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Decomposition of normal projective curvature tensor fields in Finsler manifolds

Manoj Singh Bisht and US Negi

Abstract
Takano (1966) have studied recurrent affine motion in a recurrent non-Riemannian space and later on he discussed a recurrent whose curvature tensor is decomposable. Also, Sinha and Singh (1970), has studied on decomposition of Recurrent Curvature Tensor Fields in Finsler Spaces; Pande and Khan (1973), has studied general Decomposition of Berwald's Curvature Tensor Fields in Recurrent Finsler Spaces. After that, Ram Hit (1975), have studied decomposition of Berwald's Curvature Tensor Fields; Pande and Shukla (1977) has studied a recurrent Finsler space whose curvature tensor is decomposable. In this paper we have defined and studied decomposition of normal projective curvature tensor fields in Finsler manifolds and some theorems established on its.

Keywords: Riemannian space, Kaehlerian manifold, Finsler manifold, H-projective recurrent, curvature tensors

Introduction
The Euler's theorem on homogeneous functions, accordingly to the tensor $g_{ij}(x,y)$ is positively homogeneous of degree zero in y^i and symmetric in i and j. The vector y_i satisfies the following relation:

$$g_{ij}(x,y) = \frac{1}{2}\dot{\partial}_i\dot{\partial}_j F^2(x,y). \tag{1.1}$$

$$y_i y^i = F^2, \tag{1.2}$$

Berwald covariant derivative of the function F and vector y^i vanish identically,

$$B_k F = 0 \text{ and } B_k y^i = 0. \tag{1.3}$$

The tensor H^i_{jkh} is called h-curvature tensor and defined by:

$$H^i_{jkh} = \partial_h G^i_{jk} + G^r_{jk} G^i_{rh} + G^i_{rk} G^r_j - {}^h/_k.$$

In view of Euler's theorem on homogeneous functions, we have the following relations:

$$\dot{\partial}_j H^i_{kh} = H^i_{jkh}, \tag{1.4a}$$

$$y^r \dot{\partial}_r H^i_{jkh} = y^r \dot{\partial}_j \dot{\partial}_r H^i_{kh} = 0, \tag{1.4b}$$

$$H^i_{jkh} y^j = H^i_{kh}, \tag{1.4c}$$

$$H_{ijkh} = g_{jr} H^i_{ikh} \tag{1.4d}$$

$$H^i_{kh} = \dot{\partial}_k H^i_h, \tag{1.4e}$$

$H^i_{kh} y^k = H^i_h,$ (1.4f)

$H_{jkh} = g_{ik} H^i_{jh},$ (1.4g)

$H_{jk} = H^r_{jkr}$ (1.4h)

$H^r_{rkh} = H_{kh} - H_{hk},$ (1.4i)

$H_k = H^r_{kr},$ (1.4*l*)

$H = \frac{1}{n-1} H^r_r$ (1.4m)

$H_{jk} y^k = H$ (1.4n)

Decomposition of normal Projective Curvature Tensor in Finsler Manifolds.

The connection coefficient Π^i_{jk} is positively homogeneous of degree zero in y^i and symmetric in their lower indices and defined the projective connection by [Yano (1957)],

$\Pi^i_{jk} = G^i_{jk} - \frac{1}{n+1} y^i G^r_{jkr}..$

The covariant derivative $B_k T^i_j$ of an arbitrary tensor field T^i_j with respect to x^k in the sense of Berwald is given by:

$B_k T^i_j = \partial_k T^i_j - (\dot{\partial}_r T^i_j) \Pi^r_{ks} y^s + T^r_j \Pi^i_{kr} - T^i_r \Pi^r_{kj}.$ (2.1a)

The communication formula for the above covariant derivative is given by

$B_k B_h T^i_j - B_h B_k T^i_j = T^r_j N^i_{rkh} - T^i_r N^r_{jkh} - \dot{\partial}_r T^i_j N^r_{skh} y^s$ (2.1b)

The relation between normal projective curvature tensor N^i_{jkh} and Berwald curvature tensor H^i_{jkh} obtained [Pandey (1981)], as follows:

$N^i_{jkh} = H^i_{jkh} - \frac{1}{n+1} y^i \dot{\partial}_r H^r_{rkh}.$ (2.2)

The normal projective curvature tensor N^i_{jkh} is homogeneous of degree zero in y^i. For the tensor N^i_{jkh} , we have the following identities:

$N^r_{rkh} = H^r_{rkh},$ (2.3a)

$N^i_{jkh} y^i = H^i_{kh},$ (2.3b)

$N^i_{jkh} = -N^i_{jhk}$ (2.3c)

$N^i_{jkh} + N^i_{khj} + N^i_{kjh} = 0,$ (2.3d)

$N_{jk} = N^r_{jkr}.$ (2.3e)

Let us consider the decomposition of the normal projective curvature tensor N^i_{jkh} of a Finsler space of the type (1, 3) as follows:

$N^i_{jkh} = y^i Y_{jkh}$ (2.4)

Where Y_{jkh} non-zero tensor field is called decomposition tensor field.
Further considering the decomposition of the tensor field (2.4) in the form

$N^i_{jkh} = y^i y_j Y_{kh} \; and \; N^i_{jkh} = y^i y_h Y_{jk}$ (2.5)

Let us define

$y^i \lambda_j = \sigma,$ (2.6)

Such λ_j as recurrence vector and σ is decomposition scalar. Therefore, we have

Theorem (2.1). If the normal projective curvature tensor N^i_{jkh} of a Finsler manifolds is decomposable in the form (2.4), then the normal projective curvature tensor N^i_{jkh} and the $h(v)$-torsion tensor H^i_{kh} are decomposable and the tensor H^r_{rkh} is also decomposable in the form (2.4) and the decomposable tensor field Y_{jkh} satisfies.

Proof. In view of (2.4), the identities (2.3c) and (2.3d), can be written as

$$Y_{jkh} + Y_{jhk} = 0 \; and \; Y_{jkh} + Y_{khj} + Y_{hjk} = 0 \tag{2.7}$$

In view of (2.3a), the contraction of the indices i and j in (2.4) gives

$$H^r_{rkh} = y^r Y_{rkh}. \tag{2.8}$$

Using (1.4i) in (2.8), we get

$$Y_{rkh} = \frac{1}{y^r}(H_{hk} - H_{kh}). \tag{2.9}$$

In view of (2.4), equation (2.9) can be written as

$$N^i_{jkh} = \frac{y^i}{y^j}(H_{hk} - H_{kh}). \tag{2.10}$$

Transecting (2.10) by y^j and using (2.3b), we get

$$H^i_{kh} = y^i(H_{hk} - H_{kh}). \tag{2.11}$$

Contraction of the indices i and j in (2.5), using (2.3a) and (1.2), we get

$$H^r_{rkh} = F^2 Y_{kh}. \tag{2.12}$$

Using (1.4i) in (2.12), we get

$$Y_{kh} = \frac{1}{F^2}(H_{hk} - H_{kh}). \tag{2.13}$$

In view of (2.5), equation (2.13) can be written as

$$N^i_{jkh} = y^i y_j \frac{1}{F^2}(H_{hk} - H_{kh}) \tag{2.14}$$

Transvecting (2.14) by y^j and using (2.3b), we get

$$H^i_{kh} = y^i(H_{hk} - H_{kh}). \tag{2.15}$$

Hence, we get

Theorem (2.2). If the normal projective curvature tensor N^i_{jkh} of a Finsler manifolds is decomposable in the form (2.5), then the normal projective curvature tensor N^i_{jkh} and the $h(v)$-torsion tensor H^i_{kh} are defined by (2.14) and (2.15) respectively, the tensor H^r_{rkh} is decomposable in the form (2.12) and the decomposable tensor field Y_{kh} satisfies (2.13).
Now, we have obtained,

Theorem (2.3). In a normal Projective Curvature Tensor Finsler manifold, under the decomposition (2.4), the decomposition tensor field Y_{jkh} behaves like a recurrent tensor field.

Proof. A Finsler manifolds for which the normal projective tensor N^i_{jkh} satisfies the recurrence property with respect to Berwald's connection coefficient Projective Curvature Tensor Finsler manifold is characterized by [Pandey (1981)].

$$B_m P^i_{jkh} = \lambda_m P^i_{jkh}, P^i_{jkh} \neq 0, \tag{2.16}$$

Where $\lambda_m$is non-zero covariant vector field is recurrence vector field. Let us consider a Finsler manifolds whose normal projective curvature tensor N^i_{jkh} satisfies the condition (2.16). Transecting (2.16) by y^j, using (1.3) and (2.3b), we get

$$B_m H^i_{kh} = \lambda_m H^i_{kh}. \tag{2.17}$$

Contraction of the indices i and j in (2.16) and using (2.3a), we get

$B_m H^r_{rkh} = \lambda_m H^r_{rkh}$. (2.18)

Transvecting (2.17) by y^k, using (1.3b) and (1.4f), we get

$B_m H^i_h = \lambda_m H^i_h$. (2.19)

Contraction of the indices i and h in (2.17) using (1.4i), we get

$B_m H_k = \lambda_m H_k$ (2.20)

Transvecting (2.20) by y^k, using (1.3) and (1.4n), we get

$B_m H = \lambda_m H$. (2.21)

We know that the projective curvature tensor P^i_{jkh} satisfying the following:

$\lambda_m P^i_{jkh} + \lambda_k P^i_{jhm} + \lambda_h P^i_{jmk} = 0$. (2.22)

Differentiating (2.4) covariantly with respect to x^m in the sense of Berwald, using (2.16), (2.4) and (1.3), we get

$B_m Y_{jkh} = \lambda_m Y_{jkh}$, (2.23)

Where λ_m is non-zero vector field. Again, Differentiating (2.21) covariantly with respect to x^l in the sense of Berwald and using (2.21), we get

$B_l B_m Y_{jkh} = (B_l \lambda_m) Y_{jkh} + \lambda_m \lambda_l Y_{jkh}$. (2.24)

Interchanging the indices *m* and *l* in (2.24) and subtracting the equation obtained from (2.24), we get

$B_l B_m Y_{jkh} - B_m B_l Y_{jkh} = (B_l \lambda_m - B_m \lambda_l) Y_{jkh}$. (2.25)

Using the commutation formula (2.1b) in (2.25), we get

$(B_l \lambda_m - B_m \lambda_l) Y_{jkh} = -\left(Y_{rkh} N^r_{jml} + Y_{jrh} N^r_{kml} + Y_{jkr} N^r_{hml} + \dot{\partial}_r Y_{jkh} N^r_{sml} y^s\right)$. (2.26)

In view of (2.4) and the homogeneity property of Y_{jkh}, equ. (2.26) can be written as:

$(B_l \lambda_m - B_m \lambda_l) Y_{jkh} = -\left(Y_{rkh} Y_{jml} + Y_{jrh} Y_{kml} + Y_{jkr} Y_{hml} - Y_{jkh} Y_{rml}\right) y^r$. (2.27)

Differentiating (2.27) covariantly with respect to x^n in the sense of Berwald, using (1.3b) and (2.27), we get

$B_n (B_l \lambda_m - B_m \lambda_l) = \lambda_n (B_l \lambda_m - B_m \lambda_l)$. (2.28)

Thus, we get obtained,

Theorem (2.4). In a Projective Curvature Tensor Finsler manifolds under the decomposition (2.4), the recurrence $B_l \lambda_m - B_m \lambda_l$ behaves like a recurrent tensor field.

Again, We have considering the decomposition of the tensor field Y_{jkh} in the form

$Y_{jkh} = \lambda_j Y_{jkh}$. (2.29)

Differentiating (2.29) covariantly with respect to x^m in the sense of Berwald, using (2.23) and (2.29), we get

$\lambda_m \lambda_j Y_{kh} = \left(B_m \lambda_j\right) Y_{kh} + \lambda_j B_m Y_{kh}$. (2.30)

Transvecting (2.30) by y^j, using (2.6) and (1.3a), we get

$B_m Y_{kh} = \lambda_m Y_{kh}$. (2.31)

Thus, we get obtained,

Theorem (2.5). In a Projective Curvature Tensor Finsler manifolds, under the decomposition (2.4) and (2.29), the tensor field Y_{kh} behaves like a recurrent tensor field.

Again we have From (2.27) and (2.29), we get

$$\lambda_j\{(B_l\lambda_m - B_m\lambda_l)Y_{kh} - Y_{ml}(\lambda_k Y_{rh} + \lambda_h Y_{kh})y^r\} = 0. \tag{2.32}$$

Using (2.29) in (2.7), we get

$$-\lambda_j Y_{kh} = \lambda_k Y_{hj} + \lambda_h Y_{jk}. \tag{2.33}$$

From (2.32), (2.33), using (2.6), the fact that the vector field λ_m and the tensor field Y_{jkh} are non-zero, we get

$$B_l\lambda_m - B_m\lambda_l + \sigma Y_{ml} = 0. \tag{2.34}$$

Thus, we get obtained,

Theorem (2.6). In a Projective Curvature Tensor Finsler manifold, under the decomposition (2.4) and (2.34), the necessary and sufficient condition that $B_l\lambda_m = B_m\lambda_l$ is that $\sigma = 0$.

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