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Analysis of fluid flow through channels with slip Boundary: Mathematical study

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Abstract
In this paper, we present explanatory solutions for transient flow of Newtonian fluid through miniature channels with Navier slip limit. The induction of the arrangements depends on Fourier arrangement development in space. We at that point research the impact of the slip boundaries on the marvels of the transient flow and show the transient weight field and speed profile at different moments of time.

Keywords: Component, transient flow, slip boundary, rectangular micro channels

Introduction

In the last ten years, one of the significant examination centers worldwide has been around the investigation of materials at miniature and nanoscales (Wu, *et al*., 2008) [3]. The advances here have prompted the improvement of many designing gadgets and frameworks in microscale and nanoscale. These gadgets and frameworks typically include liquid course through microchannel, alluded to as micro flows (Bourlon *et al*., 2007; Huang *et al*., 2007; Wu, 2008) [1-3]. Application models incorporate medication conveyance frameworks Su and organic detecting and energy change gadgets. As the practical attributes of the framework relies upon the stream conduct of liquid in the framework, the investigation of miniature streams is drawing in increasingly more consideration from the examination networks to infer a superior comprehension of the system of microflows and grow better models (Wu and Wiwatanapataphee, 2008) [3].

The administering field conditions for the progression of incompressible Newtonian liquids are the incompressible coherence condition and the Navier–Feeds conditions. Likewise, a limit condition must be forced on the field conditions. The no-slip condition is generally utilized. Be that as it may, it is a theory instead of a condition reasoned from material science standard, and accordingly its legitimacy has been constantly bantered in the logical writing Wu and Wiwatanapataphee, (2008) [3]. Shifts confirmations of slip stream of a liquid on a strong surface have been accounted for. For instance, contemplated the progression of polymer arrangements in permeable media and demonstrated that the evident thickness of the liquids close to the divider is lower than that in the mass and thus the liquids can display the marvel of obvious slip on the divider.

Different examinations have been made to contemplate stream issues of Newtonian and non-Newtonian liquids with Navier slip limit condition (Lee *et al*., 2007; Yousif and Melka, 1997) [2]. A few endeavors have likewise been made to infer elective formulae for the assurance of the slip length Yang and Zhu, (2006) [4]. Albeit accurate and mathematical answers for different stream issues of Newtonian liquids under the no-slip presumption have been acquired and are accessible in writing Wu. B, not many precise answers for the slip case are accessible in writing. As of late, some consistent state and transient slip answers for the courses through a line, a channel and an annulus have been acquired, (Yang, K.Q. Zhu, 2006) [4]. As of late, Wu *et al*. contemplated pressure angle driven transient progressions of incompressible Newtonian fluids in miniature annuals under a Navier slip limit condition.

They utilize Fourier arrangement as expected and Bessel capacities in space to discover careful arrangements Wu and Wiwatanapataphee, (2008) [3]. In this paper, we present another outcome for the transient progression of Newtonian liquids in rectangular miniature cylinders with a slip limit condition. The remainder of the paper is coordinated as follows. In the accompanying segment, we initially characterize the issue and afterward present its numerical definition. In Area 3, we present the new answer for the speed field and show its variety with time and its profile over the channel cross-segment.

I. Problem Description and Mathematical Formulation

We consider the flow of an incompressible Newtonian fluid through a rectangular miniature cylinder with the z-axis being the pivotal way. The differential conditions overseeing the flow incorporate the coherence condition and the Navier–Stirs conditions as follows

$$\frac{\partial U_j}{\partial x_j} = 0 \quad \text{(i)}$$

$$p\left(\frac{\partial y_j}{\partial x_j} + y_i\,\frac{\partial y_j}{\partial x_i}\right) = -\frac{\partial p}{\partial x_j} + \mu\frac{\partial^2 y}{\partial x_i \partial x_i} + pg_j,\ [i = 1,2,3;\ j = 1,2,3] \quad \text{(ii)}$$

where p and y_i are respectively the fluid pressure and velocity vector, g_j is the gravitational acceleration, ρ and μ are respectively the fluid density and viscosity and x_i denotes coordinates. As the flow is axially symmetric, the velocity components in the *x* and *y* directions vanish, namely

$y_1 = y_x = 0$ and $y_2 = y_y = 0$. Thus the continuity equation (i) becomes

$$\frac{\partial y_3}{\partial x_3} = \frac{\partial y_z}{\partial z} = 0$$

Which gives rise to $y_3 = v = v(x, y, t)$

As the flow is horizontal, $g_3 = g_z = 0$, and hence eq. (ii) becomes

$$p\left(\frac{\partial v}{\partial t}\right) = \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) - \frac{\partial p}{\partial z}$$

In this work, we consider the liquid stream driven by the weight field with a pressure angle q(t) which can be communicated by a Fourier arrangement, to be specific

$$\frac{\partial p}{\partial z} = q(t) = a_0 + \sum_{n=1}^{\infty} \left[a_n \cos\cos(n\omega t) + b_n \sin\sin(n\omega t)\right] \quad \text{(iii)}$$

As the issue is pivotally symmetric, we just need to think about a quadrant of the cross-segment in the calculation.

By applying the Navier slip conditions in the primary quadrant of the rectangular cross segment, as in the paper by Wu *et al*., (2008) [3] for each time t, we have

$$\frac{\partial v}{\partial y}(x,0) = 0;\ 0 \le x \le a$$

$$\frac{\partial v}{\partial x}(0,y) = 0;\ 0 \le y \le b$$

$$v(x,b) + l\frac{\partial v}{\partial y}(x,b) = 0;\ 0 \le x \le a$$

$$v(a,y) + l\frac{\partial v}{\partial x}(a,y) = 0;\ 0 \le y \le b \quad \text{(iv)}$$

I. Exact Solution for the Transient Velocity Field

Consider the unsteady Navier Stokes equation

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} - \frac{p\partial v}{\mu \partial t} = \frac{1}{\mu}\frac{\partial p}{\partial z} \quad \text{(v)}$$

If v_n is the solution of eq. (v) for $\frac{\partial p}{\partial z} = C_n e^{in\omega t}$ then the complete of eq. (v) for

$\frac{\partial p}{\partial z} = R_e \sum_{n=1} C_n e^{in\omega t} = \sum_{n=1} a_n \cos\cos(n\omega t) + b_n \sin(n\omega t)$ is $v = \sum_{n=1} R_e(v_n)$.

Through lengthy derivation we obtain the following exact solution for eq. (v)

Subject to conditions γ;

$$v(x,y,t) = \sum_{n=1} R_e\left[e^{in\omega t}\left(A_n \cosh\cosh(\gamma_n x)\ \cos\cos(k_n y) + B_n \cos(\alpha_n x) \cosh\cosh(\beta_n y) - \frac{C_n}{in\omega p}\right)\right]$$

Where

$\gamma_n = \sqrt{k_n{}^2 + iq_n}, q_n = \frac{n\omega p}{\mu}$

$$\beta_n = \sqrt{a_n{}^2 + iq_n}$$

$$A_n = \frac{2}{b}\int_0^b \quad R_e \left[\frac{C_U cosh(k_n y)}{cosh\ cosh\ (\gamma_n a)\ + l\gamma_n sinh(\gamma_n a)}\right] dy$$

$$B_n = \frac{2}{a}\int_0^b \quad R_e \left[\frac{C_v cosh(\alpha_n x)}{cosh\ cosh\ (\beta_n b)\ + l\beta_n sinh(\beta_n b)}\right] dx$$

$$C_U = C_V = \sum_{n=1}^{\infty} \quad R_e\left(\frac{C_n}{in\omega p}\right)$$

To demonstrate the velocity field, we present results for the case $\frac{\partial p}{\partial z} = b_1 sin(\omega t)$ with $b_1 = 5. a = b = 1$ and l = 0.1, ω = 0.2 for μ = 1.1 and p = 30.

Fig. 1 shows the variation of the driving pressure gradient in one cycle fig.2 shows the variation of axial velocity along the x-axis at varies instants of times.

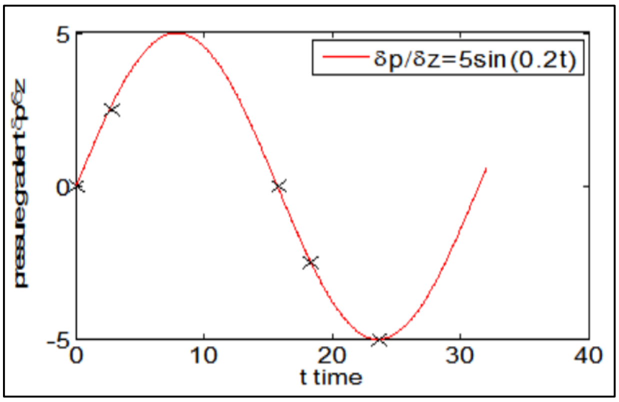

Fig 1: Pressure gradient driving the flow of the fluid

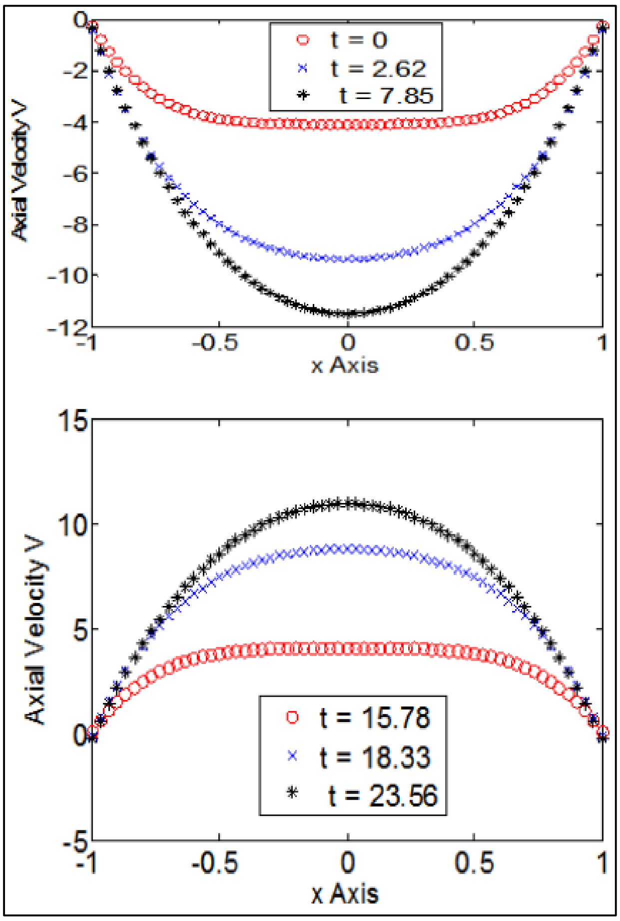

Fig 2: Variation of velocity profile along the *x*-axis at varies instants of times during one cycle.

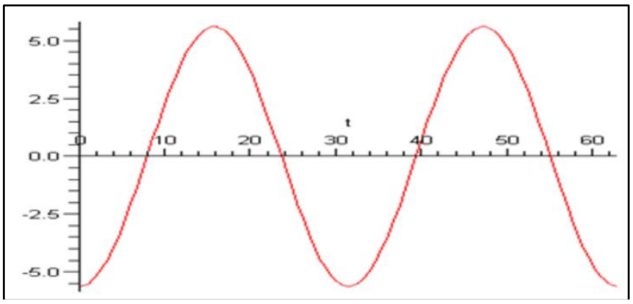

Fig 3: Variation of axial velocity with time at the point $(x, y) = (0,0)$

Conclusion

In this paper we present a careful solution for transient flow of incompressible Newtonian fluid in rectangular miniature tube with a Navier slip condition on the limit. We likewise show the speed profile in the cross area for the instance of $\partial p/\partial z = 5\sin(0.2t)$.

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