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Mathematical analysis for In-plane equations of equilibrium

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Abstract
In this paper we are studied the response of laminated plates is governed by three coupled displacement equations of equilibrium. Most of research workers prefer to take three separate function s corresponding to three displacement components. But in this article we are going to extend the method of Kushwaha then we get solution of equilibrium equation for composite plate.

Keywords: Laminated plate, coupled displacement, equilibrium, strain

1. Introduction

Composite materials have properties such as light weight, fire resistance, non-magnetic, high strength, high durability. Because of these properties, composite structures are at an important point now a days. Different features can be formed by giving desired features in line with needs. Composite materials; in plaque-based constructions, parallel to the development of material and production technologies, it began to be preferred over isotropic materials. In layered composite plates formed from orthotropic materials, the fibers forming the laminates are placed at different orientation angles to obtain the desired property. Layered composite plates, aircraft bodies and wings, space shuttle bodies, ship and car bodies, building plates, etc. many areas are used.

Turvey [30] obtained a simple exact solution for the static response of a simply supported, anti-symmetrically laminated cross and angle-ply strips resting on Winkler - Pasternak foundation and subjected to a uniform lateral load. The same author in another paper [31] analyzed a uniformly loaded, simply supported anti symmetrically laminated cross- and angle-ply rectangular plate resting on a general (Winkler-Pasternak) elastic foundation. The importance of the material coupling and foundation stiffness on the plate response has been discussed.

2. Certain assumptions

Before we proceed to the derivation of equations of equilibrium with boundary conditions we need certain assumptions.

1. We consider linear elasticity theory.
2. Body forces do not exist.
3. Small determation theory of thin plates is applicable.
4. The thickness of plate is very small in comparison to it's length, breadth and length of crack. We used thin plate theory.
5. Each layer is orthotropic. The principal material direction of each layer are aligned at an angle to the plate axes.
6. The stacking of layers is like that thick layers are in middle and less thicker at outer side. As needed for absence of coupling coefficient $B_{ij} = 0$. Derived analytically too.
7. Elastic properties of each layer is same.

3. Solution of equations of equilibrium

In-plane equations of equilibrium

The equations involving $u^0(x, y)$ and $v^0(x, y)$

$$L_1u^0 + L_2v^2 = 0, \quad L_2u^0 + L_3v^2 = 0 \qquad (3.1),(3.2)$$

with

$$\left.\begin{aligned} L_1 &= A_{11}\frac{\partial^2}{\partial x^2} + A_{66}\frac{\partial^2}{\partial y^2}, \\ L_2 &= (A_{12} + A_{66})\frac{\partial^2}{\partial x\,\partial y} \\ L_3 &= A_{22}\frac{\partial^2}{\partial y^2} + A_{66}\frac{\partial^2}{\partial x^2} \end{aligned}\right\} \qquad (3.3)$$

Eliminating v 0 between (3.1) and (3.2) we get a fourth order partial differential equation in u 0 and given as with

$$\left(a_1\frac{\partial^4}{\partial x^4} + 2a_2\frac{\partial^4}{\partial x^2\partial y^2} + a_3\frac{\partial^4}{\partial y^4}\right)u^0 = 0 \qquad (3.4)$$

$$\left.\begin{aligned} a_1 &= A_{11}A_{66}, a_3 = A_{22}A_{66} \\ 2a_2 &= A_{11}A_{22} + A_{66} - (A_{12} + A_{66})^2 \end{aligned}\right\}, \qquad (3.5)$$

While v is given a wish we took f1 (x) = o in this case.

$$v^0(x,y) = -\frac{1}{(A_{12} + A_{66})}\left[A_{11}\int u^0_{,x}dy + A_{66}\int u^0_{,y}dx\right]$$
$$f_1(x) + f_2(y) \qquad (3.6)$$

$$f_1(x) = a_0 + a_1x, f_2(y) = v_0 + v_1y \qquad (3.6)a$$

$$\left[\left(D_{11}\frac{\partial^4}{dx^4} + 2(D_{12} + D_{66})\right)\frac{\partial^4}{\partial x^2\partial y^2} + D_{22}\frac{\partial^4}{\partial y^4}\right]w(x,y)$$
$$= q(x,y) \qquad (3.7)$$

Transverse equation of Equilibrium is

4. Solutions

The solution of (4.4.4) is obtained by the method of separation of variable the substitution of (3.8) in (3.4) gives

$$\text{Let} \quad u^0(x,y) = \phi(y)\sin(\alpha_n x), \alpha_n = \frac{n\pi}{a} \qquad (4.1)$$

$$\left(a_1\alpha_n^4 - 2a_{12}\alpha_n^2\frac{d^2}{dy^2} + a_3\frac{d^4}{dy^4}\right)\phi(y) = 0 \qquad (4.2)$$

Above is ordinary differential equation. We assume

$\phi(y) = e^{my}$, then

$$a_3m^4 - 2a_2\alpha_n^2m^2 + a_1\alpha_n^4 = 0$$

$$m^2 = \frac{a_2 \pm \sqrt{a_2^2 - a_1a_3}}{a_3}\alpha_n^2$$

$$m = \pm\alpha_n\left[\frac{a_2 \pm \sqrt{a_2^2 - a_1a_3}}{a_3}\right]^{1/2}, \qquad (4.3)$$

$$\gamma_1 = \frac{a_2 + \sqrt{a_2^2 - a_1a_3}}{a_3}, \gamma_2 + \frac{a_2 + \sqrt{a_2^2 - a_1a_3}}{a_3} \qquad (4.4)(4.5)$$

$$m_1 = \gamma_1\alpha_n, \quad m_2 = -\gamma_1\alpha_n;$$

$$m_3 = \gamma_2\alpha_n, \quad m_4 = -\gamma_2\alpha_n$$

We consider such composite material whose stiffness coefficient make.

$$a_2^2 - a_1a_3 > 0$$

Then we get four distinct real values of m. Hence Solution is

$$u^0(x,y) = \sum_{n=1}^{\infty}\left[A_n\cosh(\alpha_n\gamma_1y) + \beta_n\sinh(\alpha_n\gamma_1y)\right.$$
$$\left.+C_n\cosh(\alpha_n\gamma_2b) + D_n\sinh(\alpha_n\gamma_2y)\right]$$
$$\sin(\alpha_n x) \qquad (4.6)$$

$$v^0(x,y) = -\sum_{n=1}^{\infty}\left[\beta_1A_n\sinh(\alpha_n\gamma_1y) + \beta_1B_n\cosh(\alpha_n\gamma_1y)\right.$$
$$\left.+\beta_2C_n\sinh(\alpha_n\gamma_2y) + \beta_2D_n\cosh(\alpha_n\gamma_3y)\right]$$
$$\cos(\alpha_n x) \qquad (4.7)$$

$$\beta_i = \frac{A_{11} - A_{66}\gamma_i^2}{\gamma_i(A_{12} + A_{66})}, i = 1,2 \qquad (4.8)$$

Similarly the solution of (4.7), W (X, y), is given,

$$w = w^c + w^{PI}$$

$$w^c(x,y) = \sum_{n=1}^{\infty}\left[E_n\cosh(\alpha_n\gamma_3y) + F_n\sinh(\alpha_n\gamma_3y)\right.$$
$$\left.+G_n\cosh(\alpha_n\gamma_4y) + H_n\sinh(\alpha_n\gamma_4y)\right]\ \sin(\alpha_n x)$$
$$(4.9)$$

Where, $\pm\gamma_3$ and $\pm\gamma_4$ are four values given as

$$\left.\begin{aligned}\gamma_3 &= \left[D_{12} + 2D_{66} + \sqrt{(D_{12}+2D_{66})^2 - D_{11}D_{22}}\right]^{\frac{1}{2}} / \sqrt{D_{22}} \\ \gamma_4 &= \left[D_{12} + 2D_{66} - \sqrt{(D_{12}+2D_{66})^2 - D_{11}D_{22}}\right]^{\frac{1}{2}} / \sqrt{D_{22}}\end{aligned}\right\}$$

$$w^{PI}(x, y) = \frac{1}{2} w_c(x, 0) + \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} w_{sc}(\alpha_n, \beta_m)\sin(\alpha_n x)\cos\beta_m y \qquad (4.11)$$

5. Result

We used the principle of conservation of energy, along with variational principle, and derived three partial differential equations known as equilibrium equations. SO we evaluated analytically, in plane stresses are singular i.e. They have square root (Cauchy Type) singularity at crack tip Components of bending moment or twisting couple are smooth.

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