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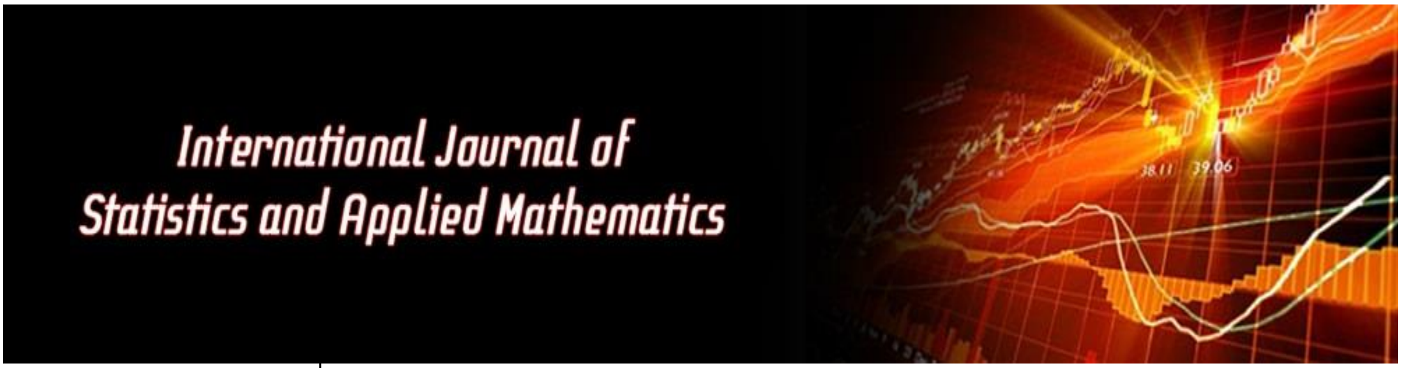

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Bayesian estimation of stress strength reliability P[X>Y] of Lomax and exponential distribution based on right censored sample

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Abstract
The present paper is concerned with the estimation of stress strength reliability R=P[X>Y] when X and Y are the random variables following Lomax and Exponential Distribution based on right censored sample. The main aim of this article is to estimate the Maximum Likelihood estimates of R and Bayesian Estimates of R under Squared Error Loss function, Linex Loss function and Entropy Loss function. Finally the performance of the estimators are evaluated by simulation study.

Keywords: lomax distribution, exponential distribution, maximum likelihood estimation, bayesian estimation, squared error loss function, linex loss function, entropy loss function

1. Introduction

In the literature the problem of estimating the stress strength reliability R= P[X>Y] has been considered as both distribution free and parametric frame works. In stress strength reliability analysis the strength X and the stress Y are considered as random variables. The system fails if at any time the applied stress is exceeds its strength. In stress strength model if the system functions only if its inherent random strength is greater than the random stress applied to it.

The stress strength reliability have wide applications in Quality control, Engineering Statistics, Medical Statistics, Bio statistics etc. The stress strength model was introduced by Birnbaum (1956) [1] and developed by Birnbaum and McCarty (1958) [2]. The term stress strength reliability was first introduced by Church and Harris (1970) [3]. The different stress strength models was studied by Kelly *et al*. (1976) [12], Owen *et al*. (1977) [17], Tong (1977) [18], Jeevan and (1997,1998 and 2016) [8, 9, 11], Kundu and Gupta (2005,2006), Jeevanand *et al*. (2008) [11], Dhanya and Jeevanand (2011, 2012, 2014, 2015 and 2018) [4-7], Neethu and Jeevanand (2021) [15-16] etc.

Let X be the strength of the random variable following Lomax distribution with parameters L(α,1), where α is the shape parameter and Y be the stress of the random variable following exponential distribution with parameter Exp (θ) and corresponding probability density functions are given below.

$$f(x,\alpha,1) = \frac{\alpha}{(1+x)^{\alpha+1}}\ ;\ x > 0, \alpha > 0 \tag{1.1}$$

$$f(y,\theta) = \theta e^{-\theta y}\ ;\ y > 0\ ,\ \theta > 0 \tag{1.2}$$

The stress strength reliability is defined as

$$R=P[X>Y] = \int_0^\infty \int_0^x \frac{\alpha}{(1+x)^{\alpha+1}} \theta e^{-\theta y}\, dxdy;\ 0 < x, y < \infty$$

$$= 1 - [E_\alpha(\theta) e^\theta],\ 0<R<1 \tag{1.3}$$

Where $E_\alpha(\theta)= \int_{-\theta}^{\infty} \frac{e^{-\alpha t}}{t} dt$ is the exponential integral, $E_\alpha(\theta)$ can be evaluated from the exponential integral table.
=This paper has been organized in the following sections.
In section 2, we estimate the Maximum Likelihood estimate of R.In section 3 we estimate the Bayesian estimates of R under Squared error loss function, Linex loss function and Entropy loss function. Finally in section 4 we illustrate the performance of the estimates by using Monte Carlo Simulation.

2. Maximum Likelihood Estimation of R
Consider a right censored sample $\underline{x} = (x_{(1)}, x_{(2)}, \ldots\ldots x_{(n-k)})$ with k observations censored on right taken from Lomax distribution $L(\alpha, 1)$ then its likelihood function is given by

$$L(\underline{x}/\alpha, 1) = \left(1 - F_{(n-k)}\right)^k \prod_{i=1}^{n-k} f(x_i) = \left(1 + x_{(n-k)}\right)^{-\alpha k} \prod_{i=1}^{n-k} \frac{\alpha}{(1+x_i)^{\alpha+1}} \tag{2.1}$$

Let $\underline{y} = (y_1, y_2, \ldots\ldots y_m)$ be the random sample of m observation taken from Exponential distribution $Exp(\theta)$ then its likelihood function is given by

$$L(\underline{y}/\theta)=\prod_{j=1}^{m} \theta\, e^{-\theta y_j} =\theta^m e^{-\theta \sum_{j=1}^{m} y_j} \tag{2.2}$$

The joint likelihood function is given by

$$L(\underline{x}, \underline{y} /\alpha, 1, \theta) =\left(1 + x_{(n-k)}\right)^{-\alpha k} \frac{\alpha^{n-k}}{\prod_{i=1}^{n-k}(1+x_i)^{\alpha+1}}\, \theta^m e^{-\theta \sum_{j=1}^{m} y_j} \tag{2.3}$$

Take logarithm on both sides of (2.3)

$$\log L (\underline{x}, \underline{y} /\alpha, 1, \theta)= -\alpha k \log(1 + x_{(n-k)})+(n-k)\log \alpha-(\alpha + 1) \sum_{i=1}^{n-k} \log (1 + x_i)+m\log \theta- \theta \sum_{j=1}^{m} y_j \tag{2.4}$$

Differentiating (2.4) partially with respect to α and θ and equate to zero we get the MLE of α and θ

$$\hat{\alpha} = \frac{n-k}{\sum_{i=1}^{n-k} \log(1+x_i)+k \log (1+x_{(n-k)})} \tag{2.5}$$

$$\hat{\theta} = \frac{m}{\sum_{j=1}^{m} y_j} \tag{2.6}$$

Using (2.5) and (2.6) in (1.3) we get the MLE of R and it is given by

$$\hat{R}= 1- \left[E_{\left[\frac{n-k}{\sum_{i=1}^{n-k} \log(1+x_i)+k \log (1+x_{(n-k)})}\right]}\left(\frac{m}{\sum_{j=1}^{m} y_j}\right)e^{\left[\frac{m}{\sum_{j=1}^{m} y_j}\right]}\right] ;\ 0< R < 1 \tag{2.7}$$

3. Bayesian Estimation of R
In this section we estimate the Bayesian estimate of R using gamma prior under Squared error loss function, Linex loss functions and Entropy loss function

3.1. Estimation when α and θ are known.
The gamma prior for α is given by

$$g(\alpha) \propto \alpha^{p-1}e^{-\alpha\tau} ,\ \alpha, \tau, p > 0 \tag{3.1}$$

Combining the prior distribution (3.1) and the likelihood function (2.1) the posterior density of α is derived as follows.

$$f(\alpha /\underline{x}) \propto \alpha^{N-k}e^{-\alpha P} \tag{3.2}$$

Where

$N= n+p-1$ and $P = \tau + \sum_{i=1}^{n-k} \log (1 + x_i)+k \log(1 + x_{(n-k)})$

The gamma prior for θ is $g(\theta) \propto \theta^{q-1}e^{-\theta\Psi}$, $\theta, q, \Psi>0$ (3.3)

Combining the prior distribution (3.3) and the likelihood function (2.2) the posterior density of θ is derived as follows.

$$f(\theta/\underline{y}) \propto \alpha^{Q-1}e^{-\theta M} \quad (3.4)$$

Where $Q = m+q$, $M = \Psi + \sum_{j=1}^{m} y_j$

Assume that α and θ are independently distributed the joint posterior density of (α, θ) is given by

$$f(\alpha,\theta/\underline{x},\underline{y}) = \alpha^{N-k}\theta^{Q-1}e^{-(\alpha P+\theta M)} \quad (3.5)$$

Applying the transformation $R = \frac{\alpha}{\alpha+\theta}$ and $S = \alpha + \theta$, S>0, 0<R<1 (3.6)

From (3.6) α = RS and θ = S(1-R)

$$f(R,S/\underline{x},\underline{y}) = (RS)^{N-k}\left(S(1-R)\right)^{Q-1}e^{-S[M(1+R)+RP]} \text{ ; } S>0,\ 0<R<1 \quad (3.7)$$

Integrating out S from (3.7)

$$f(R/\underline{x},\underline{y}) = \int_0^\infty R^{N-k}(1-R)^{Q-1}S^{N-k+Q-1}e^{-S[M(1+R)+RP]}dS = \frac{\Gamma_{(N-k+Q)}}{(M(1+R)+RP)^{N-k+Q}} R^{N-k}(1-R)^{Q-1};0<R<1 \quad (3.8)$$

Bayesian Estimate of R under squared error loss function is given by

$$\widehat{R}_{SLF} = E(R/\underline{x},\underline{y}) = \int_0^1 Rf(R/\underline{x},\underline{y})\ dR = \frac{C_1(1)}{C_1(0)} \quad (3.9)$$

Where $C_1(d) = \Gamma_{(N-k+Q)} \int_0^1 \frac{R^{N+d-k}(1-R)^{Q-1}}{(M(1+R)+RP)^{N-k+Q}} dR$

Bayesian Estimate of R under Linex loss function is given by

$$\widehat{R}_{LLF} = \frac{1}{a}\log V_1 \text{ ; } a \neq 0 \quad (3.10)$$

Where $V_1 = (C_1(0))^{-1}\,\Gamma_{(N-k+Q)} \int_0^1 e^{aR} \frac{R^{N-k}(1-R)^{Q-1}}{(M(1+R)+RP)^{N-k+Q}} dR$ (3.11)

Bayesian Estimate of R under Entropy loss function is given by

$$\widehat{R}_{ELF} = \frac{C_1(1)}{C_1(-1)} \quad (3.12)$$

4 Simulation study

In this section in the absence of real data we study the performance of the estimators obtained in the above section using Monte Carlo simulated data sets. The simulation study has been conducted by generating 1000 samples of sizes n, m=10, 25, 50 each from Lomax distribution and exponential distribution with parameter values 0.5, 2, 3.5 for α and θ. After the samples are generated the Maximum likelihood estimators and Bayesian estimators of reliability are evaluated. The bias and mean square error of estimators of reliability for various values of α and θ are given in the following table.

Table 1: Bias and MSEs (Parentheses) of the Estimates of R for Right Censored Sample

n	m	α	θ	$\widehat{R}_{MLE}$	$\widehat{R}_{SLF}$	$\widehat{R}_{LLF}$	$\widehat{R}_{ELF}$
10	10	0.5	0.5	0.03067 (0.01672)	0.03488 (0.01107)	0.08910 (0.01054)	0.05363 (0.00629)
		0.5	2	0.11301 (0.01374)	0.05196 (0.00612)	0.0087 (0.00209)	0.04146 (0.00496)
		0.5	3.5	0.02635 (0.00456)	0.01424 (0.00103)	0.02058 (0.00105)	0.00602 (0.00062)
		2	0.5	0.08914 (0.02519)	0.06720 (0.01853)	0.04799 (0.01081)	0.01851 (0.00094)
		2	2	0.06253 (0.01232)	0.02716 (0.00287)	0.00586 (0.00222)	0.02342 (0.00171)
		2	3.5	0.27628 (0.08551)	0.01334 (0.02527)	0.18949 (0.04258)	0.14146 (0.02442)
		3.5	0.5	0.03713 (0.01075)	0.04573 (0.01001)	0.07221 (0.00728)	0.05377 (0.00327)
		3.5	2	0.02415 (0.00142)	0.03812 (0.00175)	0.03133 (0.00145)	0.02013 (0.00079)
		3.5	3.5	0.08264 (0.05484)	0.01382 (0.00966)	0.02961 (0.00908)	0.07034 (0.00890)
25	25	0.5	0.5	0.05431 (0.00718)	0.05033 (0.00461)	0.0532 (0.00552)	0.03700 (0.00280)
		0.5	2	0.1097 (0.01364)	0.07127 (0.01264)	0.04904 (0.01011)	0.04515 (0.00972)
		0.5	3.5	0.00272 (0.00121)	0.00302 (0.00117)	0.01598 (0.00106)	0.02156 (0.00059)
		2	0.5	0.00631 (0.03774)	0.03462 (0.01154)	0.02550 (0.00755)	0.04113 (0.00751)
		2	2	0.01238 (0.00378)	0.03041 (0.00235)	0.02664 (0.00133)	0.00054 (0.00129)
		2	3.5	0.03408 (0.00338)	0.00183 (0.00196)	0.06956 (0.01186)	0.03084 (0.00146)
		3.5	0.5	0.02269 (0.01432)	0.01438 (0.00304)	0.01462 (0.00293)	0.01490 (0.00259)
		3.5	2	0.03812 (0.00175)	0.02845 (0.00313)	0.02739 (0.00884)	0.01767 (0.00136)

		3.5	3.5	0.03129 (0.00287)	0.00340 (0.00168)	0.02845 (0.00313)	0.00651 (0.00145)
50	50	0.5	0.5	0.09250 (0.02894)	0.05737 (0.00668)	0.06246 (0.00591)	0.05277 (0.00351)
		0.5	2	0.08974 (0.00905)	0.04973 (0.00874)	0.08381 (0.00836)	0.03014 (0.00516)
		0.5	3.5	0.01809 (0.00161)	0.0055 (0.00092)	0.00216 (0.00128)	0.00525 (0.00079)
		2	0.5	0.10887 (0.05419)	0.08289 (0.01436)	0.15131 (0.04074)	0.07863 (0.01354)
		2	2	0.00083 (0.00160)	0.01850 (0.00116)	0.00813 (0.00492)	0.00663 (0.00064)
		2	3.5	0.02423 (0.00289)	0.02845 (0.00313)	0.03561 (0.00173)	0.02974 (0.00165)
		3.5	0.5	0.01685 (0.02162)	0.06517 (0.01758)	0.0099 (0.01984)	0.06234 (0.01692)
		3.5	2	0.0204 (0.00475)	0.05586 (0.00419)	0.046074 (0.00214)	0.00695 (0.00134)
		3.5	3.5	0.01082 (0.00236)	0.00959 (0.00178)	0.01263 (0.00146)	0.01627 (0.00143)

1. From the table we can observe that when sample size increases bias and mean square error of the Maximum likelihood estimates and Bayesian estimates of reliability are decreases.
2. In Bayesian Estimation, estimates of reliability under entropy loss function has lesser bias than squared error loss function and Linex loss function.
3. From the table we can conclude that Bayesian Estimates of reliability is performed better than the Maximum Likelihood Estimates of reliability.

5. References

1. Birnbaum ZW. On a use of Mann- Whitney Statistics , Proceedings third Berkeley symposium on Mathematical Statistics and Probability 1956;1:13-17.
2. Birnbaum ZW, McCarty RC. A distribution free upper confidence bound for P(X>Y) based on independent samples X and Y. The Annals of Mathematical Statistics 1958;29:558-562.
3. Church JD, Harris B. The estimation of reliability from stress-strength relationships, Techno metrics 1970;12:49-54.
4. Dhanya M, Jeevanand ES. Baye's estimation of Pxy{(X,Y)/X>Y}for Power function distribution, Statistical Methods in Interdisciplinary Studies 2011, 115-127.
5. Dhanya M, Jeevanand ES. Quasi Bayesian Estimation of Px,y({(X,Y)/X >Y}) for the Lomax Distribution, Mathematical Modeling and Applied Soft Computing, Shanga Verlag 2012, 1089-1096.
6. Dhanya M, Jeevanand ES. Estimation Of P(Y<X) for the Power Function Distribution, Proceeding of International conference on Mathematics and its application, Publisher Shanga Verlag 2014.
7. Dhanya M, Jeevanand ES. Semi-Parametric Estimation Of Px,y({(X,Y)/X >Y}) For The Power Function Distribution, International Journal of Engineering, Science and Mathematics 2014;3(2):94-101.
8. Jeevanand ES. "Bayes Estimation of $P(X_2<X_1)$ for a bivariate Pareto distribution", The Statistician, Journal of the Royal Statistical Society D 1997;46(1):93-99.
9. Jeevanand ES. Bayes estimate of the reliability under stress-strength model for the Marshal-Olkin bivariate exponential distribution, IAPQR Transactions 1998;23(2):133-136.
10. Jeevanand ES. Bayes estimation of reliability under stress-strength model when stress is censored at strength, Advances in Mathematical Modelling and its Application, Proceedings of the UGC sponsored National Seminar on Mathematical Modelling and its Application, U.C. College, Aluva, 2016, 11-27.
11. Jeevanand ES, Alice PM, Hitha N. Semi-Parametric Estimation of $P_{X,Y}(X>Y)$, Economic Quality Control (Stochastic and Quality Control) 2008;23(2):171-180.
12. Kelley GD, Kelley JA, Schucany WR. Efficient estimation of P(Y < X) in the exponential case, Technimetrics, 1976, 359 – 360.
13. .Kundu D, Gupta RD. Estimation of P(Y < X) for the Generalized Exponential Distribution, Metrika 2005;61:291-308.
14. Kundu D, Gupta RD. Estimation of P(Y < X) for Weibull distributions, IEEE Transactions on Reliability 2006;55(2):270-280.
15. Neethu Jacob, Jeevanand ES. Semi parametric estimation of stress strength reliability P[X>Y] of Lomax distribution, Far East Journal of Mathematical Sciences 2021;61(2):95-107.
16. Neethu Jacob, Jeevanand ES. Bayesian Estimation of Stress Strength Reliability P[X>Y] of Lomax and Exponential Distribution, Journal of Information Storage and Processing Systems 2021;20:182-189.
17. Owen DB, Craswell KJ, Hanson DL. Non-parametric upper confidence bounds for P(Y < X) and confidence limits for P(Y < X) when X and Y are normal, Journal of the American Statistical Association 1977;59:906-924.
18. Tong H. On the estimation of P(Y < X) exponential families, IEEE Transaction on Reliability 1977;26(1):54-56.