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Transforming demand function to linear function

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Abstract

The research aimed to transform the nonlinear demand function to linear function using statistical and mathematical models by using logarithms and dummy variables. The research resulted in that the linear model is a good alternative for nonlinear model.

Keywords: demand function, nonlinear function, linear function, generalized least square

Introduction

Demand for any commodity or services referring to a consumer's desire to purchase and willingness to pay a price for it. And it has an inverse relationship with price i.e., increasing in price of any commodity or service decreases the demand for it and vice versa. And the graph for demand and price which is called demand curve has a nonlinear function that describes the demand as a function of price as follows [3].

$$D_{it} = \alpha_i P_{it}^b \mu_{it}$$

$$i = 1, 2, 3, \dots, N.$$

$$t = 1, 2, 3, \dots, T \quad (1)$$

Where:

D_{it} : Is the quantity demanded from commodity i at time period t .

P_{it} : Is the price of commodity i at time t .

U_{it} : Is the error term for commodity i at time period t , with the following assumptions:

$$E(\mu_{it}) = 0$$

$$E(\mu_{it}^2) = \sigma_\mu^2$$

$$E(\mu_{it} \mu_{is}) = E(\mu_{it} \mu_{jt}) = E(\mu_{it} \mu_{js}) = 0 \quad \text{for } i \neq j \text{ and } t \neq s$$

α_i : Is the intercept term, and the index " i " indicate that the intercepts are differ from commodity to another.

b : Is the slope coefficient, and it represents the elasticity of demand, which is defined as the relative change in the quantity demanded (ΔD) divided by the relative change in the price (ΔP), i.e.

$$b = \frac{\Delta D}{\Delta P}$$

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The aim of this paper is to transform nonlinear demand function to linear function, - butting in consideration that data is available for many commodities or services - so that it can be estimated by using ordinary least squares (OLS).

Methods and procedures

First, introducing log to the two sides of equation (1):^[3]

$$\text{Log } D_{it} = \text{Log } \alpha_i + b \log P_{it} + \log \mu_{it} \quad (2)$$

When estimating this equation, it's useful to leave it in its log form than to translate it back into exponential form, because change in (say log'x') is approximately equal to the relative change in (x) itself.

This result follows from the elementary differentiation formula:

$$\frac{\partial \text{Log}(x)}{\partial x} = \frac{1}{x}$$

$$\Rightarrow \partial \text{Log}(x) = \frac{\partial x}{x}$$

i.e., change in $\log(x)$ = relative change in (x)

Now, let:

$$\begin{aligned} \text{Log}(D_{it}) &= Y_{it} & \text{Log}(P_{it}) &= X_{it} \\ \text{Log}(\alpha_i) &= a_i & \text{Log}(\mu_{it}) &= e_{it} \end{aligned} \quad (3)$$

Substituting equation (3) into equation (2) we obtain:

$$Y_{it} = a_i + bX_{it} + e_{it} \quad (4)$$

Second, Introducing Dummy Variables to capture valuation in parameters comes from that using least square or generalized least square models must have a constant parameter to be estimated and then using least square or generalized least square is not clear what they are estimators of therefore:^[2]

Now equation (4) can be written as:

$$Y_{it} = \sum_{j=1}^N a_j D_j + bX_{it} + e_{it} \quad (5)$$

Where D_{it} is the dummy variable and defend as:

$$D_{jt} = \begin{cases} 0 & \text{if } j \neq i \\ 1 & \text{if } j = i \end{cases}$$

Equation (5) can be written as:

$$\begin{aligned} Y_{it} &= a_1 D_{1t} + \sum_{j=2}^N a_j D_{jt} + bX_{it} + e_{it} \\ &= a_1 D_{1t} + \sum_{j=2}^N (a_1 + \delta_j) D_{jt} + bX_{it} + e_{it} \quad , a_j = a_1 + \delta_j \\ &= a_1 \sum_{j=1}^N D_{jt} + \sum_{j=2}^N \delta_j D_{jt} + bX_{it} + e_{it} \\ &= a_1 + \sum_{j=2}^N \delta_j D_{jt} + bX_{it} + e_{it} \quad \left[\sum_{j=1}^N D_{jt} = 1 \right] \end{aligned} \quad (6)$$

[The last step obtained by taking the constraint
Third, estimating the parameters:^[4]

We can write equation (5) as follows:

$$Y_{it} = \sum_{j=1}^N a_j D_{jt} + bX_{it} + e_{it}$$

In matrix notation as:

$$\underline{Y}_i = a_j j_T + \underline{X}_i b + \underline{e}_i \quad i = 1, \dots, N \tag{7}$$

Where:

$$\underline{Y}_i = [Y_{i1}, Y_{i2}, \dots, Y_{iT}]'$$

$$j_T = [1, 1, \dots, 1]' \quad (T \times 1)$$

$$\underline{X}_i = [X_{i1} \quad X_{i2} \dots \dots \dots X_{iT}]'$$

$$\underline{e}_i = [e_{i1} \quad e_{i2} \dots \dots \dots e_{iT}]'$$

Or in more compact form as:

$$Y = [I_N \otimes J_T \quad X] \begin{bmatrix} a \\ b \end{bmatrix} + e \quad \text{or} \quad \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_N \end{bmatrix} = \begin{bmatrix} J_T & 0 & X_1 \\ & J_T & X_2 \\ & & \ddots \\ & 0 & J_T & X_N \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \\ b \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_N \end{bmatrix} \tag{8}$$

Now equation (8) takes the form of the general linear mode:

$$Y = X\beta + e \tag{9}$$

Where,

$$Y = [Y_1 Y_2 \dots Y_N]'$$

$$X = [I_N \otimes J_T \quad X]$$

$$\beta = \begin{bmatrix} a \\ b \end{bmatrix}$$

$$e = [e_1 e_2 \dots e_N]'$$

Then, Applying ordinary least square method to estimates $\hat{\beta}$:

$$\hat{\beta} = (X'X)^{-1} X'Y$$

$$ie \quad \hat{\beta} = \begin{bmatrix} \hat{a} \\ b \end{bmatrix} = \left[\begin{bmatrix} (I_N \otimes J_T)' \\ X' \end{bmatrix} \begin{bmatrix} I_N \otimes J_T \\ X \end{bmatrix} \right]^{-1} \begin{bmatrix} (I_N \otimes J_T)' Y \\ X' Y \end{bmatrix}$$

$$= \begin{bmatrix} T_{I_N} & (I_N \otimes J_T)' X \\ X'(I_N \otimes J_T) & X'X \end{bmatrix}^{-1} \begin{bmatrix} (I_N \otimes J_T)' Y \\ X'Y \end{bmatrix} \tag{10}$$

But if N is large the inversion of X in equation (9) will be unreliable, so an alternative expression is suggested for estimating "a" and "b": [2]

$$\hat{b} = [(X'(I_N \otimes D_T)X)]^{-1} X'(I_N \otimes D_T)Y \tag{11}$$

Where:

$$D_T = I_T - \frac{J_T J_T'}{T}$$

and

$$\hat{a}_i = \bar{Y}_i - \bar{X}_i \hat{b}$$

where :

$$\bar{Y}_i = \frac{1}{T} \sum_{t=1}^T Y_{it} \quad , \quad \bar{X}_i = \frac{1}{T} \sum_{t=1}^T X_{it}$$

(These alternative expressions are obtained from equation (10) by using the partition inverse)

And to simplify the computation procedure in practice, it can be shown that D_T is idempotent and hence $(I_N \otimes D_T)$ is also idempotent. Therefore, equation (11) can be written as:

$$\hat{b} = (X'(I_N \otimes D_T)(I_N \otimes D_T)X)^{-1} X'(I_N \otimes D_T)(I_N \otimes D_T)Y = (Z'Z)^{-1} Z'w \tag{12}$$

Where:

$$Z = (I_N \otimes D_T)X \tag{13}$$

$$w = (I_N \otimes D_T)Y \tag{14}$$

Then, transform the observation on the independent variable and the dependent variable as follows: equation (13) will be:

$$Z = (I_N \otimes D_T)X = \begin{bmatrix} D_T & & & O \\ & D_T & & \\ & & \ddots & \\ O & & & D_T \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_N \end{bmatrix} = \begin{bmatrix} D_T X_1 \\ D_T X_2 \\ \vdots \\ D_T X_N \end{bmatrix}$$

Examining the i^{th} vector:

$$D_T X_i = \left(I_T - \frac{J_T J_T'}{T} \right) X_i = X_i - \bar{X}_i J_T$$

$$\text{Since } \frac{J_T X_i}{T} = \bar{X}_i$$

$$D_T X_{it} = X_{it} - \bar{X}_i = X^*_{it} \tag{15}$$

And similarly, equation (14) will be:

$$w = (I_N \otimes D_T)Y \Rightarrow D_T Y_{it} = Y_{it} - \bar{Y}_i = Y^*_{it} \tag{16}$$

Now the transformed model is:

$$Y^*_{it} = X^*_{it} b + e^*_{it} \quad \text{where } e^*_{it} = e_{it} - \bar{e}_i \tag{17}$$

Then the generalized least square estimate for b is:

$$\hat{b} = (X^* X^*)^{-1} X^* Y^* \tag{18}$$

The linear model in (17) has no intercept, because if price is zero there is no demand.

Comparing results between the two models

Applying equation (1) and equation (18) to the following data which show demand (in Packages) and price (in thousand Sudanese pounds) for dates in Atbara market for 20 months:

demand	20	25	23	24	21	32	35	33	31	36	38	27	29	28	25	24	28	27	26	33
price	10	9	11	10	12	8	7	8	9	5	6	9	10	11	8	12	8	9	10	6

Applying equation (1) results in significant model, then applied the alternative model of equation (18) we get the following results:

Table 1: Model Summary^b

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.797 ^a	.635	.615	3.40333
a. Predictors: (Constant), p				
b. Dependent Variable: d				

Table 2: ANOVA^a

Model	Sum of Squares	df	Mean Square	F	Sig.	
1	Regression	363.263	1	363.263	31.363	.000 ^b
	Residual	208.487	18	11.583		
	Total	571.750	19			
a. Dependent Variable: d						
b. Predictors: (Constant), p						

Table 3: Coefficients^a

Model	Unstandardized Coefficients		Standardized Coefficients	t	Sig.	
	B	Std. Error	Beta			
1	(Constant)	-7.991E-16	.761		.000	1.000
	p	-2.249	.402	-.797	-5.600	.000
a. Dependent Variable: d						

The above results shows that the model is highly significant with P-value = 0 in the ANOVA table and R² =63.5% and in the estimated model the price (independent variable) is highly significant (P-value = 0) while the constant is not significant because if price is zero there is no demand which is logical result. Also, Price coefficient is negative which support the inverse relationship between demand and price.

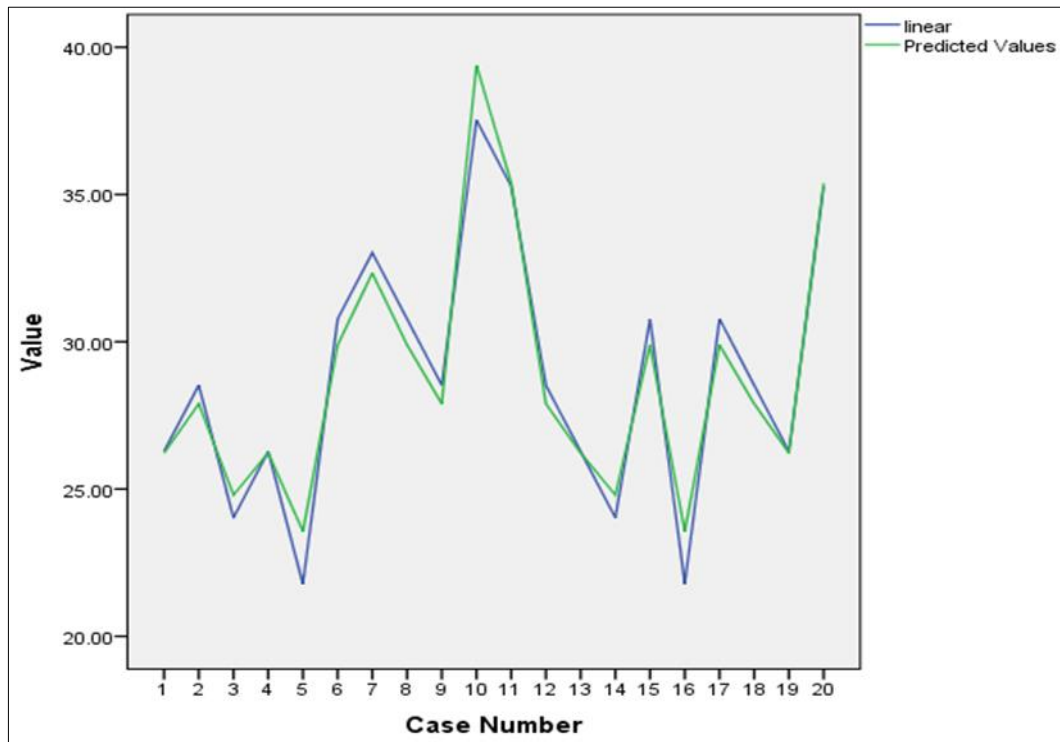


Fig 1: Graphing predicted values for both models we get:

Which indicate that the predicted values for two models are approximately equal.

Conclusion

From the results obtained in this study, we can conclude that the model of equation (17) can be alternative for the model of equation (1).

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