Modelling the effects of temperature on the flow of fluid through loamy soil with spatial dependent thermo-physical property

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Abstract
The modelling of effects of temperature on the flow of fluid through loamy soil with spatial dependent thermo-physical property was studied. Navier-Stoke equations were modified with Boussinesq’s approximation under necessary conditions for the flow of the fluid through the porous medium considered (loamy soil). These set of equations formed the governing equations for the flow in dimensional form. After reducing the equations into non-dimensional form employing some relevant dimensionless parameters, they were converted to ordinary differential equations and solved analytically which formed the desired model. Data retrieved concerning specific information about the loamy soil were imputed into the model and Matlab R2009b was then used to display the results on graphs. The effects of relevant physical parameters were examined on the velocity of the fluid. Increasing thermal Grashof number (Gr), Internal heat (Q) and solar radiation enhance the velocity of the fluid while the Prandtl number reduces the rate of the flow.

Keywords: Boussinesq’s approximation, dimensionless parameters, loamy soil, solar radiation, thermo-physical property, thermal Grashof number

1. Introduction
In power industry, which involves the method of generating electricity through the heat of the fluid that flows in the ground, the study of the fluid dynamics is of a great importance. Moreover, because of the flow of these fluids through the ground, the temperature of both the ground and the fluid are affected by a number of factors. This of no doubt has drawn the attentions of researchers toward this subject. For example, Akinpelu et al. [1] examined how solar radiation affects both the clay-loam soil and sandy-loam soil using convectional boundary condition. They reported in their findings that the solar radiation generally affects the temperature of both soil samples. In other word, the temperatures of these soils rise as the intensity of the radiation also increases. In addition, they found out that as the moisture content of these soils increased, their thermal conductivities also increased, this in turn increased their temperature when the solar radiation is involved; though this is at different levels and rates. Mohammed [2] in his own work includes chemical reaction and heat generation while studying the effects of radiation on mass flow passing via a very high porous medium. Having examined the physical parameters that are involved in the work, he showed in his results that the thermal and solutal buoyancy forces, heat generation and the permeability of the medium all increased the rate at which the fluid flows. However, since the fluid considered was emitting radiation, the temperature and the speed of the fluid flow reduced as the radiation increased. Olaleye et al. [3] modelled the temperature of loamy soil as being affected by solar radiation. Having examined some appropriate heat related physical quantities that surfaced in the study, they revealed that the solar radiation alongside heat generation are capable of shooting up the warmth of loamy soil when they are enlarged. On the other hand, when all the material parameters are kept constant, after some time, the temperature of the soil will begin to drop, especially as the depth of the soil increases.
In their own study, Mahender and Srikanth [4] includes viscous dissipation and magnetic field
while researching into the free convection flow of the fluid pass a vertical plate that is porous in nature. They concluded that as a result of the temperature produced by the source of heat and viscous dissipation, the temperature and the velocity of the fluid also increased. Meanwhile, the Schmidt number reduced the thickness of the concentration boundary layer. Alabison et al. [5] studied the effects of warmth generation and radiation on convective fluid movement through a permeable medium using an intermittent temperature boundary state. It was evident in their result that the increasing buoyancy forces alongside the warm generation boost the temperature and increase the speed of the flow. But the increasing Schmidt number, Chemical parameter and Prandtl number lowers the temperature and the fluid’s velocity.

Some others include Hud et al. [6] who modelled heat transport in greenhouse amid a surface soil warming structure. Sharma and Bismaeta [7] considered the way uneven thermal conductivity and viscosity together with Dufour and Soret influence the movement of fluid through an upright cone in free convective way.

Considering the above works among host of others, this research focuses on modelling the effects of temperature on the flow of fluid through loamy soil using spatial dependent thermo-physical property. Since the soil is a porous medium which allow the passage of heat and fluid, the choice of spatial dependent thermo-physical property makes the work more relevant in application.

2. Mathematical analysis

The Navier-Stoke heat equations were modified under the following conditions. The flow is considered infinite along the horizontal axis leaving the equation to be a function of the vertical axis (z) and time, t (being unsteady). The soil is optically thin environment. That is, it allows the passage of fluid and heat. At the boundary layer, the temperature is considered to be variable due to different factors. Also, both the suction velocity and the thermal conductivity are taken to be free stream temperature, \( z \), is the vertical axis, \( \omega \) is absorption coefficient.

In line with Nwaigwe [9], variable suction velocity is known to be:

\[
\tilde{w} = -w_{0}(1 + \varepsilon Ae^{i\omega \tilde{t}})
\]  
(7)

Where \( w_{0} \) is initial suction velocity, \( A \) is suction parameter and \( \omega \) is frequency of oscillation. In agreement with Akinpelu et al. [1], Mohammed [2] and Sharma [10], non-dimensional quantities used are:

\[
\omega = \frac{w_{0}}{w_{0}}, z = \frac{w_{0}z}{w}, \nu = \frac{\tilde{v}}{v_{0}}, t = \frac{fw_{0}t}{w}, \tau = \frac{\tilde{t}}{w_{0}-\tilde{t}_{0}}
\]
\[ Gr = \frac{v \beta (T_w - T_0)}{w_0}, \quad P_r = \frac{\nu \rho C_p}{k_\infty}, \quad \theta = \frac{T - T_\infty}{T_w - T_0} \] (8)

Moreover, in agreement with the work of Olaleye et al. [3] and Akinpelu et al. [11], the thermal conductivity is considered to be spatial dependent, varying linearly with position, and given as:

\[ k = k_0(1 + \gamma z) \] (9)

\[ k_0 \text{ and } \gamma \text{ are constant and variable thermal conductivity (parameter) respectively.} \]

Using equations (6)–(9), equations (2)–(5) were reduced to:

\[ \frac{\partial \theta}{\partial t} - \frac{\partial \theta}{\partial z} - \varepsilon \gamma e^{i\omega t} \frac{\partial \theta}{\partial z} = \frac{\partial^2 \theta}{\partial z^2} + Gr \theta - \frac{\theta}{k} \] (10)

\[ \frac{\partial \theta}{\partial t} - \frac{\partial \theta}{\partial z} - \varepsilon \gamma e^{i\omega t} \frac{\partial \theta}{\partial z} = \frac{1}{\nu} \frac{\partial^2 \theta}{\partial z^2} + \gamma^2 \frac{\partial^2 \theta}{\partial z^2} - R^2 + Q \theta \] (11)

Subject to:

\[ v = V_p, \theta = 1 + re^{i\omega t} \text{ at } z = 0 \] (12)

\[ v \to 1, \theta \to 0 \text{ as } z \to \infty \] (13)

Where \( P_r \) is Prandtl number, \( Gr \) is thermal Grashof number, \( \gamma \) is thermal conductivity parameter, \( R \) is radiation parameter and \( Q \) is heat generation parameter.

### 3. Method of solution

Equations (10)–(11) are partial differential equations of second order. In order to reduce them into ordinary differential equations using perturbation method, the assumed solution is given as:

\[ v(z, t) = v_0(z) + \varepsilon e^{i\omega t} v_1(z) + \cdots \] (14)

\[ \theta(z, t) = \theta_0(z) + \varepsilon e^{i\omega t} \theta_1(z) + \cdots \] (15)

Differentiating equations (14)–(15) and substituting them into equations (10)–(11), while taking the order of epsilon and ignoring higher order \( o(\varepsilon^2) \), we have

\[ \frac{\partial^2 v_0}{\partial z^2} + \frac{\partial v_0}{\partial z} - \frac{1}{k} v_0 = -Gr \theta_0 \] (16)

\[ \frac{\partial^2 v_1}{\partial z^2} + \frac{\partial v_1}{\partial z} - \left( i\omega + \frac{1}{k} \right) v_1 = -A \frac{\partial v_0}{\partial z} - Gr \theta_1 \] (17)

\[ \frac{\partial^2 \theta_0}{\partial z^2} + \gamma^2 \frac{\partial^2 \theta_0}{\partial z^2} + P_r \frac{\partial \theta_0}{\partial z} + P_r Q \theta_0 = P_r R^2 \] (18)

\[ \frac{\partial^2 \theta_1}{\partial z^2} + \gamma^2 \frac{\partial^2 \theta_1}{\partial z^2} + P_r \frac{\partial \theta_0}{\partial z} + P_r (Q - i\omega) \theta_1 = -P_r A \frac{\partial \theta_0}{\partial z} \] (19)

Subject to:

\[ v_0 = V_p, v_1 = 0, \theta_0 = 1, \theta_1 = 1 \text{ at } z = 0 \] (20)

\[ v_0 \to 1, v_1 \to 0, \theta_0 \to 0, \theta_1 \to 0 \text{ as } z \to \infty \] (21)

In order to obtain the solution to equations (18)–(19), as employed by Umavathi [12], perturbation method of second phase is required. The variable thermal conductivity parameter is used as the perturbation factor:

\[ \theta_0 = \theta_{00} + \gamma \theta_{01} \] (22)

\[ \theta_1 = \theta_{10} + \gamma \theta_{11} \] (23)

Differentiating above equations (22)–(23) and put them into equations (18)–(19), while taking order of gamma and disregard higher order \( o(\gamma^2) \),

\[ \frac{\partial^2 \theta_{00}}{\partial z^2} + P_r \frac{\partial \theta_{00}}{\partial z} + P_r Q \theta_{00} = P_r R^2 \] (24)
\[
\frac{\partial^2 \theta_{01}}{\partial z^2} + P_r \frac{\partial \theta_{01}}{\partial z} + P_r Q \theta_{01} = -z \frac{\partial^2 \theta_{01}}{\partial z^2} 
\]
(25)

\[
\frac{\partial^2 \theta_{10}}{\partial z^2} + P_r \frac{\partial \theta_{10}}{\partial z} + P_r (Q - i \omega) \theta_{10} = -P_r A \frac{\partial \theta_{01}}{\partial z} 
\]
(26)

\[
\frac{\partial^2 \theta_{11}}{\partial z^2} + P_r \frac{\partial \theta_{11}}{\partial z} + P_r (Q - i \omega) \theta_{11} = -z \frac{\partial^2 \theta_{11}}{\partial z^2} - P_r A \frac{\partial \theta_{01}}{\partial z} 
\]
(27)

Solving equations (24)–(27) analytically subject to (20)–(21),

\[
\theta_{00} = C_1 e^{m_{12}z} + C_2 e^{m_{22}z} + C_3 
\]
(28)

\[
\theta_{01} = C_5 e^{m_{32}z} + (C_6 z + C_7) e^{m_{32}z} + (C_8 z + C_9) e^{(m_{32} - m_{23})z} 
\]
(29)

\[
\theta_{10} = C_{11} e^{m_{12}z} + C_{12} e^{m_{22}z} + C_{13} e^{(m_{22} - m_{12})z} 
\]
(30)

\[
\theta_{11} = C_{13} e^{m_{12}z} + (C_{17} z + C_{18}) e^{m_{22}z} + (C_{19} z + C_{20}) e^{(m_{22} - m_{12})z} 
\]
(31)

Moreover, solving equations (16)–(17) subject to (20)–(21),

\[
v_0 = C_{14} e^{m_{12}z} + C_{15} e^{m_{22}z} + C_{16} e^{m_{32}z} + C_{17} e^{m_{33}z} + C_{18} e^{m_{23}z} + C_{19} e^{m_{13}z} + C_{20} e^{m_{33}z} + C_{21} e^{m_{32}z} + C_{22} e^{m_{23}z} + C_{23} e^{m_{31}z} + C_{24} e^{m_{23}z} + C_{25} e^{m_{22}z} + C_{26} e^{m_{23}z} + C_{27} e^{m_{21}z} + C_{28} e^{m_{31}z} + C_{29} e^{m_{31}z} + C_{30} e^{m_{31}z} + C_{31} e^{m_{31}z} + C_{32} 
\]
(32)

Therefore, the velocity and temperature distributions respectively are obtained to be:

\[
v = v_0 + \varphi e^{i \omega t} v_1 
\]
(34)

\[
\theta = \theta_{00} + \gamma \theta_{01} + \varphi e^{i \omega t} (\theta_{10} + \gamma \theta_{11}) 
\]
(35)

Where

\[
m_4 = m_3 = -\frac{P_r}{2} + \frac{P_r^2}{4} - P_r Q z \\
m_2 = m_4 = -\left(\frac{P_r}{2} + \frac{P_r^2}{4} - P_r Q\right) \\
m_6 = m_7 = -\frac{P_r}{2} + \frac{P_r^2}{4} - P_r (Q - i \omega) \\
m_6 = m_8 = -\left(\frac{P_r}{2} + \frac{P_r^2}{4} - P_r (Q - i \omega)\right) \\
m_9 = -\left(\frac{1}{2} + \sqrt{\frac{K+4}{4K}}\right) \\
m_10 = -\left(\frac{1}{2} + \sqrt{\frac{K+4+4i \omega K}{4K}}\right) \\
C_3 = \frac{K^2}{Q}, \quad C_4 = -C_3 e^{-m_{22}z}, \\
C_2 = 1 + \tau e^{i \omega t} - C_1 - C_3 \\
C_6 = \frac{(C_3 - 1 - \tau e^{i \omega t}) m_3^2}{m_3^2 + P_r m_2 + P_r Q}, \\
C_7 = \frac{-2 m_2 C_8 - P_r C_9}{m_3^2 + P_r m_2 + P_r Q} \\
C_8 = \frac{(m_2 m_3 + m_2 (m_2 - m_3) - m_3^2) C_3}{(m_2 - m_3)^2 + P_r (m_2 - m_1) + P_r Q} \\
C_9 = \frac{-2 (m_2 - m_3) C_3 - P_r C_9}{(m_2 - m_3)^2 + P_r (m_2 - m_1) + P_r Q} \\
C_{10} = -(C_7 + C_9), \\
C_{12} = \frac{P_r A (m_2 - m_1) C_3 - P_r (m_2 - m_3) C_4}{(m_2 - m_1)^2 + P_r (m_2 - m_1) + P_r (Q - i \omega)} \\
C_{13} = \frac{-P_r A (m_2 - m_1) C_3}{(m_2 - m_1)^2 + P_r (m_2 - m_1) + P_r (Q - i \omega)} \\
C_{11} = -(C_{12} + C_{13}) \\
C_{14} = \frac{-m_2^2 C_1 - P_r A m_2 C_6}{m_2^2 + P_r m_3 + P_r (Q - i \omega)} \\
C_{15} = \frac{-P_r A (m_2 - m_1) C_3 - 2 m_2 C_14 - P_r C_{14}}{m_2^2 + P_r m_3 + P_r (Q - i \omega)} \\
C_{16} = \frac{-m_2^2 C_1 - P_r A m_2 C_6}{m_2^2 + P_r m_3 + P_r (Q - i \omega)} \\
C_{17} = \frac{-m_2^2 C_{11}}{m_2^2 + P_r m_3 + P_r (Q - i \omega)} \\
C_{18} = \frac{-2 m_1 C_1 - P_r C_{15}}{m_2^2 + P_r m_3 + P_r (Q - i \omega)} \\
C_{19} = \frac{-m_2^2 C_1 - P_r A (m_2 - m_3) C_9}{(m_2 - m_3)^2 + P_r (m_2 - m_1) + P_r (Q - i \omega)} \\
C_{20} = \frac{-m_2^2 C_{11}}{(m_2 - m_3)^2 + P_r (m_2 - m_1) + P_r (Q - i \omega)} 
\]
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\[ C_{20} = -\frac{p_{\text{R}}(m_2 - m_1)c_0 + c_8 - 2(m_2 - m_1)c_9 p_{\text{R}} c_9}{m_1^2 + p_{\text{R}}(m_2 - m_1) + p_{\text{R}}(q - i\omega)} \]

\[ C_{21} = (1 - C_{27} - C_{25}e^{m_1z} - C_{25}e^{m_1z})e^{-m_1z} \]

\[ C_{22} = V_p - C_{21} - C_{23} - C_{24} - C_{25} - C_{26} - C_{27} \]

\[ C_{23} = -\frac{Gr C_2}{m_2^2 + m_2 - \frac{p_{\text{R}}}{m_1}} \]

\[ C_{24} = \frac{Gr C_2}{m_2^2 + m_2 - \frac{p_{\text{R}}}{m_1}} \]

\[ C_{25} = \frac{Gr C_2}{m_2^2 + m_4 - \frac{p_{\text{R}}}{m_3}} \]

\[ C_{26} = \frac{Gr C_2}{m_2^2 + m_4 - \frac{p_{\text{R}}}{m_3}} \]

\[ C_{27} = Gr K(C_3 + DC_6) \]

\[ C_{28} = -(C_{30}e^{m_1z} + C_{32}e^{m_1z} + C_{34}e^{m_1z} + C_{36}e^{m_1z} + C_{38}e^{m_1z} + C_{40})e^{-m_1z} \]

\[ C_{29} = -(C_{28} + C_{30} + C_{31} + C_{32} + C_{33} + C_{34} + C_{35} + C_{36} + C_{37} + C_{38} + C_{39} + C_{40}) \]

\[ C_{30} = -\frac{Am C_2}{m_2^2 + m_2 - \frac{p_{\text{R}}}{m_1}} \]

\[ C_{31} = \frac{Am C_2}{m_2^2 + m_2 - \frac{p_{\text{R}}}{m_1}} \]

\[ C_{32} = -\frac{Am C_2}{m_2^2 + m_2 - \frac{p_{\text{R}}}{m_1}} \]

\[ C_{33} = \frac{Am C_2}{m_2^2 + m_2 - \frac{p_{\text{R}}}{m_1}} \]

\[ C_{34} = -\frac{Am C_2}{m_2^2 + m_2 - \frac{p_{\text{R}}}{m_1}} \]

\[ C_{35} = \frac{Am C_2}{m_2^2 + m_2 - \frac{p_{\text{R}}}{m_1}} \]

\[ C_{36} = -\frac{Gr C_2}{m_2^2 + m_2 - \frac{p_{\text{R}}}{m_1}} \]

\[ C_{37} = \frac{Gr C_2}{m_2^2 + m_2 - \frac{p_{\text{R}}}{m_1}} \]

\[ C_{38} = -\frac{Gr C_2}{m_2^2 + m_2 - \frac{p_{\text{R}}}{m_1}} \]

\[ C_{39} = \frac{Gr C_2}{m_2^2 + m_2 - \frac{p_{\text{R}}}{m_1}} \]

\[ C_{40} = \frac{Gr K(C_3 + DC_6)}{10 K + 1} \]

4. Results and Discussion

In the previous section, the model for the work has been developed which are the velocity and the temperature profiles (equations 34 and 35). The following data/information about the loamy soil were retrieved so as to be specific about the soil working on and to be able to examine the physical parameters that emerged in the model which has to do with temperature/heat. Namely: Solar radiation parameter (R), buoyancy force parameter (Gr), heat generation parameter (Q) and the prandtl number.

Table 1: Thermo-physical properties of Loamy soil

<table>
<thead>
<tr>
<th>Description</th>
<th>Permeability (cm/hour)</th>
<th>Thermal conductivity (Btu/ft hr°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loamy Soil</td>
<td>1.30</td>
<td>0.52</td>
</tr>
</tbody>
</table>

Gary [13], Soil Permeability [14]

Moreover, making use of other parameters involved in the study as used by some existing works like Mohammed [2] among host of others unless otherwise stated,

Table 2: Default parameters

<table>
<thead>
<tr>
<th>R</th>
<th>Q</th>
<th>P_R</th>
<th>(\omega)</th>
<th>(\varepsilon)</th>
<th>t</th>
<th>A</th>
<th>(\tau)</th>
<th>(V_p)</th>
<th>Gr</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.50</td>
<td>0.10</td>
<td>0.71</td>
<td>0.01</td>
<td>1.00</td>
<td>0.50</td>
<td>1.00</td>
<td>10.00</td>
<td>5.00</td>
<td></td>
</tr>
</tbody>
</table>

With the aid of Matlab R2009b software, the effects of these temperature based physical parameters were examined on the velocity of the fluid as it passed through the loamy soil. The results are presented on graphs below.
**Fig 1:** The effect of increasing radiation parameter (R) on the velocity of fluid’s flow through loamy soil.

**Fig 2:** The effect of rising heat generation parameter (Q) on the velocity of fluid’s flow through loamy soil.

**Fig 3:** The effect of mounting buoyancy force (thermal Grashof number – Gr) on the velocity of fluid’s flow through loamy soil.
Figure 1 depicts the effect of increasing radiation on the velocity of fluid’s flow through loamy soil. As the radiation increases, it is seen that the speed of flow of the fluid also increases. This is because both the soil and the fluid absorb the electromagnetic energy which afterward enhances their temperature. As the temperature of the fluid is raised, its viscosity is reduced, and hence increases the speed of flow of the fluid.

In figure 2, which is the effect of rising heat generation on the velocity of fluid’s flow through loamy soil, the fluid’s velocity also increased as the internal heat mounted. This follows from the above; the internal heat increased the fluid’s temperature which lowers its viscosity and in turn increased the flow rate.
Figure 3 represents the effect of thermal Grashof number on the velocity of fluid’s flow through the soil. The increasing buoyancy force boosted the temperature of the fluid and thereafter speeds up the flow velocity as a result of the decreasing viscosity. Figure 4 refers to the effect of increasing Prandtl number (Pr) on the velocity of fluid’s flow through loamy soil. As the Prandtl number increases, the thermal conductivity of the fluid reduces and its temperature also reduces. As a result, the viscosity increases which reduces the velocity of the flow.

When all other parameters are kept constant, as time goes on, at increasing depth of the soil, the flow of the fluid decreased. This is shown in figure 5.

In figure 6, it is clearly seen that the increasing permeability of the soil increases the velocity of the fluid through the soil.

5. Conclusion

Unsteady one dimensional heat equation with spatial dependent thermal conductivity with some relevant conditions was used to model the effects of temperature on the flow of fluid through loamy soil with spatial dependent thermo-physical property. The model was formed after solving the equations governing the study which are the transient temperature and velocity. According to the results, it is evident that the solar radiation, thermal buoyancy and internal heat boost will increase the flow of a fluid through the loamy soil. But when these factors remain unchanged, the flow speed will gradually decline with time.

6. References


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