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Study of MHD boundary layer flow of a casson fluid due to an exponentially stretching sheet with radiation effect

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Abstract
The effect of radiation on magneto-hydrodynamic (MHD) boundary layer flow of a Casson Fluid over an exponentially stretching sheet was studied. Casson Fluid model is used to characterize the non-Newtonian fluid. The governing system of partial differential equations was transformed into ordinary differential equations before being solved numerically. The effects of the governing parameters on the flow field and heat transfer characteristics were obtained and discussed. It was found that the local heat transfer rate at the surface decreases with increasing values of the magnetic and radiation parameters.

Keywords: exponentially stretching, casson fluid, thermal radiation, MHD

Introduction
The boundary layer flow on a continuous stretching sheet has attracted considerable attention during the last few decades due to its numerous applications in industrial manufacturing processes such as hot rolling, wire drawing, glass-fiber and paper production, drawing of plastic films, metal and polymer extrusion and metal spinning. Both the kinematics of stretching and the simultaneous heating or cooling during such processes has a decisive influence on the quality of the final products. The study of laminar flow and heat transfer occurring over a stretching sheet in a viscous fluid is of considerable interest because of their ever increasing industrial applications and important bearings on several technological processes. Most of the available literature deals with the study of boundary layer flow over a stretching sheet where the velocity of stretching sheet is assumed to be linearly proportional to the distance from the fixed origin. However, it is believed that Gupta (1997) [6] stretching of plastic sheet may not necessarily be linear. A few years later, several researchers like Magyari and Keller (1999) [2, 18], Elbashbeshy (2001) [4], Partha *et al*. (2005) [19] focused on heat and mass transfer on boundary layer flow due to the presence of an exponentially stretching sheet under different thermo-physical conditions. The radiative effects have important applications in physics and engineering processes. The radiations due to heat transfer effects on different flows are very important in space technology and high temperature processes. But very little is known about the effects of radiation on the boundary layer. Thermal radiation effects may play an important role in controlling heat transfer in polymer processing industry where the quality of the final product depends, to some extent to the heat controlling factors. High temperature plasmas, cooling of nuclear reactors, liquid metal fluids, and power generation systems are some important applications of radiative heat transfer from a vertical wall to conductive gray fluids. Convective heat transfer plays a vital role during the handling and processing of non-Newtonian fluid flows. Mechanics of non-Newtonian fluid flows present a special challenge to engineers, physicists, and mathematicians. Because of the complexity of these fluids, there is not a single constitutive equation which exhibits all properties of such non-Newtonian fluids. In the process, a number of non-Newtonian fluid models have been proposed. Amongst these, the fluids of viscoelastic type have received much attention.

The non-Newtonian fluids are mainly classified into three types, namely differential, rate, and integral. The simplest subclass of the rate type fluids is the Maxwell model which can predict the stress relaxation. This rheological model, also, excludes the complicated effects of shear-dependent viscosity from any boundary layer analysis. There is another type of non-Newtonian fluid known as Casson fluid introduced by Casson fluid exhibits yield stress. It is well known that Casson fluid is a shear thinning liquid which is assumed to have an infinite viscosity at zero rate of shear, a yield stress below which no flow occurs, and a zero viscosity at an infinite rate of shear, i.e., if a shear stress less than the yield stress is applied to the fluid, it behaves like a solid, whereas if a shear stress greater than yield stress is applied, it starts to move. The examples of Casson fluid are of the type are as follows: jelly, tomato sauce, honey, soup, concentrated fruit juices, etc. Human blood can also be treated as Casson fluid. Due to the presence of several substances like, protein, fibrinogen, and globulin in aqueous base plasma, human red blood cells can form a chainlike structure, known as aggregates or rouleaux. If the rouleaux behave like a plastic solid, then there exists a yield stress that can be identified with the constant yield stress in Casson's fluid. Casson fluid can be defined as a shear thinning liquid which is assumed to have an infinite viscosity at zero rate of shear, a yield stress below which no flow occurs, and a zero viscosity at an infinite rate of shear.

The purpose of this present work is to extend the flow and heat transfer analysis in boundary layer flow of a Casson fluid over an exponentially stretching sheet. Combined effects of suction/blowing and thermal radiation are investigated. Using similarity transformations, a third order ordinary differential equation corresponding to the momentum equation and a second order differential equation corresponding to the heat equation are derived. Numerical calculations up to desired level of accuracy were carried out for different values of dimensionless parameters of the problem. The results have been discussed thoroughly. It is found that the flow field and heat transfer are influenced appreciably by other parameters in presence of suction or injection at the wall. Finally, estimation of skin friction which is very important from the industrial application points of view is also discussed

Problem Formulation

Consider a steady two-dimensional flow of an incompressible viscous and electrically conducting fluid caused by a stretching sheet, which is placed in a quiescent ambient fluid of uniform temperature T , as shown in Figure 1. We consider that a variable magnetic field *B(x)* is applied normal to the sheet and that the induced magnetic field is neglected, which is justified for MHD flow at small magnetic Reynolds number. Under the usual boundary layer approximations, the flow and heat transfer with the radiation effects are governed by the following equations:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \qquad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B^2}{\rho}u, \qquad (2)$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}, \qquad (3)$$

Where s^* and k^* are the Stefan-Boltzmann constant and the mean absorption coefficient, respectively. This approximation is valid at points optically far from the boundary surface, and, it is good only for intensive absorption, which is for an optically thick boundary layer. It is assumed that the temperature differences within the flow such that the term T^4 may be expressed as a linear function of temperature. Hence, expanding T^4 in a Taylor series about T and neglecting higher-order terms gives:

$$T=T_w=T_\infty+T_0e^{x/(2L)} \quad \text{at } y=0,$$
$$u\to 0,\ T\to T_\infty \quad \text{as } y\to\infty, \qquad (4)$$

$$q_r=-\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y}, \qquad (5)$$

$$T^4\approx 4T_\infty^3T-3T_\infty^4. \qquad (6)$$

Using Eqs. (5) and (6), Eq. (3) reduces to:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{16\sigma^* T_\infty^3}{3\rho c_p k^*}\frac{\partial^2 T}{\partial y^2}. \quad (7)$$

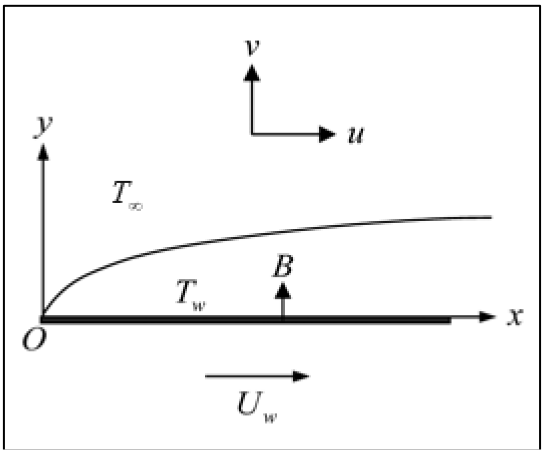

Fig 1: Physical model and coordinate system

To obtain similarity solutions, it is assumed that the magnetic field *B(x)* is of the form:

$$B=B_0 e^{x/(2L)}, \quad (8)$$

Where *B*0 is the constant magnetic field. The continuity equation (1) is satisfied by introducing a stream function y such that:

$$u=\frac{\partial \psi}{\partial y} \quad \text{and} \quad v=-\frac{\partial \psi}{\partial x}. \quad (9)$$

Where *u* and *v* are the velocities in the *x*- and *y*-directions, respectively, *r* is the fluid density, *v* the kinematic viscosity, *k* the thermal conductivity, *cp* the specific heat, *T* the fluid temperature in the boundary layer and *qr* is the radiative heat flux. The boundary conditions are given by:

$$u=U_w=U_0 e^{x/L}, \quad v=0,$$

The momentum and energy equations can be transformed into the corresponding ordinary differential equations by the following transformation (Sajid & Hayat 2008) [21]:

$$\eta=\left(\frac{U_0}{2vL}\right)^{1/2} e^{\frac{x}{2L}} y,$$

$$u=U_0 e^{x/L} f'(\eta), \quad v=-\left(\frac{vU_0}{2L}\right)^{1/2} e^{\frac{x}{2L}}\left(f(\eta)+\eta f'(\eta)\right),$$

$$T=T_\infty+T_0 e^{x/(2L)}\theta(\eta), \quad (10)$$

Where h is the similarity variable, *f(h)* is the dimensionless stream function, *q(h)* is the dimensionless temperature Where *U*0 is the reference velocity, *T*0 the reference and primes denote differentiation with respect to *h*. The transformed ordinary differential equations are:

Most of the effort in understanding fluid radiation is devoted to the derivation of reasonable simplifications. One of these simplifications was made who assumed that the fluid is in the optically thin limit and, accordingly, the fluid does not absorb its own radiation but it only absorbs radiation emitted by the boundaries. For an optically thick gas, the gas self-absorption rises and the situation become difficult. However, the problem can be simplified by using the Rosseland approximation (Rosseland 1936 [20] which simplifies the radiative heat flux as:

$$f''' + ff'' - 2f'^2 - Mf' = 0, \tag{11}$$

$$\left(1 + \frac{4}{3}K\right)\theta'' + \Pr\left(f\theta' - f'\theta\right) = 0, \tag{12}$$

$$M = \frac{2\sigma B_0^2 L}{\rho U_0}$$

$$\Pr = \frac{\rho v c_p}{k}$$

in which is the magnetic parameter, $K = \frac{4\sigma^* T_\infty^3}{k^* k}$ the radiation parameter and is the Prandtl number. The transformed boundary conditions are:

$$f(0) = 0,\ f'(0) = 1,\ \theta(0) = 1,$$
$$f'(\eta) \to 0,\ \theta(\eta) \to 0 \quad \text{as} \quad \eta \to \infty. \tag{13}$$

The main physical quantities of interest are the skin friction coefficient $f''(0)$ and the local Nusselt number $-q'(0)$, which represent the wall shear stress and the heat transfer rate at the surface, respectively. Our task is to investigate how the values of $f''(0)$ and $-q'(0)$ vary with the radiation parameter K, magnetic parameter M and Prandtl number Pr.

Results and Discussion

The system of ordinary differential equations (11) – (13) has been solved numerically using the Keller box method described in the book by Cebeci and Bradshaw (1988) [9]. This method has been successfully used by the present author to solve various boundary layer problems along with the concept of similarity solution (see Ishak (2009 a, b) [14, 15] and Ishak *et al*. (2008 a, b, 2009 a, b)) [14, 15]. Comparison with the existing results from the literature shows a favourable agreement, as presented in Table 1.

The velocity profiles for different values of the magnetic parameter M presented in Figure 2 show that the rate of transport is considerably reduced with the increase of M. It clearly indicates that the transverse magnetic field opposes the transport phenomena. This is because the variation of M leads to the variation of the Lorentz force due to the magnetic field, and the Lorentz force produces more resistance to the transport phenomena. We note that the Prandtl number Pr and the radiation parameter K have no influence on the flow field, which is clear from Equation (11). The velocity gradient at the surface $f''(0)$ which represents the surface shear stress increases with increasing M. Thus, the magnetic parameter M acts as a controlling parameter to control the surface shear stress.

The temperature profiles for different values of M, K and Pr with other parameters are fixed to unity are presented in Figures 3, 4 and 5, respectively. Figures 2 to 5 show that the far field boundary conditions are satisfied asymptotically, thus supporting the accuracy of the numerical results obtained. It is evident from Figures 3 to 5 that the thermal boundary layer thickness increases as M and K increase but opposite trends are observed for increasing values of Pr. This results in decreasing manner of the local Nusselt number $-q'(0)$, which represents the heat transfer rate at the surface, with increasing M and K but opposite trends are observed for increasing values of Pr. This is because, when Pr increases, the thermal diffusivity decreases and thus the heat is diffused away from the heated surface more slowly and in consequence increase the temperature gradient at the surface.

Table 1: Values of $q'(0)$ for different values of K, M and Pr

K	*M*	Pr	Magyariand Keller (1999) [2, 18]	El Azi (2009)	Bidin and Nazar (2009)	Present results
0	0	1	-0.954782	-0.954785	-0.9548	-0.9548
		2			-1.4714	-1.4715
		3	-1.869075	-1.869074	-1.8691	-1.8691
		5	-2.500135	-2.500132		-2.5001
		10	-3.660379	-3.660372		-3.6604
	1	1				-0.8611
1	0				-0.5315	-0.5312
	1					-0.4505

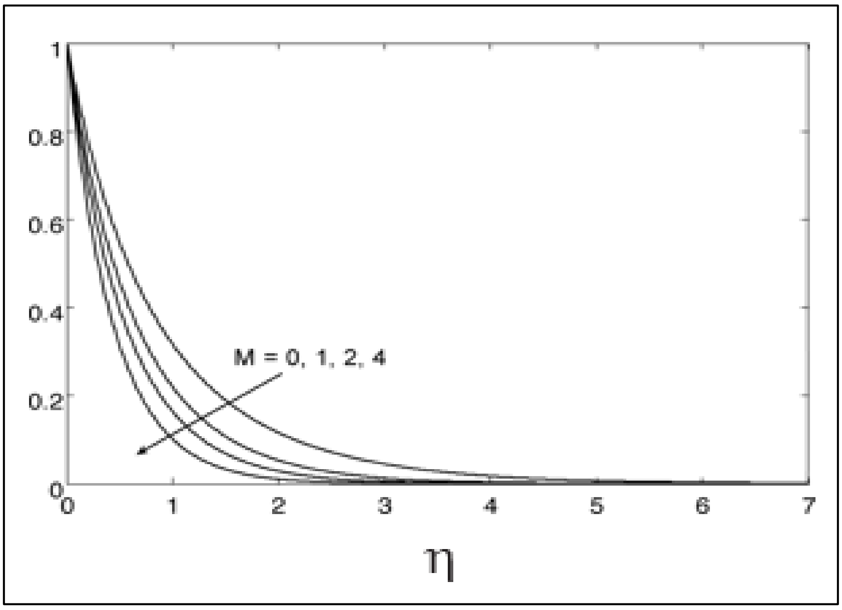

Fig 2: Velocity profiles for different values of M

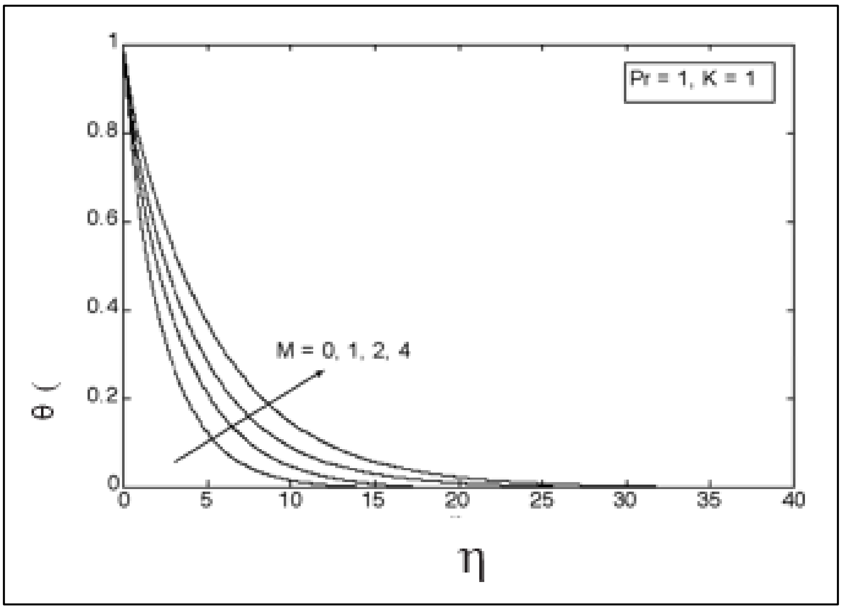

Fig 3: Temperature profiles for different values of *M* when Pr = 1 and *K* = 1

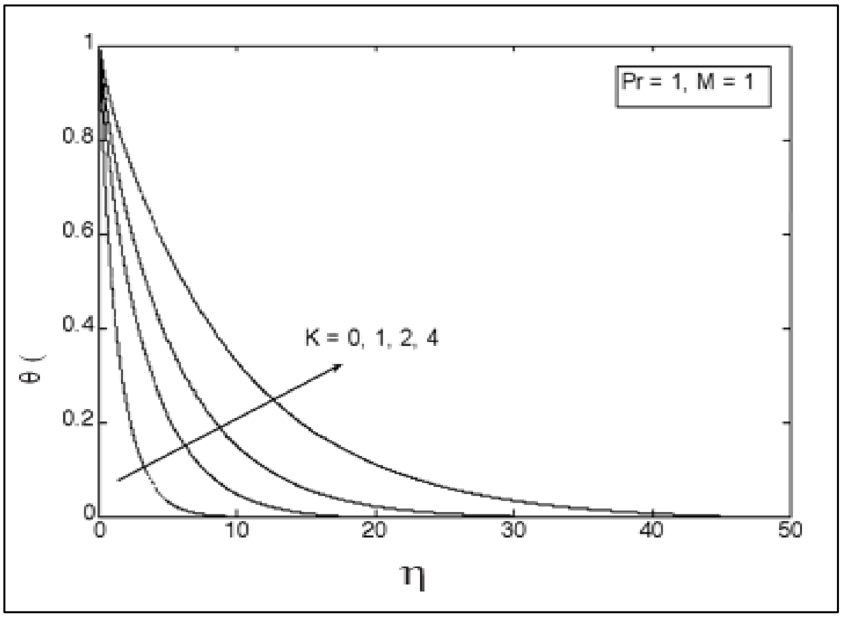

Fig 4: Temperature profiles for different values of *K* when Pr = 1 and *M* = 1

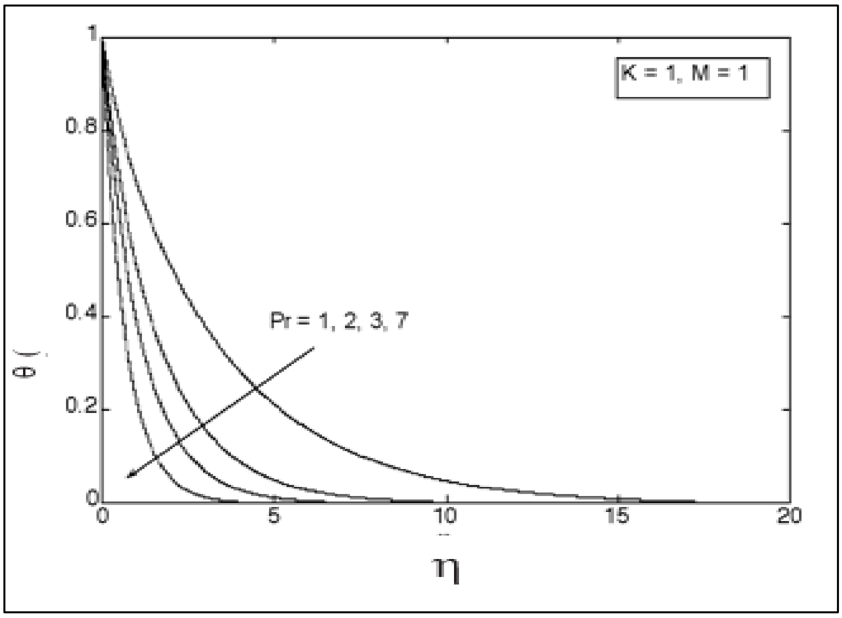

Fig 5: Temperature profiles for different values of Pr when *K* = 1 and *M* = 1

Conclusions

The effect of radiation on steady MHD boundary layer flow over an exponentially stretching sheet was investigated. The numerical results obtained agreed very well with previously reported cases available in the literature. It was found that the surface shear stress increases with the magnetic parameter *M* and Casson parameter, while the heat transfer rate increases with Prandtl number Pr, but decreases with both magnetic parameter *M* and radiation parameter *K*. Momentum boundary layer thickness decreases with increasing Casson parameter but the thermal boundary layer thickness increases in this case

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