumer as well.

Prof. Dodge (1969) have classified acceptance sampling into four broad categories. They are lot-by-lot sampling by attributes, lot-by-lot sampling by variables, Continuous Sampling by attributes or variables and Special purpose sampling plans. Hamaker (1960), insist during designing a sampling plan one has to exert pressure on the producer when the quality of the lot is unreliable. Peach and Littauer (1946) have given tables to determine the parameters of the single sampling plan for the known producer and consumer risks. Guenther (1978) [8] has developed a search procedure for finding the parameters of sampling plans. Golub (1953) [6] has given a method for finding the sampling plans by minimizing the sum of the risks. Cameron (1952) [1], has constructed tables for selection of single sampling plans. Dodge and Romig (1959) have constructed tables for single sampling plans indexed through LTPD. Schilling (1967) [2] has contributed to new type of designing procedure.

Hald (1981) [9], has given construction of sampling plans for attribute quality characteristics. Wald (1947) [14], has developed sequential sampling protocol and determined the parameters. Suresh and Devaarul (2012), have developed mixed sampling plans for chain of lots. Devaarul and Edna (2011) [3], have developed sampling plans for costly products. Devaarul and Vijila (2018), have developed new type of Relational Chain Sampling Plans. Devaarul and Ramya (2018), have developed reliability sampling plans based on censoring of time.

Formulation and Algorithm

Let
N= Lot size
n= Sample size in the first stage.
c= Acceptance constant
d= No. of defectives in the first stage sample
d_m= no. of defectives at m^{th} stage of sequential sampling
p_1=AQL p_2=LTPD.

SSS-algorithm for sentencing the lots.

First stage

Step 1: Draw a random sample of size n from the lot. Let it be first stage.
Step 2: Count the number of defectives d.
Step 3: If $d \leq c$, then accept the lot.
Step 4: If $d > c$, do not reject the lot, but go to the second stage of inspection.

Second stage

Step 5: Now sequential sampling inspection is implemented in the second stage.
Step 6: Select and examine one item at a time. At m^{th} stage of inspection determine.

$$\lambda_m = \frac{{p_2}^{d_m}(1-p_2)^{m-d_m}}{{p_1}^{d_m}(1-p_1)^{m-d_m}},$$

m= 1,2,3,…

Where p_1, p_2 be the probabilities of getting d_m defectives in the sample of size m under AQL and LQL respectively. Let A=(1-ß)/α and B=ß/(1-α)
Step 7: If $\lambda_m \leq$ B, accept the lot.
Step 8: If $\lambda_m \geq$ A, reject the lot.
Step 9: B < λ_m < A, continue the item by item inspection and follow step5 onwards.
Step 10: For computational points of view, it would be much easier to deal with log λ_m rather than with λ_m.

Thus, sequential sampling can be restated as follows
i) If $\log \lambda_m \geq \log A$, reject the lot
ii) If $\log \lambda_m \leq \log B$, accept the lot, and
iii) If $\log B < \log \lambda_m < \log A$, continue sampling by taking one more observation
iv) $\log \lambda_m = d_m \log\left(\frac{p_1}{p0}\right) + (m-d_m)\log\left(\frac{1-p_1}{1-p0}\right)$

Hence accept the lot if

$$d_m \log\left(\frac{p_1}{p0}\right) + (m-d_m)\log\left(\frac{1-p_1}{1-p0}\right) \leq \log B$$

$$\Rightarrow d_m \leq \frac{\log B - m\log\left(\frac{1-p_1}{1-p0}\right)}{\log\left(\frac{p_1}{p0}\right) - \log\left(\frac{1-p_1}{1-p0}\right)} = a_m$$

Reject the lot if

$$d_m \log\left(\frac{p_1}{p0}\right) + (m-d_m)\log\left(\frac{1-p_1}{1-p0}\right) \geq \log A$$

$$\Rightarrow d_m \geq \frac{\log A - m\log\left(\frac{1-p_1}{1-p0}\right)}{\log\left(\frac{p_1}{p0}\right) - \log\left(\frac{1-p_1}{1-p0}\right)} = r_m$$

Continue sampling if $a_m < d_m < r_m$
For each m, acceptance number is a_m and rejection number is r_m.

Theorem

The Operating Characteristics function of a SSS plan is
$P_a(p) = P(d \leq c) + P(d > c)\, P[\lambda_m \leq B]$

Where, B=ß/(1-α)
In terms of h(p), we have
$P_a(p) = P(d \leq c) + P(d > c)\, L(p)$

Where L(p)=[A $^{h(p)}$ – 1]/[A $^{h(p)}$ – B $^{h(p)}$], A=(1-ß)/α and B=ß/(1-α)
Here, ß$_1$” = (1-α) and ß$_2$” = ß

Proof:
The lot may be accepted in following cases

Case (i)
Take a sample of size “n” from the lot
i) If d=0, accept the lot
ii) If d=1, accept the lot

If d=c, accept the lot Otherwise case (ii) is followed.

Case (ii)
Suppose if d>c, inspect item by item sequentially.
i) Check for first unit if d=0 or 1
ii) Check for second unit if d=0 or 1
…..
(…) At mth stage check if d=0 or 1
Now determine log λ_m
If log$\lambda_m \leq$ B accept the lot at any m ≥ 1.
Case (i) and case (ii) are mutually exclusive, Therefore we get
P$_a$(p)=P(d=0)+P(d=1) + ……+P(d=c) + P(d>c) P($\lambda_m \leq$ B)

In general,
P$_a$(p)=P(d≤ c)+P (d>c)L(p) … (1)

If one uses Poisson distribution then

$$P_a(p) = \sum_{x=0}^{c} e^{-np} \frac{np^x}{x!} + \left(\sum_{c+1}^{n} e^{-np} \frac{np^x}{x!}\right) [A^{h(p)} - 1]/ [A^{h(p)} - B^{h(p)}] \quad \text{… (2)}$$

$$P_a(p) = \sum_{x=0}^{c} e^{-np} \frac{np^x}{x!} + \left(\sum_{c+1}^{n} e^{-np} \frac{np^x}{x!}\right) L(p) \quad \text{… (3)}$$

Hence the proof.

Average sample number
The average sample number (ASN) is the expected value of sample size required for making a decision about the acceptance or rejection of the lot.

Theorem
The ASN of new sampling plan is as follows:
ASN = n+E(n$_m$)

$$= n + \frac{L(p) logB + [1-L(p)] \log A}{p \log(\frac{p_2}{p_1}) + (1-p) log\left(\frac{1-p_2}{1-p_1}\right)} (1-p_{a1})$$

Proof:
We know that
ASN=E(n)
Two cases, case(i) and case(ii)
Therefore ASN = E (case(i)) + E (case(ii))
= n + E(n$_m$)

$$= n + \frac{L(p) logB + [1-L(p)] \log A}{p \log(\frac{p_2}{p_1}) + (1-p) log\left(\frac{1-p_2}{1-p_1}\right)} (1-p_{a1})$$

Average outgoing quality
AOQ = p.P$_a$(p)

$$= p\{ \sum_{x=0}^{c} e^{-np} \frac{np^x}{x!} + \left(\sum_{c+1}^{n} e^{-np} \frac{np^x}{x!}\right) [A^{h(p)} - 1]/ [A^{h(p)} - B^{h(p)}]\}$$

Designing the (SSS) sampling plans indexed through two points on the OC curve
Let
β$_1$ = Probability of Acceptance of the lot at AQL.
β$_2$ = Probability of acceptance of the lot at LQL.
β$_1$' = Probability of Acceptance of the lot at AQL in the first stage inspection.
β$_2$' = Probability of acceptance of the lot at LQL in the first stage of inspection.
β$_1$” = Probability of Acceptance of the lot at AQL in the second stage of inspection

β_2'' = Probability of acceptance of the lot at LQL in the second stage of inspection

Step 1: The designing procedure of (SSS) indexed through AQL and LTPD are given

$P_a(p_1) \geq \beta_1$ and $P_a(p_2) \leq \beta_2$

$\Rightarrow \sum_{x=0}^{c} e^{-np_1} \frac{np_1^x}{x!} + \sum_{x=c+1}^{n} e^{-np_1} \frac{np_1^x}{x!}) \, L(p_1) \geq 0.95$

$\Rightarrow \sum_{x=0}^{c} e^{-np_2} \frac{np_2^x}{x!} + \sum_{x=c+1}^{n} e^{-np_2} \frac{np_2^x}{x!}) \, L(p_2) \leq 0.10$

Step 2: Using Schilling's (1967) [2] designing procedure, split the probability of acceptance for the first and second stages. Let it be β_1' and β_1'' at AQL and β_2' and β_2'' at LQL.

Step 3: Determine the first stage inspection parameters for the known probability of acceptance.

$\Rightarrow \sum_{x=0}^{c} e^{-np_1} \frac{np_1^x}{x!} \geq \beta_1'$

$\Rightarrow \sum_{x=0}^{c} e^{-np_2} \frac{np_2^x}{x!} \leq \beta_2'$

Step 4: Determine the second stage inspection parameter based on second stage probability of acceptance.

$\sum_{x=c+1}^{n} e^{-np_1} \frac{np_1^x}{x!}) \, L(p_1) \geq \beta_1''$

$\sum_{x=c+1}^{n} e^{-np_2} \frac{np_2^x}{x!}) \, L(p_2) \leq \beta_2''$

Step 5: If p_1 and p_2 are known, then by using the sequential sampling protocol one can determine the process average p.

Using the above designing procedure, tables are constructed for easy implementation in industries.

Table 1: Probability of acceptance for the known sample size n, c=2 and h for various fraction defectives

AQL		n=100	n=200	n=400
p	h	Pa(p)	Pa(p)	Pa(p)
0.001	-9	0.997839	0.997839	0.997839
0.002233	-8.5	0.998429	0.989339	0.938256
0.003082	-8	0.996119	0.975226	0.872263
0.004242	-7.5	0.990713	0.945372	0.758081
0.005817	-7	0.978652	0.887318	0.588924
0.006593	-6.8	0.970619	0.852794	0.509101
0.007468	-6.6	0.959919	0.810455	0.42607
0.008453	-6.4	0.945851	0.759726	0.343415
0.00956	-6.2	0.927614	0.700489	0.265054
0.010804	-6	0.904341	0.633267	0.194684
0.012199	-5.8	0.875147	0.559354	0.135173
0.013762	-5.6	0.839216	0.480871	0.088079

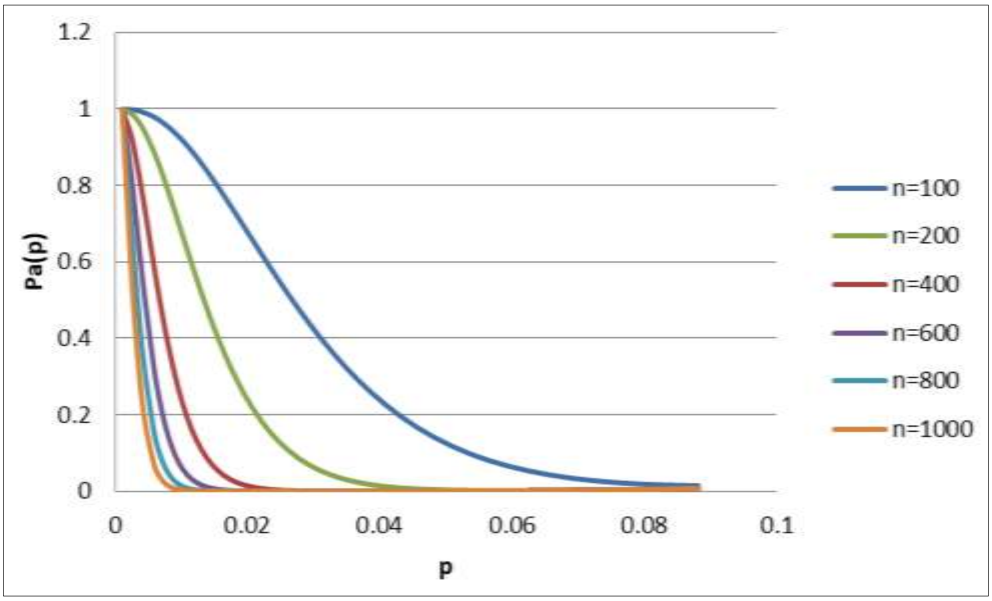

Fig 1: OC Curves for different 'n' values

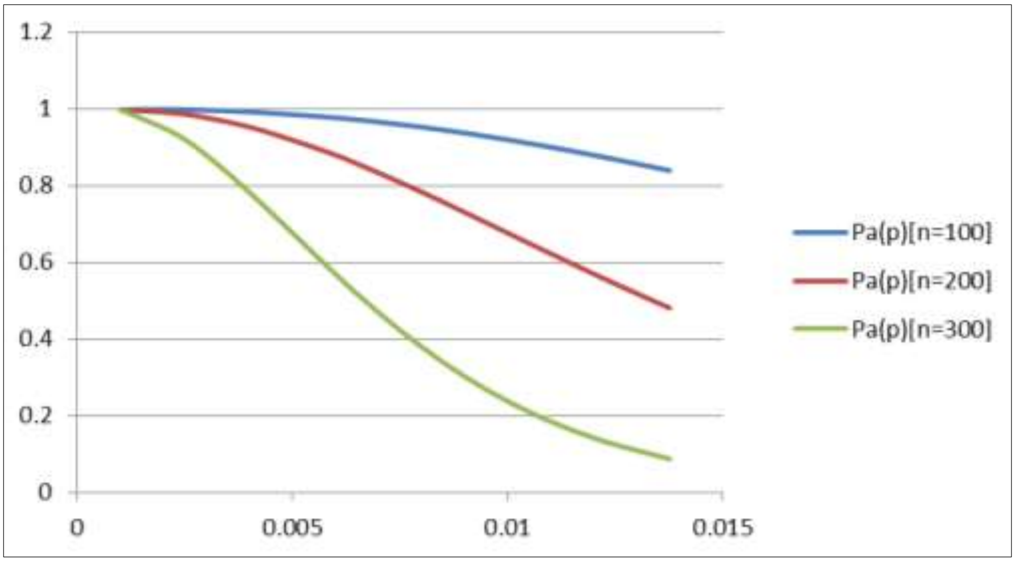

Fig 2: OC curve for various process fraction defectives

Table 2: Sample size n and sequential parameter h for the known fraction defectives with β_1=0.99, β_1'=0.80 and β_1''=0.95

p	h	C: 0	1	2	3	4	5
0.001	1	n=200	700	1400	2000	2800	3700
0.002	0.99	100	350	700	1000	1400	1850
0.003	0.98	66	233	466	666	933	1233
0.004	0.97	50	175	350	500	700	925
0.005	0.96	40	140	280	400	560	740

Table 2A: Values of p for the known h, AQL and LQL

p_1	p_2	h	p
0.001	0.01	1	0.001
0.002	0.02	0.99	0.002
0.003	0.03	0.98	0.003
0.004	0.04	0.097	0.004
0.005	0.05	0.096	0.005

Interpretation of figures

From the figure (1) and (2), it is found that Probability of acceptance has good shoulder effect when the sample size is lesser. It is advantageous for the producer. Similarly when the inspection of items is more, then OC curve is steep which indicates the sampling plan discriminate between good and bad lots. This may be appreciated and welcome by the consumer.

Illustration

A company's quality control section has to select a SSS sampling plans. The process average is 0.2% and the allowable defective in the first stage of inspection is one. Determine the parameters of SSS plans.

Solution:

Since the AQL = p = 0.2%, from table (2), the SSS plans parameters are
n = 350
c=1
h =0.99

Operating procedure of SSS plans

First stage

Step 1: Draw a random sample of size n=350 from the lot. Let it be first stage.
Step 2: Count the number of defectives d.
Step 3: If $d \le 1$, then accept the lot.
Step 4: If d >1, do not reject the lot, but go to the second stage of inspection.

Second stage

Step 5: Now sequential sampling inspection is implemented in the second stage.
Step 6: Select and examine one item at a time. At mth stage of inspection, determine

$$\lambda_m = \frac{p_2{}^{d_m}(1-p_2)^{m-d_m}}{p_1{}^{d_m}(1-p_1)^{m-d_m}},$$

m= 1,2,3,…

Where p_1, p_2 be the probabilities of getting d_m defectives in the sample of size m under AQL and LQL respectively.
Let
$\alpha = \beta_1" = 0.10$ and $ß = \beta_2" = 0.05$ in the second stage of inspection
Let
A=(1-ß)/α = 0.95 /0.10 = 9.5
And
B=ß/(1-α) = 0.05/0.90 = 0.05
Since h = 0.99, from table (2A), the corresponding AQL and LQL are p_1= 0.002, p_2 =0.02
Suppose if d =0 during sequential sampling inspection, when m=1,

Then
λ_m = (1-0.02)/ (1-0.002) = 0.9819

Step 7: Since, $0.05 < \lambda_m < 9.5$, continue the item by item inspection and follow step5 onwards until the lot is either accepted or rejected.

Conclusion

In this paper, a new sampling algorithm by blending single and sequential sampling techniques is developed. The proposed novel algorithm sentence the lot very effectively with an ease of producer risk. This plan also reduces the burden of consumer since item by item inspection is followed if the first stage inspection fails to accept the lot. Based on the novel algorithm, the efficiency measures such as operating characteristics function, Average Sample Number, Average Total Inspection and its Average Outgoing Quality are derived. When the quality deteriorates the probability of acceptance of the lot is lesser compared to ordinary Single Sampling Plans. This gives more pressure on the producer to maintain the quality. But at the same circumstances, if the lot is not accepted based on the first stage sample, then another chance is given for the producer through item by item inspection. Hence this SSS sampling plan is advantage for both producer and a well as consumer. Since the sampling protocol is simple, it is can be implemented in modern industries.

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