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Predicting the spread of COVID-19 epidemic using the SIR model and optimal intervention policy for Nigeria

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Abstract

The spread of the COVID-19 epidemic has made the Nigerian economy vulnerable to both domestic and external shocks. Currently, the second wave of the epidemic is resonating around the world and may further depress the Nigerian economy. To guide policymakers, implement an optimal intervention policy, this study used daily data on 23 November 2020 and simulated a Susceptibles-Infected-Recovery (SIR) model to predict the spread of Covid-19 in Nigeria. The simulated model predicts that the COVID-19 epidemic will reach its peak around 31 March 2021, with a worst-case scenario of approximately 269,444,1223 ($\times 100,000$) infected persons and 46.12 % of the entire Nigerian population will be infected in the days ahead. Further results indicate that an optimal intervention policy for Nigerian, is for policy makers to considerably reduce the transmission rate by increasing public health expenditure.

Keywords: Epidemic, prediction, policy intervention, SIR model, Nigerian economy JEL: H5, E17, E61, C63

Introduction

The outbreak of the Covid-19 epidemic and the slump in crude oil prices have triggered a global economic depression, leaving African economies vulnerable to a combination of unprecedented domestic and external shocks. The spread of the pandemic has caused a Keynesian-type supply shock that has triggered a demand shortage leading to a contraction in output and employment, while the slump in crude oil prices has considerably reduced the foreign earnings of African countries, whose mainstay is crude oil. OECD (2020) ^[16] report indicates that the continuous spread of the Covid-19 epidemic and the persistence of economic shocks will affect Africa's growth through domestic and external channels, impacting significantly on the well-being of the population.

To mitigate the spread of Covid-19, most African governments responded agilely by adopting the national lockdown strategy that encouraged movements/border restrictions and social distancing. To cushion the burgeoning effect of economic shocks, Central Banks across the continent adopted aggressive monetary policy initiatives aimed at boosting liquidity in their economies, increasing the availability of funds to commercial banks and introducing debt repayment holidays (Flecher, 2020). The fiscal measures are varied, but centre on key aspects: upgrading healthcare facilities, providing economic stimulus packages to leverage economic activities, and tax reliefs to encourage businesses and protect jobs (see Asiedu, & Adoteye, 2020; Bedasso & Cole, 2020; Bounang & Tchafack, 2020; Olomola, 2020; Oyedele, 2020) ^[3, 5, 6, 17, 18]. Regardless of the aggressive measures adopted by African governments to mitigate the spread of Covid-19, the disease keeps spreading unabatedly. The lockdown strategy and other mitigation policies have rapidly slowed down economic activities and disrupted the supply chain of African economies. This submission plausibly justifies World Bank's (2020) projections that the epidemic will shrink the economic growth in sub-Saharan Africa from 2.4 percent in 2019 to between -2.1 percent and -5.1 percent in 2020.

Statistics from different organisations show that the spread of Covid-19 is growing exponentially, despite the aggressive measures adopted and thus, underscores the need for policymakers in Africa to have a fore knowledge of its evolution and trend before adopting

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any mitigating strategy. The trend of Covid-19 has been efficiently tracked and analysed by most researchers (Eichenbaum, Rebelo & Trabandt, 2020; Drozd & Tavares, 2020; Alvarez, Argente & Lippi, 2020; Stock, 2020; Kissler, Tedijanto, Lipsitch & Grad, 2020, etc.)^[8, 7, 2, 19, 13] using various mathematical models. The major attribute of these models in the case of Covid-19, is to guide policymakers to properly allocate health care expenditures and effectively employ available human resources.

Already, some countries around the world have started re-enforcing some existing measure as the second wave of COVID-19 epidemic seems imminent. The Nigerian government is contemplating to re-enforce a second lockdown, which will further affect the economy that has already gone into recession. Hence, this study proposes a mathematical-epidemiological model that predicts the course of the epidemic to help the government plan for an effective control strategy, and how to implement Public health care/economic measures.

Thus, a timely decision predicated on projections from mathematical models will guide policy makers to implement various health care measures to mitigate the spread of the epidemic and reduce both internal and external shocks. One of such mathematical models adopted for this study is the basic Susceptible-Infected-Recovered (SIR) epidemic model, which is currently being applied to project the trend and evolution of Covid-19. Thus, the main objective of this study is to predict the spread of Covid-19 and a plausible optimal intervention policy for the country.

The rest of the paper is as follows, Section 2 reviews related studies on SIR model, Section 3 describes the basic SIR framework. Section 4 presents the predictions of Covid-19 and policy implications, while Section 5 advances the conclusion and recommendation.

Literature Review

The Susceptible-Infected-Recovered (SIR) model is a simple compartmental model introduced by Kermack and McKendrick (1927)^[12] to predict the outbreak of infectious diseases over time. Basically, it is a mathematical model, governed by a set of differential equations, which shows how a susceptible population may become infected through contact with other infected individuals and finally assume a non-contiguous state (recovery/death). The basic assumptions of the SIR model are: the population is divided into three compartments (Susceptibles, Infectives and Recovered); the rates of transfer from one compartment to the next are expressed mathematically as derivatives with respect to time; the population size N is constant over time. A major advantage of the SIR model is that it considers few parameters, which are convenient for estimation when data is limited (Toda, 2020)^[20].

Based on its simple estimation approach, Economists are increasingly calibrating the parameters of the SIR model to analyse the economic effect of Covid-19 and to determine the optimal shutdown/social distancing policy. For instance, Atkeson (2020)^[4] employed the SIR model to simulate different isolation scenarios, while (Eichenbaum, Rebelo & Trabandt, 2020)^[8] simulated the SIR model with a representative macroeconomic agent to analyse the macroeconomic outcomes of Covid-19. They all concluded that economic outcomes depend on SIR parameters and that the entire model can be calibrated using data selected under historical testing guidelines. Drozd and Tavares (2020)^[7]; Alvarez, Argente and Lippi (2020)^[2]; Stock (2020)^[19]; Kissler, Tedijanto, Lipsitch and Grad (2020)^[13] and Toda (2020)^[20] have extended the SIR model to advance optimal policies that could tackle Covid-19. Drozd and Tavares (2020)^[7] discussed basic implications derived from various simulated SIR models and the need for adequate response to the ensuing economic crisis caused by the Covid-19.

Alvarez, Argente and Lippi (2020)^[2] derived an optimal lockdown policy for covid-19 from an SIR model. Stock (2020)^[19] in his analysis, indicated that the transmission coefficient (β) is key to designing an optimal social distancing/shutdown policy, and can also be used to determine the trade-off between economic cost and cost of excess lives lost by overwhelming the health care system. Toda (2020)^[20] affirmed the use of the estimates of the transmission coefficient to compare various social distancing policies of a group of countries. This approach is also accentuated in the work of (Kissler, Tedijanto, Lipsitch & Grad, 2020)^[13] who compared various lockdown policies using the SIR model. Kantner (2020)^[11] employed an extended Susceptible-Exposed-Infected-Recovered (SEIR) model to show that social distancing policies can minimize disease-related deaths alongside a desired degree of herd immunity. Staying within the context of the SIR model, Kruse and Strack (2020)^[14] concluded that optimal social distancing can be an effective measure in substantially reducing the death rate of the pandemic.

SIR models have been used to guide policymakers on various health and safety measures by making projections regarding the spread of infectious diseases. Almeshal, Almazrouee, Alenizi and Alhajeri (2020)^[1] predicted the size of the Covid-19 epidemic in Kuwait. The simulation results showed that the epidemic is yet to reach its peak value. Wu, Leung, Leung, (2020)^[23] forecasted the extent of domestic and global public health risks in Wuhan, based on the estimated size of the of the Covid-19 pandemic. Employing the SEIR model, Kuniya (2020)^[15] predicted the peak value of Covid-19, which provided an insight of the feasibility of conducting the Summer Olympics of 2020 in Japan.

Even though the SIR model is simple to estimate, it assumes that all individuals in a population have an equal probability of coming in contact with one another (homogeneous mixing of the population). This does not reflect human social structures, in which the majority of contact occurs within limited networks, hence, constituting a major setback of the SIR model (Tolles & Luong, 2020)^[21].

Methodology

Framework for the Susceptible-Infected-Recovered (SIR) Model

The SIR model is extensively being used in literature to predict the spread of infectious diseases.

First, the population (N) at time (t) is divided into three compartments: Susceptibles $S(t)$ - People who are prone to the virus. It is considered that everybody is susceptible at the initial outbreak of the epidemic, implying that $S(t)=S_0$.

Infectives $I(t)$ - People who are already infected and can infect others. The number of infected persons at the start of the outbreak is such that $I(t)=I_0$.

Removed $R(t)$ - People who have recovered or died. However, nobody recovers or dies at the start of the outbreak implying that $R(t) = 0$.

Thus, the sum of the different compartments $S(t)$, $I(t)$ and $R(t)$ at time t is:

$$N(t) = S(t) + I(t) + R(t) \tag{1}$$

Basic Assumptions

1. At the initial outbreak of the epidemic, the population is constant.
2. The rate of increase of infectives is proportional to the contact between susceptibles and infectives, and occurs at a constant rate.
3. Infectives recover or die at a constant rate say (γ) .

Beta (β) measures the rate of spread, specifically, the probability of transmitting disease between a susceptible and infectious individual, gamma (γ) measures the rate of recovery/death.

Hence, a system of differential equations that governs S , I and R as time changes is:

$$\frac{dS}{dt} = -\beta SI \tag{2}$$

Equation 2 shows the rate of change of Susceptibles, which decrease at a constant (β) depending on the number of contacts between the susceptibles and infectives.

$$\frac{dI}{dt} = \beta SI - \gamma I \tag{3}$$

Equation 3 shows the rate of change of Infectives, which increase as the contact rate (β) between susceptibles and infectives increases. Also, the infectives will reduce at a constant rate (γ) , when people begin to recover or die with time.

$$\frac{dR}{dt} = \gamma I \tag{4}$$

Equation 4 describes the rate of change of Recovery/Death, which increases at a constant rate (γ) depending on the number of infectives.

At the initial outbreak of a pandemic, $S(t) = S_0$; $I(t) = I_0$ and $R_0 = 0$:

$$S + I + R = S_0 + I_0 = N \tag{5}$$

The total population at initial outbreak of the disease $S + I + R$ is constant, implying that:

$$\frac{dS}{dt} + \frac{dI}{dt} + \frac{dR}{dt} = -\beta SI + (\beta SI - \gamma I) + \gamma I \Rightarrow -\beta SI + \beta SI - \gamma I + \gamma I = 0 \tag{6}$$

From equation 4 above, S is monotone decreasing, which implies that at some point, $(S \leq S_0)$, and the disease will spread if $I \geq I_0$ (S will be less than or equal to its initial value S_0 , and I is growing more than I_0).

Substituting in equation 3

$$\text{above: } \frac{dI}{dt} < \beta S_0 - \gamma I; \Rightarrow \frac{dI}{dt} < I(\beta S_0 - \gamma) \Rightarrow S_0 > \frac{\gamma}{\beta}$$

If $S_0 > \frac{\gamma}{\beta}$, the virus will spread, and if $S_0 < \frac{\gamma}{\beta}$, the virus will die off (not spread).

Alternatively, if we let $S_0 > \frac{\gamma}{\beta} = \frac{1}{q}$, where q is the contact ratio: $q = \frac{\beta}{\gamma}$ i.e., the fraction of the population that comes into contact with an infected individual.

$\frac{\gamma}{\beta}$, is called "threshold value", for which the epidemic will spread or peter out.

Basic Reproduction Number

The SIR model can be used to estimate the basic reproductive number (R_0), which informs policy makers whether the pandemic will continue to spread (and how fast) or peter out.

$\beta S_0 = \gamma$. In actual sense, γ is the rate at which the disease is reproducing with respect to a particular value R_0 . This implies that:

$$\beta S_0 = \gamma R_0, \Rightarrow R_0 = \frac{\beta S_0}{\gamma} \text{ is the basic reproductive ratio.}$$

Thus, if $R_0 = \frac{\beta S_0}{\gamma} > 1$, then disease will spread, and if $R_0 = \frac{\beta S_0}{\gamma} < 1$, then disease will peter out.

Future values are calculated thus

$$S_{n+1} = S_n + \frac{dS}{dt} * \Delta t; I_{n+1} = I_n + \frac{dI}{dt} * \Delta t; R_{n+1} = R_n + \frac{dR}{dt} * \Delta t,$$

Depend on the transmission coefficient β .

Secondary data for COVID-19 used in the study were obtained from worldmeter.inof/coronavirus. The study considered the initial daily values for COVID-19 in Nigeria on 23 November 2020.

Simulation of SIR model and Prediction

The evolution of the SIR model, which shows the behaviour and future trend of the Covid19 pandemic is represented graphically. The green curve is the proportion of the population who are susceptible to the virus, the blue curve is the proportion of the population that are infected and the red curve is the proportion of the population that has recovered.

Preliminary Calculations

This analysis begins with some preliminary calculations to obtain the initial parameters and initial conditions for the prediction.

Total Population N = 206,139,589; Infectives = 66,383; Recovered = 62076;

For convenience, all the values were divided by 100, 000.

1. Population N = 2061.393; Infectives I = 0.66383; Recovery R= 0.62076
2. $N = S + I + R \Rightarrow$ Suceptibles $S = N - (I+R) = 2061.393 - 1.28459 = 2060.1113$
3. Total Susceptibles to COVID-19 in Nigeria is $2060.1113 \times 100000 = 206,011,130$
4. Transmission Coefficient (β) = $\alpha \times p$; p = is transmission probability.
5. Average Household size in Nigeria is 5, implying each individual has a 0.2 chance of transmitting the virus. Thus, $p = 1/5 = 0.2$; contact rate: $\beta = I/S = 0.66383/2060.1113 \approx 0.0003223$
6. Transmission Coefficient $\alpha = 0.0003223 * 0.2 \approx 6.4446E-05$ i.e., the rate at which the virus is transmitted.
7. According to WHO, it takes a minimum of 2 weeks (14 days) for a patient to recover.

Hence, $\gamma = 1/14 = 0.0714$, i.e., the rate at which individuals get removed (recovered/die).

$$R_o = \beta S_o / \gamma = 0.0003223 * 2060.1113 / 0.0714 \approx 1.8587 > 1$$

Basic Reproductive Rate

Hence, COVID19 will spread very well in Nigeria and the fraction of the population that will be affected in the days ahead is:

$$1 - 1/1.8587 = 46.12 \% R_{new} = \beta S_o / \gamma = 0.0003223 * 1545.08347 / 0.0714 \approx 1.3940 > 1$$

Hence, COVID19 will spread very well in Nigeria and the fraction of the population that will affected in the days ahead is:

$$1 - 1/1.3940 = 28.27 \%$$

Analysis of the SIR Curve

The parameters are substituted in the models and simulated to obtain the SIR curves which predict the evolution of COVID-19 in Nigeria. The curve spans 200 days with an initial susceptible population of 206,011,130, an initial infected population of 66,383 and an initial recovered population of 62076. Figure 1 reveals that the proportion of recovered people increases with time, while the proportion of susceptible and infected persons decrease gradually until they eventually peter out at zero. The simulated SIR graph predicts that COVID-19 will reach its peak around 31 March 2021, with a worst-case scenario of approximately 269.4441223 ($\times 100,000$) infected persons. The basic reproductive rate of approximately 1.8587 shows that COVID19 will continue to spread and 46.12 % of the entire Nigerian population will be infected in the days ahead.

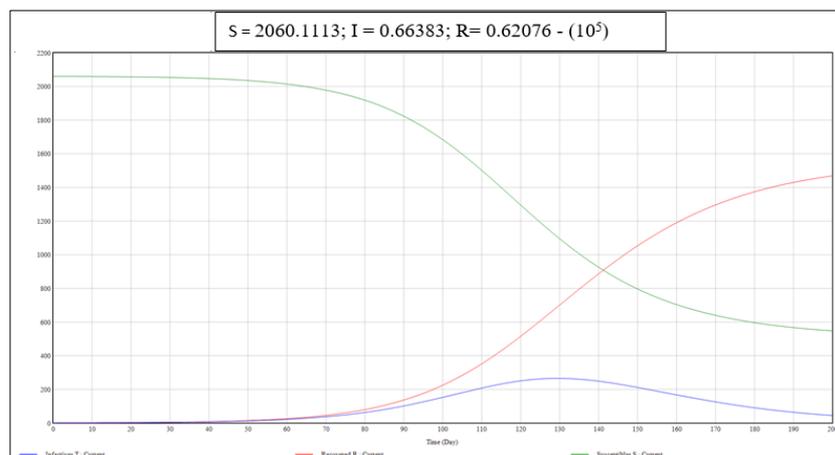


Fig 1: Simulated SIR graph of COVID-19 in Nigeria with initial susceptible value ~212~

Policy Options

Policy implementation and its economic consequences are based on two criteria. One, the government can decide to flatten the curve (contain the virus) by enforcing social discipline and national lockdown. Hence, the fraction of population that are susceptible and those infected will reduce, while the number of recovered cases will increase over time. Two, the government can decide to quarantine all infected persons, hence reducing the rate of transmission α with health care interventions.

Case 1: Policy to enforce social discipline and national lockdown

If the government decides to reduce the rate of susceptibility S by $\frac{3}{4}$, then the fraction of people susceptible to COVID-19 will drop to 1545.083475 ($\times 100,000$) and the worst-case scenario will peak at an approximate value of 199.1765289 ($\times 100,000$) infected persons around 23 March 2021 as reported in Figure 2. The basic reproductive rate will also drop to approximately 1.394 and only 28.27 % of the entire Nigerian population will be infected in the days ahead. This policy is plausible and worth implementing.

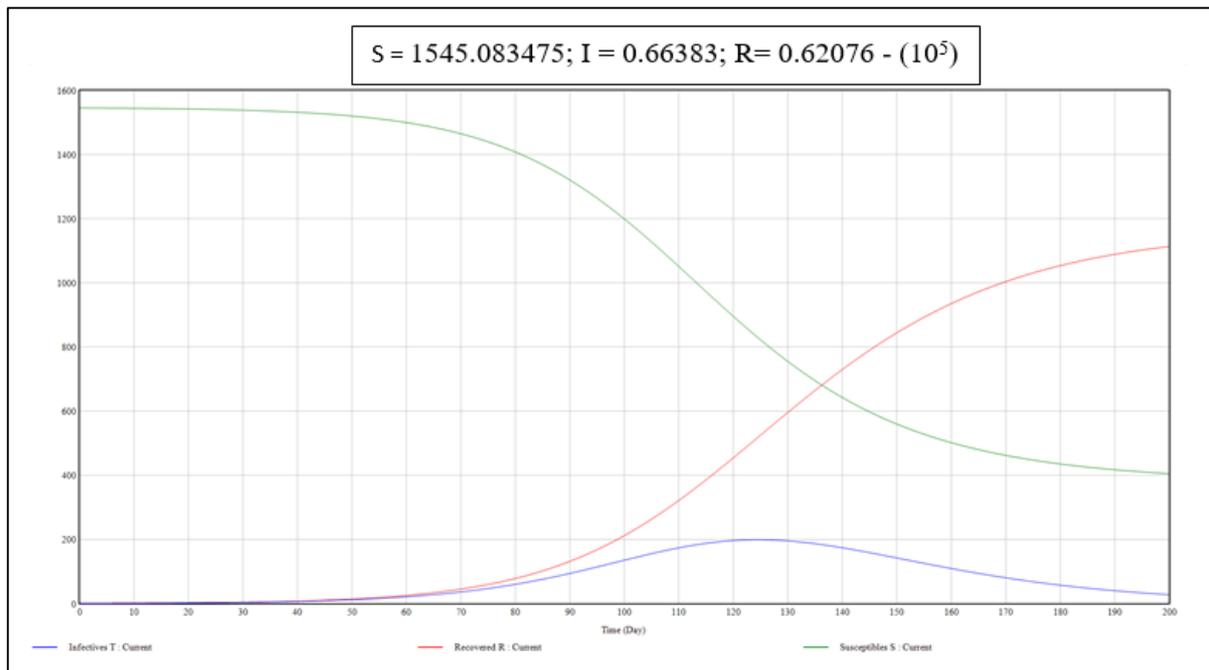


Fig 2: Simulated SIR graph of COVID-19 in Nigeria with a $\frac{3}{4}$ reduced susceptible value

Case 2: Policy to reduce the transmission rate (α)

If the government decides to quarantine all infected persons in order to reduce the rate of transmission α , then health expenditure must increase so that recovery rate will increase rapidly. Thus, α is a critical policy variable that determines the level and speed of health intervention as well as the recovery rate. Figure 3 shows that it takes fewer days for more people to be infected as the rate of transmission α increases. For instance, the worst-case scenario will peak at 934.92 ($\times 100,000$) after 37 days when the transmission coefficient α is 1.61E-04 and will peak at 264.075 ($\times 100,000$) after 128 days when α is 6.44E-05.

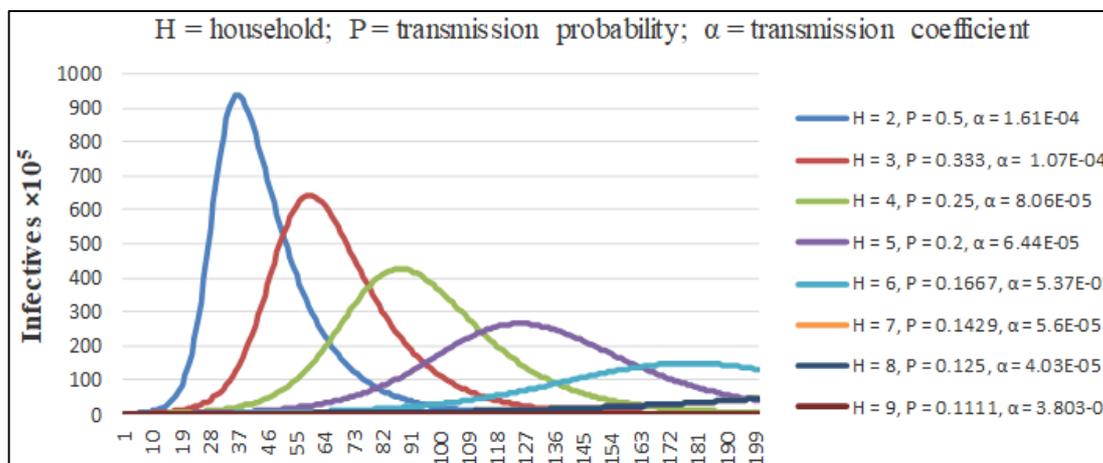


Fig 3: Simulated policy graph of infectives with different transmission coefficients

Figure 4 shows that it takes fewer days for the susceptible population to become infected as the transmission rate α decreases. For instance, it will take 15 days for approximately 2059.364014 ($\times 100,000$) susceptible individuals to become infected when α is 1.61E-04, and 82 days for approximately 2055.70146 ($\times 100,000$) susceptible individuals to be infected with COVID-19 when α is 5.37E-05.

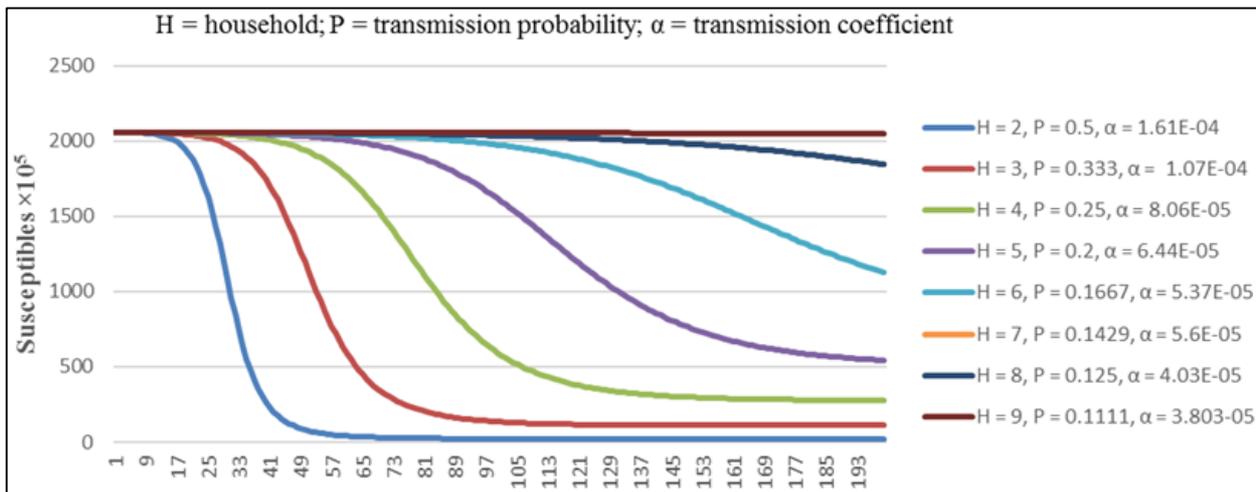


Fig 4: Simulated policy graph of susceptibles with different transmission coefficients

Figure 5 reveals that the recovery time is shorter as transmission rate α reduces i.e., it takes fewer days for the more infected individuals to fully recover when the mitigated rate of transmission is reduced. For instance, recovery starts on day 20 with a total of 2040.699219 ($\times 100,000$) recovered individuals at the end of the study period (200 days) when α is 1.61E-04. But when α is say 6.44E-05, recovery rate starts on day 62 with a total of only 1478.630127 ($\times 100,000$) recovered individuals at the end of the study period. Hence α is critical in determining public health expenditure that seeks to mitigate the spread of the COVID-19 epidemic.

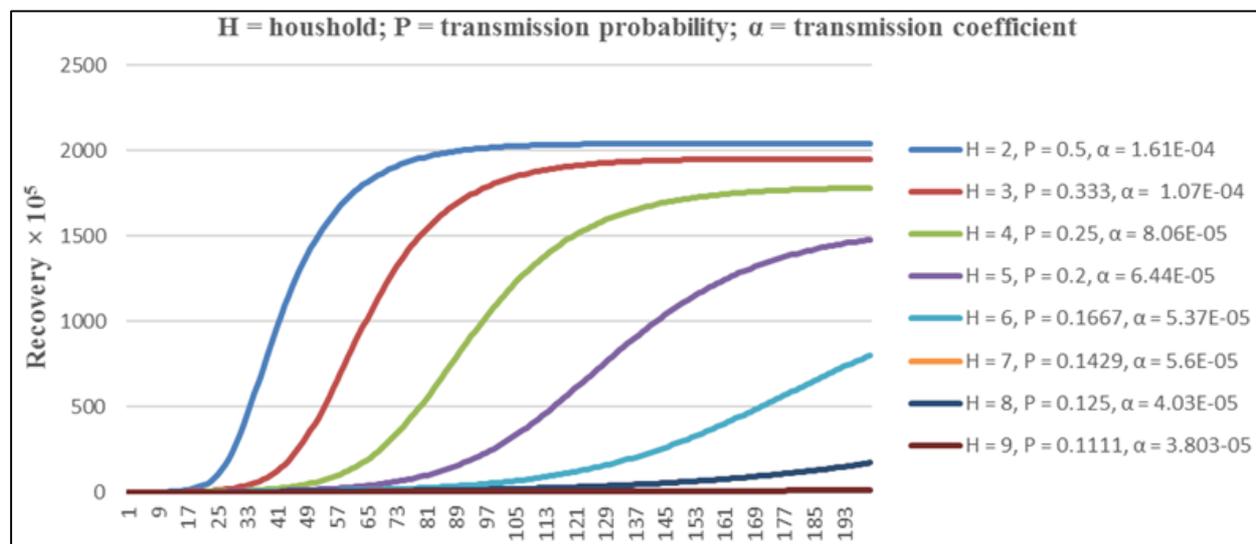


Fig 5: Simulated policy graph of Recovered with different transmission coefficients

Conclusion

This study simulated an SIR model to predict the COVID-19 epidemic outbreak in Nigeria using daily infected and recovered cases, on 23 November 2020. The study reveals that a worst-case scenario will occur around 31 March 2021, with an approximate of 269.4441223 ($\times 100,000$) infected persons. The basic reproductive rate shows that 46.12 % of the entire population will be infected by epidemic. The study reveals that two policy outcomes can actually mitigate the spread of COVID-19 in Nigeria. First, the fraction of the Nigerian population that is susceptible to COVID-19 will actually reduce if social discipline and national lockdown are properly enforced. Hence, COVID-19 will infect only 28.27 % of the entire Nigeria population. Secondly, public health expenditure policy that provide health facilities for the already infected/quarantined persons in order to hasten recovery and health palliatives for the vulnerable population to protect them from being infected will reduce the transmission rate α and rapidly increase the rate of recovery. Hence, an optimal policy for Nigerian, is for policymakers to considerably reduce the transmission rate α by increasing public health expenditure.

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