International Journal of Statistics and Applied Mathematics 2021; 6(1): 153-156

International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
Maths 2021; 6(1): 153-156

www.mathsjournal.com
Received: 30-10-2020
Accepted: 28-12-2020

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Quasi-Bayesian estimation of time to test transform under asymmetric loss functions

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Abstract
In the present paper, Quasi-Bayesian estimation of Total Time on Test transform (TTT) for the Lomax distribution. A two parametric Lomax distribution is considered for the analysis. Estimators are obtained by using extension of Jeffrey's prior and Gamma prior under Entropy loss function and Precautionary loss function. Both the classical and Bayes estimators have been developed.

Keywords: entropy loss function, informative and non-informative priors, Lomax distribution, precautionary loss function, Quasi-Bayesian estimation, total time on test transform

1. Introduction

In the 1970s the total time on test plot, and its theoretical counterpart, the scaled TTT transform, was presented by R. E. Barlow *et al.* as a tool for model identification based on data representing lives of non-repairable equipment. Later, Klefsjo (1982) [4] presented some relationship between the TTT transform and other ageing properties (with their duals) of random variable. Chacko *et al.* (2010) [2] discussed use of TTT transform in identifying failure rate model of semi-Markov reliability system. Total Time on Test transform plots are ALSO useful for analyzing non-negative data. The plots help in choosing a mathematical model for the data and provide information about failure rate. Also incomplete data can be analyzed and there is a theoretical basis for such an analysis. The TTT-Transform has also been found quite useful in theoretical applications such as looking for test statistics for particular purposes and to study their power. Kochar *et al.* (2002) [5] defined TTT transform order and Shaked and Shanthikumar (2007) [8] studied it explicitly. Nair *et al.* (2008) [6] provided applications of TTT of order n in reliability analysis. The scaled total time on test (TTT) transform of F is defined as

$$\emptyset(t) = \frac{1}{\mu}\int_0^{F^{-1}(t)} \overline{F}(x)dx \text{ for } 0 \le t \le 1 \tag{1.1}$$

$$\overline{F} = 1 - F,\ \mu = \int_0^{\infty} \overline{F}(x)dx \text{ and } F^{-1}(y) = \inf\{x: F(x) \ge y\} \text{ for } 0 \le y \le 1$$

The main common types of Pareto distribution are known as Pareto Type I, II, III, IV, and Feller Pareto distributions. One of the popular hierarchies of Pareto distribution is Pareto Type II which has been named as Lomax distribution. Lomax distribution is an advantageous lifetime distribution in reliability analysis. The applicability of the Lomax distribution is not restricted only to reliability field, but it has broad application in the field of economics, actuarial statistics, queuing problems, biological sciences, etc. Lomax distribution has been applied in a variety of fields such as engineering and reliability and life testing. Lomax distribution has been used as an alternative to the exponential, gamma and Weibull distributions for heavy tailed data by Bryson (1974) [1]. Golaup *et al.* (2005) [3] introduced the size distribution of computer files on servers using Lomax distribution. Nasiri and Hosseini (2012) [7] also studied Lomax distribution regarding the MLE and various Bayesian estimation

based on record values.
The probability density function of Lomax distribution is given by

$$f(x{:}\theta,\lambda) = \frac{\theta\lambda^{\theta}}{(\lambda+x)^{\theta+1}}\ x,\theta,\lambda > 0 \tag{1.2}$$

Where θ and λ are shape and scale parameters respectively.
For the above model, the TTT simplifies to

$$\emptyset(t) = 1-(1-t)^{\frac{\theta-1}{\theta}} \tag{1.3}$$

2. Quasi-bayesian estimation

The quasi-likelihood function was introduced by Wedderburn (1974), to be used for estimating the unknown parameters in generalized linear models. If the underlying distribution comes from a natural exponential family the maximum quasi-likelihood estimate maximizes the likelihood function and so it has full asymptotic efficiency; under more general distributions there is some loss of efficiency.

Wedderburn defined the quasi-likelihood function as

$$Q(x,\mu) = \int_{\mu} \frac{x-\mu}{V(\mu)} d\mu + o(x) \tag{2.1}$$

Where $\mu = E(x), V(\mu) = Var(x)$ and $o(x)$ is some function of x only. The variance assumption is generalized to $Var(x) = \phi Var(\mu)$ where the variance function V(.) is assumed to be known and the parameter ϕ may be unknown.
For a sample $\underline{x} = (x_1, x_2, \dots x_n)$ of size n from (1.2), the quasi-likelihood function simplifies to

$$Q(x,\theta,\lambda) = \log\left[\frac{\theta-1}{\lambda}\right]^{n} - \left[\frac{\theta-1}{\lambda}\right] v, \text{ Where } v = \sum_{i=1}^{n} x_i \tag{2.2}$$

The natural exponent of $Q(x,\theta,\lambda)$ is the likelihood function and is given as

$$l(\underline{x}|\theta,\lambda) = \left[\frac{\theta-1}{\lambda}\right]^{n}.exp\left[-\left(\frac{\theta-1}{\lambda}\right)v\right] \tag{2.3}$$

In this section, we derive the quasi Bayesian estimates of TTT under The extension of Jeffrey's prior and conjugate prior by using different loss function.

2.1 Quasi-Bayesian estimation of TTT under The Extension of Jeffrey's prior by using different loss function

i) λ is known

Using the likelihood function as given in (2.3) and the extended Jeffrey's prior, the posterior density of θ is derived as follows:

$$f(\theta,\lambda|\underline{x}) \propto \left[\frac{\theta-1}{\lambda}\right]^{n}.exp\left[-\left(\frac{\theta-1}{\lambda}\right)v\right]\frac{1}{\theta^{2c}} \tag{2.4}$$

Let $\emptyset(t)$ be a parameter itself denoted by $\emptyset$ for simplicity. Replacing θ in (2.4) in terms of $\emptyset$ by that (1.3), we get the posterior of the TTT as

$$f(\emptyset|\underline{x}) = \frac{{R_{\emptyset}}^{2(1-c)}(1-\emptyset)^{-1}\left(\frac{R_{\emptyset}-1}{\lambda}\right)^{n}.exp\left[-\left[\frac{R_{\phi}-1}{\lambda}\right]v\right]}{C_1(t,0)} \tag{2.5}$$

$$\text{Where } C_1(t,d) = \int_0^t \emptyset^d {R_{\emptyset}}^{2(1-c)}(1-\emptyset)^{-1}\left(\frac{R_{\emptyset}-1}{\lambda}\right)^{n}.exp\left[-\left[\frac{R_{\phi}-1}{\lambda}\right]v\right]d\emptyset \tag{2.6}$$

$$\text{With } R_{\emptyset} = \left[1-\frac{log[1-\emptyset(t)]}{\log(1-t)}\right]^{-1} \tag{2.7}$$

The symbol C with suffixes stands for the normalizing constants.

- The Quasi Bayes estimator of TTT under Entropy Loss Function is given by

$$\hat{\emptyset}_{QBE1} = \left[E(\emptyset^{-1}|\underline{x})\right]^{-1} = \frac{C_1(0)}{C_1(-1)} \tag{2.8}$$

- The Quasi Bayes estimator of TTT under Precautionary Loss function is given by

$$\hat{\emptyset}_{QBP1} = \sqrt{E(\emptyset^2|\underline{x})} = \left[\frac{C_1(2)}{C_1(0)}\right]^{\frac{1}{2}} \tag{2.9}$$

ii) λ is unknown

Using the likelihood function as given in (2.3) and the extended Jeffrey's prior, the posterior density of θ is derived as follows,

$$f(\theta/\underline{x}) \propto \frac{1}{\theta^{2c}} \int_0^\infty \left[\frac{\theta-1}{\lambda}\right]^n . exp\left[-\left(\frac{\theta-1}{\lambda}\right) v\right] d\lambda \qquad (2.10)$$

Replacing θ in (2.4) in terms of $\emptyset$ by that (1.2), we get the posterior of the TTT as

$$f(\emptyset|\underline{x}) = \frac{{R_\emptyset}^{2(1-c)} (1-\emptyset)^{-1} \int_0^\infty \left(\frac{R_\emptyset-1}{\lambda}\right)^n .exp\left[-\left[\frac{R_\phi-1}{\lambda}\right] v\right] d\lambda}{C_2(t,0)} \qquad (2.11)$$

Where $C_2(t,d) = \int_0^t \int_0^\infty \emptyset^d {R_\emptyset}^{2(1-c)} (1-\emptyset)^{-1} \left(\frac{R_\emptyset-1}{\lambda}\right)^n . exp\left[-\left[\frac{R_\phi-1}{\lambda}\right] v\right] d\lambda\, d\emptyset$ (2.12)

- The Quasi Bayes estimator of TTT under Entropy Loss Function is given by

$$\hat{\emptyset}_{QBE2} = \left[E\left(\emptyset^{-1}|\underline{x}\right)\right]^{-1} = \frac{C_2(0)}{C_2(-1)} \qquad (2.13)$$

- The Quasi Bayes estimator of TTT under Precautionary Loss function is given by

$$\hat{\emptyset}_{QBE2} = \sqrt{E\left(\emptyset^2|\underline{x}\right)} = \left[\frac{C_2(2)}{C_2(0)}\right]^{\frac{1}{2}} \qquad (2.14)$$

2.2 Quasi-Bayesian estimation of TTT under Conjugate prior by using different loss function

i) λ is known

Here we suggest the conjugate prior distribution for the parameters and is given by

$$g(\theta) = \frac{\tau^r}{\Gamma r} \theta^{r-1} e^{-\theta\tau} \qquad (2.15)$$

Combining (2.3) and (2.15), the joint posterior density is obtained as

$$f(\theta/\underline{x}) \propto \frac{\tau^r}{\Gamma r} \theta^{r-1} \left[\frac{\theta-1}{\lambda}\right]^n . exp\left[-\theta\tau - \left(\frac{\theta-1}{\lambda}\right) v\right] \qquad (2.16)$$

Replacing θ in (3.3) in terms of $\emptyset$ by that (1.2), we get the posterior of the TTT as

$$f(\emptyset|\underline{x}) = \frac{{R_\emptyset}^{r+1} (1-\emptyset)^{-1} \left(\frac{R_\emptyset-1}{\lambda}\right)^n .exp\left[-R_\emptyset\tau - \left[\frac{R_\phi-1}{\lambda}\right] v\right]}{C_3(t,0)} \qquad (2.17)$$

Where $C_3(t,d) = \int_0^t \emptyset^d {R_\emptyset}^{r+1} (1-\emptyset)^{-1} \left(\frac{R_\emptyset-1}{\lambda}\right)^n . exp\left[-R_\emptyset\tau - \left[\frac{R_\phi-1}{\lambda}\right] v\right] d\emptyset$ (2.18)

- The Quasi Bayes estimator of TTT under Entropy Loss Function is given by

$$\hat{\emptyset}_{QBE3} = \left[E\left(\emptyset^{-1}|\underline{x}\right)\right]^{-1} = \frac{C_3(0)}{C_3(-1)} \qquad (2.19)$$

- The Quasi Bayes estimator of TTT under Precautionary Loss function is given by

$$\hat{\emptyset}_{QBP3} = \sqrt{E\left(\emptyset^2|\underline{x}\right)} = \left[\frac{C_3(2)}{C_3(0)}\right]^{\frac{1}{2}} \qquad (2.20)$$

ii) λ is unknown

In the inference problem considered here we assume that the parameter λ is unknown and in this case the joint prior distribution for the parameters as

$$g(\theta,\lambda) = \frac{\tau^r}{\Gamma r} \frac{\theta^{r-1} e^{-\theta\tau}}{\lambda} \qquad (2.21)$$

Combining (2.3) and (2.21), the joint posterior density is obtained as

$$f(\theta/\underline{x}) \propto \frac{\tau^r}{\Gamma r} \int_0^\infty \frac{\theta^{r-1}}{\lambda} \left[\frac{\theta-1}{\lambda}\right]^n . exp\left[-\theta\tau - \left(\frac{\theta-1}{\lambda}\right) v\right] d\lambda \qquad (2.22)$$

Replacing θ in (2.22) in terms of ∅ by that (1.2), we get the posterior of the TTT as

$$f(\emptyset\,|\underline{x}) = \frac{{R_\emptyset}^{r+1}\,(1-\emptyset)^{-1}\int_0^\infty\left(\frac{R_\emptyset-1}{\lambda}\right)^n exp\left[-R_\emptyset\tau-\left[\frac{R_\emptyset-1}{\lambda}\right]v\right]d\lambda}{C_4(t,0)} \quad (2.23)$$

Where $C_4(t,d) = \int_0^t\int_0^\infty \emptyset^d {R_\emptyset}^{r+1}\,(1-\emptyset)^{-1}\left(\frac{R_\emptyset-1}{\lambda}\right)^n exp\left[-R_\emptyset\tau-\left[\frac{R_\emptyset-1}{\lambda}\right]v\right]d\lambda\, d\,\emptyset$ (2.24)

- The Quasi Bayes estimator of TTT under Entropy Loss Function is given by

$$\widehat{\emptyset}_{QBE4} = \left[E\left(\emptyset^{-1}|\underline{x}\right)\right]^{-1} = \frac{C_4(0)}{C_4(-1)} \quad (2.25)$$

- The Quasi Bayes estimator of TTT under Precautionary Loss function is given by

$$\widehat{\emptyset}_{QBP4} = \sqrt{E\left(\emptyset^2|\underline{x}\right)} = \left[\frac{C_4(2)}{C_4(0)}\right]^{\frac{1}{2}} \quad (2.26)$$

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