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Derivation of confidence intervals for the difference, sum and ratio of coefficients of variation of normal distribution with a known ratio of variances

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Abstract
This research paper proposes the confidence interval for the difference, sum and ratio of coefficients of variation of normal distribution with a known ratio of variance. The modified confidence intervals perform well for both the coverage probability and the expected length. These results are shown via Monte-Carlo simulation.

Keywords: Coverage probability, expected length, Monte-Carlo simulation, confidence interval, coefficients of variation

1. Introduction

The coefficient of variation (CV) of a distribution is a dimensionless number that quantifies the degree of variability relative to the mean. It is a statistical measure for comparing the dispersion of several variables obtained by different units. The population coefficient of variation is defined as a ratio of the population standard deviation (σ) to the population mean (μ) given by $k = \sigma/\mu$. The typical sample estimate of k is given as $\hat{k} = S/\bar{X}$, where S is the sample standard deviation, the square root of the unbiased estimator of population variance, and $\bar{X}$ is the sample mean.

The coefficient of variation has been widely used in many areas such as science, medicine, engineering, economics, and others. For example, the coefficient of variation was employed by Ahn (1995) [1] to analyze the uncertainty of fault trees. Gong and Li (1999) [6] assessed the strength of ceramics by using the coefficient of variation. Faber and Korn (1991) [4] applied the coefficient of variation as a way of including a measure of variation in the mean synaptic response of the central nervous system. The coefficient of variation has also been used to assess the homogeneity of bone test samples to help determine the effect of external treatments on the properties of bones as researched by Hammer *et al* (1995). Billings *et al.* (1993) [7, 2] used the coefficient of variation to study the impact of socioeconomic status on hospital use in New York City. In finance and actuarial science, the coefficient of variation can be used as a measure of relative risk and a test of the equality of the coefficients of variation for two stocks as studied by Millerand and Karson (1977) [9]. Furthermore, Pyne *et al*. (2004) [10] studied the variability of the competitive performance of Olympic swimmers by using the coefficient of variation.

The ratio of two independent coefficients of variation (CVs) is a particular problem in the estimation of two independent normally distributed random variables. This problem has previously appeared in the literature of Verrill and Johnson (2007) [15]. Their simulation results indicated that the performance of Verrill and Johnson confidence interval for the ratio of coefficients of variation by using the normal approximation worked quite well for small samples and the coverage probability of Verrill and Johnson confidence interval is not at least the nominal confidence levels.

Recently, Wongkho *et al.* (2015) [17] have shown that the new confidence intervals based on the generalized confidence interval (GCI) of Weerahandi (1993) [16] and the new confidence interval based on the method of variance estimates recovered from the confidence limits

(Mover) described by Donner and Zou (2010) [3] are better than Verrill and Johnson's confidence interval. As a result, they recommended both methods for constructing the confidence interval for the ratio of coefficients of variation.

In this paper, this research study considers not only the ratio of the CVs of normal distribution but also the difference and sum between CVs when we know a ratio of variances. Schechtman and Sherman (2007) [12] described a situation of a known ratio of variances arises in practice when two instruments report (averaged) response of the same object based on a difference number of replicates. If the two instruments have the same precision for a single measurement, then the ratio of the variance of the responses is known, and it is simply the ratio of the number of replicates going into each response. They finally proposed a t-test statistic, which has an exact t-distribution with n+m-2 degrees of freedom, compared to the Satterthwaite's t-test statistic. They found that their proposed test has more power than an existing Satterthwaite's test. However, they did not investigate the coverage probability and the expected length of the confidence intervals for the difference and the ratio of CVs when the ratio of variances is known.

2. Confidence intervals for the difference, sum and ratio of coefficients of variation of normal distribution with a known ratio of variance

Let $X_i \sim N(\mu_1, \sigma^2{}_1), i = 1,2,\dots,n, Y_j \sim N(\mu_2, \sigma^2{}_2), j = 1,2,\dots,m$, and $\mu_1, \mu_2, \sigma^2{}_1$ and $\sigma^2{}_2$ be respectively the population means and population variances of X and Y. Also, let $\bar{X} = n^{-1}\sum_{i=1}^{n} X_i, \bar{Y} = m^{-1}\sum_{j=1}^{m} Y_j, S_1^2 = (n-1)^{-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$ and $S_2^2 = (m-1)^{-1}\sum_{j=1}^{m}(Y_j - \bar{Y})^2$ be estimators of the population means and population variances of X and Y respectively. Then the construction of the confidence interval for the difference, sum and ratio of coefficients of variation are supposed to be $\eta = \theta_1/\theta_2, \delta = \theta_1 - \theta_2$ and $\Omega = \theta_1 + \theta_2$ $where$ $\theta_1 = \sigma_1/\mu_1$ and $\theta_2 = \sigma_2/\mu_2$. In some cases, when the ratio of variances are known as explained by Shechman and Sherman (2007) [12] and further expanded by the author, the parameter of interest, $\eta = \theta_1/\theta_2$ reduces to $\eta = \frac{\sigma_1/\mu_1}{\sigma_2/\mu_2} = \frac{\mu_2}{\mu_1}.\frac{\sigma_1}{\sigma_2} = \sqrt{\frac{n}{m}}.\frac{\mu_2}{\mu_1} = c\frac{\mu_2}{\mu_1}, c = \sqrt{\frac{n}{m}} = \frac{\sigma_1}{\sigma_2}$. Also, the parameter of $\delta = \theta_1 - \theta_2$ reduces to $\delta = \sigma_1/\mu_1 - c^{-1}\sigma_1/\mu_2$. Similarly, the parameter of $\Omega = \theta_1 + \theta_2$ reduces to $\Omega = \sigma_1/\mu_1 + c^{-1}\sigma_1/\mu_2$

2.1 The generalized confidence interval (GCI)

The Generalized Confidence Interval (GCI) for the difference, sum and ratio between the coefficients of variation of normal distributions is being presented in this subsection based on the works of Weerahandi (1993) [16]. Weerahandi (1993) [16] defined a generalized pivot (GP) as a statistic that has a distribution free of unknown parameters and an observed value of a generalized pivotal quantity does not depend on nuisance parameters. In a general setting, a generalized pivot can be defined as follows: Let X be a random quantity having a density function $f(X,\xi)$ $where$ $\xi = \{\theta, \tau\}$ are unknown parameters; θ is the parameter of interest and τ is a nuisance parameter. Let x be the observed value of X. In the procedure to construct the confidence interval for θ, we start with a generalized pivotal quantity $R(X,x,\xi)$ which is a function of random variable X, its observed value x and the parameter ξ. Also, $R(X,x,\xi)$ is required to satisfy the following conditions:

Condition 1: For fix x, a probability distribution of $R(X,x,\xi)$ is free of unknown parameters.

Condition 2: The observed pivot, which is defined as $R(x,x,\xi)$,, does not depend on nuisance parameter.

Now, we construct confidence interval for $\eta = \theta_1/\theta_2$ where $\theta_1 = \sigma_1/\mu_1$ and $\theta_2 = \sigma_2/\mu_2$, and the ratio of variances $\frac{\sigma_1}{\sigma_2} = \sqrt{\frac{n}{m}}$. It is straightforward to see that:

$$\begin{aligned} \delta &= \frac{\sqrt{\frac{(n-1)S_1^2}{U_1}}}{\bar{X}-T_1S_1/\sqrt{n}} - \frac{c^{-1}\sqrt{\frac{(n-1)S_1^2}{U_1}}}{\bar{Y}-T_2S_2/\sqrt{m}} \\ \Omega &= \frac{\sqrt{\frac{(n-1)S_1^2}{U_1}}}{\bar{X}-T_1S_1/\sqrt{n}} + \frac{c^{-1}\sqrt{\frac{(n-1)S_1^2}{U_1}}}{\bar{Y}-T_2S_2/\sqrt{m}} \\ \eta &= c\frac{\bar{Y}-T_2S_2/\sqrt{m}}{\bar{X}-T_1S_1/\sqrt{n}} \end{aligned} \quad (1)$$

where $T_1 \sim t_{n-1}, T_2 \sim t_{m-1}, U_1 = \frac{(n-1)S_1^2}{\sigma_1^2} \sim \chi_{n-1}^2$ and $U_2 = \frac{(m-1)S_2^2}{\sigma_2^2} \sim \chi_{m-1}^2$

Using equation (1), we can define generalized pivotal quantity as:

$$\begin{aligned} Q(X,Y,x,y,\xi) &= \frac{\sqrt{\frac{(n-1)s_1^2}{U_1}}}{\bar{x}-T_1s_1/\sqrt{n}} - \frac{c^{-1}\sqrt{\frac{(n-1)s_1^2}{U_1}}}{\bar{y}-T_2s_2/\sqrt{m}} \\ P(X,Y,x,y,\xi) &= \frac{\sqrt{\frac{(n-1)s_1^2}{U_1}}}{\bar{x}-T_1s_1/\sqrt{n}} + \frac{c^{-1}\sqrt{\frac{(n-1)s_1^2}{U_1}}}{\bar{y}-T_2s_2/\sqrt{m}} \\ R(X,Y,x,y,\xi) &= c\frac{\bar{y}-T_2s_2/\sqrt{m}}{\bar{x}-T_1s_1/\sqrt{n}} \end{aligned} \quad (2)$$

where $s_1, s_2, \bar{x}$ and $\bar{y}$ are the observed values of $S_1, S_2, \bar{X}$ and $\bar{Y}$ respectively.

It is easy to see that $Q(X,Y,x,y,\xi), P(X,Y,x,y,\xi)\ and\ R(X,Y,x,y,\xi)$ satisfy Conditions 1 and 2 above. Therefore, the $100(1-\alpha)\%$ generalized confidence interval for $\eta, \Omega\ and\ \delta$ are respectively:

$$
\begin{aligned}
CI_{gci1} &= [Q_{\alpha/2}, Q_{1-\alpha/2}] \\
CI_{gci2} &= [P_{\alpha/2}, P_{1-\alpha/2}] \\
CI_{gci3} &= [R_{\alpha/2}, R_{1-\alpha/2}]
\end{aligned} \tag{3}
$$

where $Q_{1-\alpha/2}, P_{1-\alpha/2}\ and\ R_{1-\alpha/2}$ are the $(1-\alpha/2)$th percentile for $Q(X,Y,x,y,\xi)$, $P(X,Y,x,y,\xi)$ and $R(X,Y,x,y,\xi)$ respectively.

2.2 The method of variance estimates recovery (Mover)

Zou, Huo and Taleban (2009), and Donner and Zhou (2010) [18, 3] as researchers proposed a simple confidence interval for the sum of the parameters $\theta_1\ and\ \theta_2\ (that\ is, \theta_1 + \theta_2)$ is in the form of:

$$CI_0 = \left\{\widehat{\theta_1} + \widehat{\theta_2} - Z_{1-\alpha/2}\sqrt{var(\widehat{\theta_1}) + var(\widehat{\theta_2})}, \widehat{\theta_1} + \widehat{\theta_2} + Z_{1-\alpha/2}\sqrt{var(\widehat{\theta_1}) + var(\widehat{\theta_2})}\right\}$$

where $\widehat{\theta_1}\ and\ \widehat{\theta_2}$ are the estimates of the parameters $\theta_1\ and\ \theta_2$ respectively. Suppose $CI_0 = [L, U]$ is the confidence interval for $\theta_1 + \theta_2$. Furthermore, Zou, Huo and Taleban (2009), and Donner and Zhou (2010) [18, 3] also found that:

$$
\begin{aligned}
L &= \widehat{\theta_1} + \widehat{\theta_2} - \sqrt{(\widehat{\theta_1} - l_1)^2 + (\widehat{\theta_2} - l_2)^2} \\
U &= \widehat{\theta_1} + \widehat{\theta_2} + \sqrt{(u_1 - \widehat{\theta_1})^2 + (u_2 - \widehat{\theta_2})^2}
\end{aligned} \tag{4}
$$

where $\theta_1 \epsilon (l_1, u_1)\ and\ \theta_2 \epsilon (l_2, u_2)$, and the confidence interval for $\theta_1 - \theta_2$ is $\theta_1 + (-\theta_2)$. It is noted that the confidence interval for $-\theta_2$ is $(-u_2, l_2)$ and it replaced $\theta_2 \epsilon (-u_2, l_2)$ in equation (4). Therefore the confidence interval for $\theta_1 - \theta_2$ is:

$$
\begin{aligned}
L_1 &= \widehat{\theta_1} - \widehat{\theta_2} - \sqrt{(\widehat{\theta_1} - l_1)^2 + (\widehat{\theta_2} - l_2)^2} \\
U_1 &= \widehat{\theta_1} - \widehat{\theta_2} + \sqrt{(u_1 - \widehat{\theta_1})^2 + (u_2 - \widehat{\theta_2})^2}
\end{aligned} \tag{5}
$$

Now, we set $\theta_1 = K_1 = \sigma_1/\mu_1$, $\theta_2 = K_2 = \sigma_2/\mu_2$, $\hat{\theta}_1 = \widehat{K}_1 = S_1/X$, $\hat{\theta}_2 = \widehat{K}_2 = S_2/Y$, and putting these in equation (5), the confidence interval for $\theta_1 - \theta_2$ is as follows:

$$CL_G = [L_1,\ U_1]$$

Donner and Zhou (2010) [3] again proposed the confidence interval for the ratio of parameters, θ_1/θ_2, by using the method of variance estimates recovery in 2010. They constructed the confidence interval for θ_3/θ_4 by using the confidence limits $l_i, u_i, i = 3,4$ where l_i, u_i are the $100\ (1-\alpha/2)\%$ two-sided confidence intervals for θ_i. Then, the confidence interval for θ_3/θ_4 is given by η_L, η_U, where:

$$\eta_L = \frac{\hat{\theta}_3\hat{\theta}_4 - \sqrt{(\hat{\theta}_3\hat{\theta}_4)^2 - l_3 u_4\ 2\hat{\theta}_3 - l_3\ 2\hat{\theta}_4 - u_4}}{u_4\ 2\hat{\theta}_4 - u_4} \tag{6}$$

$$\eta_U = \frac{\hat{\theta}_3\hat{\theta}_4 + \sqrt{(\hat{\theta}_3\hat{\theta}_4)^2 - u_3 l_4\ 2\hat{\theta}_3 - u_3\ 2\hat{\theta}_4 - l_4}}{l_4\ 2\hat{\theta}_4 - l_4} \tag{7}$$

Following the research of Donner and Zhou (2010) [3], the confidence interval for $\delta = \sigma_1/\mu_1 - c^{-1}\,\sigma_1/\mu_2$ is starting from the following:

A confidence interval for $\theta_1 = \sigma_1/\mu_1$ which as Mahmoudvand and Hassani (2009) [8] is:

$$l_1, u_1 = \left[\frac{\widehat{K}_1}{2 - c_n + Z_{1-\alpha/2}\sqrt{1-c_n^2}}, \frac{\widehat{K}_1}{2 - c_n - Z_{1-\alpha/2}\sqrt{1-c_n^2}}\right]$$

$$c_n = \sqrt{\frac{2}{n-1}}\frac{\Gamma(n/2)}{\Gamma((n-1)/2)}$$

and $\hat{\theta}_1 = \hat{k}_1, \widehat{K}_1 = S_1/\bar{X}$

Following with a confidence interval for $c^{-1}\,\sigma_1/\mu_2$ which is:

$l_2.u_2 = \eta_L, \eta_U, \theta_3 = c^{-1}\sigma_1, \theta_4 = u_2$ and confidence intervals for $\theta_3 = c^{-1}\sigma_1, \theta_4 = u_2$ are respectively $l_3.u_3 = \left(c^{-1}\sqrt{\frac{(n-1)S_1^2}{\chi^2_{1-\alpha/2,(n-1)}}} \le c^{-1}\sigma_1 \le c^{-1}\sqrt{\frac{(n-1)S_1^2}{\chi^2_{\alpha/2,(n-1)}}}\right)$ and

$l_4.u_4 = \bar{Y} - z_{1-\alpha/2}S_2/\sqrt{m} \le \mu_2 \le \bar{Y} + z_{1-\alpha/2}S_2/\sqrt{m}, \hat{\theta}_2 = c^{-1}S_1/\bar{Y}, \hat{\theta}_3 = c^{-1}S_1,\ \hat{\theta}_4 = \bar{Y}$, putting these $l_3.u_3$ and $l_4.u_4$ in equations (6) and (7), we then have $l_2.u_2 = \eta_L, \eta_U$, and finally putting $l_1.u_1$ and $l_2.u_2$ in equation (5), we then have a confidence interval for $\delta = \sigma_1/\mu_1 - c^{-1}\,\sigma_1/\mu_2$. This confidence interval is called CI_{rov1}.

The confidence interval for $\eta = \frac{\sigma_1/\mu_1}{\sigma_2/\mu_2} = \frac{\mu_2}{\mu_1}.\frac{\sigma_1}{\sigma_2} = \sqrt{\frac{n}{m}}.\frac{\mu_2}{\mu_1} = c\frac{\mu_2}{\mu_1}, c = \sqrt{\frac{n}{m}}, c^2 = \frac{n}{m}$ is therefore constructed from equations (6) and (7) by replacing:

$l_3 = c(\bar{Y} - z_{1-\alpha/2}S_2/\sqrt{m}), u_3 = c(\bar{Y} + z_{1-\alpha/2}S_2/\sqrt{m})$ and

$l_4 = \bar{X} - z_{1-\alpha/2}S_1/\sqrt{n}, u_4 = \bar{X} + z_{1-\alpha/2}S_1/\sqrt{n}$

$\hat{\theta}_3 = c\bar{Y}, \hat{\theta}_4 = \bar{X}$. This confidence interval is called CI_{rov1}.

2.3. The confidence interval for θ based on fieller's method

In the construction of the confidence interval for $\eta = c\,\mu_2/\mu_1$ by the application of Fieller's method (1954) [5]:

$$T = \frac{(c\bar{Y} - \theta\bar{X})(\mu_1 - c\mu_2)}{\sqrt{\left[\frac{\theta^2}{n} + \frac{c}{m}\right]S^2}}, S^2 = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2 + \sum_{i=1}^{n}(Y_i - \bar{Y})^2}{n+m} \quad (8)$$

a statistics $\sqrt{(n+m-2)/(n+m)}\,T$ follows t-distribution with (n+m-2) degrees of freedom.
A 100(1-α)% confidence interval for θ, by solving equation (8), which is:

$$CI_{F^{**}} = \left[\theta: \sqrt{\frac{(n+m-2)}{n+m}}/T/< t_{1-\alpha/2}\right] = \left[\theta:/c\bar{y} - \theta\bar{x}/\le \sqrt{\frac{(n+m)(c/m + \theta^2/n)}{n+m-2}S^2}\right]$$

where $t_{1-\alpha/2}$ is the $(1-\alpha/2)$th percentile of t-distribution with $n+m-2$ degrees of freedom. Note that T has an exact t-distribution with $n+m-2$ degrees of freedom. The confidence interval $CI_{F^{**}}$ can be transformed into:

$$CI_F = \left[\frac{c\bar{Y}}{\bar{X}} - d\frac{\sqrt{w}}{\bar{X}}, \frac{c\bar{Y}}{\bar{X}} + d\frac{\sqrt{w}}{\bar{X}}\right] \text{ where } d = t_{1-\alpha/2, n+m-2} \text{ and } w = \sqrt{\frac{(n+m)(c/m + \theta^2/n)}{n+m-2}S^2}, \hat{\theta} = \frac{\bar{Y}}{\bar{X}}$$

3. Results of simulation studies

Extensive studies and evaluations were done on the performance of all confidence intervals: $CI_{gci1}, CI_{gci2}, CI_{rov1}, CI_{rov2}, CI_F$ via Monte-Carlo simulation. The simulation studies are carried out to evaluate the coverage probabilities and expected lengths of each confidence interval. The number of simulation runs is equal to 10,000 and the design value of the nominal confidence level is 0.95. For the generalized pivotal approach, in each of 10,000 simulations, 500 pivotal quantities are used. All data are generated from normal distribution with means $\mu_1 = \mu_2 = 1$ and standard deviations $\sigma_1 = \sigma_2 = 1$, $c = [1,2,4,8,16]$ and sample sizes $(n, m) = (10,20,40,100)$. The results were performed using a program written in the R statistic software. The simulation results are shown in Tables 1 and 2. As seen in Table 1, the confidence interval CI_{gci1} has coverage probabilities approximately nominal confidence level of 0.95 in all cases, but the coverage probabilities of the confidence interval CI_{rov1} are slightly decreasing from the nominal value as c is increasing. This means the coverage probabilities of the confidence interval CI_{rov1} is not stable, that is, it is depending on the values of c. Therefore, the confidence interval CI_{gci1} is chosen for the ratio of coefficient of variations (CVs) with a known ratio of variances $\eta = \theta_1/\theta_2$. From Table 2, the confidence interval CI_F using the Fieller's method has produced a wider confidence interval as usual since coverage probabilities of this confidence interval are over the nominal level at 0.95. Therefore, the confidence interval CI_F is not considered. The confidence interval CI_{gci2} have much coverage probabilities than the confidence interval for MOVER method, CI_{rov2} for all sample sizes and values of c. Therefore, the confidence interval CI_{gci2} is chosen for the difference between CVs when we know a ratio of variances. The simulation studies was as well extended to the difference and sum of variances.

Table 1: Simulation results of 95% confidence intervals for the ratio, difference and sum of two independent coefficient of variation in normal distribution

N, M	C	GCI		Mover		Ratio (Mover/GCI)	Difference (Mover - GCI)	SUM (Mover + GCI)
		COV	Ex. Length	COV	Ex. Length			
(10, 10)	1	0.9524	1.9095	0.9982	0.9754	0.5108	-0.9341	2.8849
	2	0.9382	0.2344	0.9863	0.6992	2.9829	0.4648	0.9336
	4	0.9497	0.4532	0.9525	0.5897	1.3012	0.1365	1.0429

	8	0.9542	0.6940	0.9256	0.5496	0.7919	-0.1444	1.2436
	16	0.9546	0.4397	0.9045	0.5419	1.2324	0.1022	0.9816
(10, 20)	1	0.9507	0.2573	0.9992	0.8981	3.4905	0.6408	1.1554
	2	0.9440	0.1966	0.9859	0.6687	3.4013	0.4721	0.8653
	4	0.9482	0.3370	0.9544	0.5756	1.7080	0.2386	0.9126
	8	0.9536	1.6735	0.9238	0.5467	0.3267	-1.1268	2.2202
	16	0.9594	0.9662	0.9057	0.5433	0.5623	-0.4229	1.5095
(20, 10)	1	0.9490	0.5249	0.9930	0.6615	1.2602	0.1366	1.1864
	2	0.9455	0.4208	0.9850	0.4560	1.0837	0.0352	0.8768
	4	0.9464	0.4974	0.9517	0.3789	0.7618	-0.1185	0.8763
	8	0.9458	0.3702	0.9210	0.3513	0.9489	-0.0189	0.7215
	16	0.9521	0.6224	0.9061	0.3446	0.5537	-0.2778	0.9670
(20, 40)	1	0.9471	0.4302	0.9996	0.5403	1.2559	0.1101	0.9705
	2	0.9447	0.6600	0.9873	0.4058	0.6148	-0.2542	1.0658
	4	0.9509	0.3359	0.9518	0.3585	1.0673	0.0226	0.6944
	8	0.9542	0.4422	0.9275	0.3451	0.7804	-0.0971	0.7873
	16	0.9480	0.3574	0.8986	0.3434	0.9608	-0.0140	0.7008
(40, 20)	1	0.9494	0.2106	0.9933	0.4168	1.9791	0.2062	0.6274
	2	0.9455	0.2436	0.9829	0.2931	1.2032	0.0495	0.5367
	4	0.9472	0.1952	0.9523	0.2475	1.2679	0.0523	0.4427
	8	0.9481	0.3081	0.9248	0.2344	0.7608	-0.0737	0.5425
	16	0.9526	0.3373	0.9084	0.2307	0.6840	-0.1066	0.5680
(40, 40)	1	0.9489	0.1933	0.9984	0.3752	1.9410	0.1819	0.5685
	2	0.9428	0.2265	0.9861	0.2760	1.2185	0.0495	0.5025
	4	0.9442	0.2949	0.9478	0.2417	0.8196	-0.0532	0.5366
	8	0.9474	0.2825	0.9204	0.2328	0.8241	-0.0497	0.5153
	16	0.9494	0.3027	0.9085	0.2301	0.7602	-0.0726	0.5328
(20, 20)	1	0.9465	0.3104	0.9982	0.5768	1.8582	0.2664	0.8872
	2	0.9441	0.2985	0.9877	0.4195	1.4054	0.1210	0.7180
	4	0.9476	0.4294	0.9528	0.3627	0.8447	-0.0667	0.7921
	8	0.9437	0.3933	0.9204	0.3461	0.8800	-0.0472	0.7394
	16	0.9500	0.3161	0.9042	0.3427	1.0842	0.0266	0.6588
(50, 50)	1	0.9489	0.2298	0.9982	0.3303	1.4373	0.1005	0.5601
	2	0.9432	0.2412	0.9840	0.2424	1.0050	0.0012	0.4836
	4	0.9438	0.3625	0.9489	0.2138	0.5898	-0.1487	0.5763
	8	0.9476	0.2106	0.9210	0.2056	0.9763	-0.0050	0.4162
	16	0.9522	0.2290	0.9095	0.2037	0.8895	-0.0253	0.4327
(100, 100)	1	0.9517	0.1658	0.9985	0.2261	1.3637	0.0603	0.3919
	2	0.9470	0.1905	0.9838	0.1674	0.8787	-0.0231	0.3579
	4	0.9443	0.1664	0.9498	0.1481	0.8900	-0.0183	0.3145
	8	0.9464	0.2155	0.9210	0.1427	0.6622	-0.0728	0.3582
	16	0.9527	0.1581	0.9101	0.1417	0.8963	-0.0164	0.2998

Table 2: Simulation results of 95% confidence intervals three independent coefficient of variation in normal distribution

N, M	C	Filler		GCI		Mover	
		COV	Ex. Length	COV	Ex. Length	COV	Ex. Length
(10, 10)	1	0.9968	1.4486	0.9324	1.3459	0.8877	1.0275
	2	0.9976	2.9047	0.9327	2.7029	0.8846	2.0654
	4	0.9969	5.8039	0.9355	5.3996	0.8893	4.1368
	8	0.9966	11.6020	0.9361	10.9313	0.8918	8.1375
	16	0.9965	23.1812	0.9316	21.4523	0.8847	17.1837
(10, 20)	1	0.9974	1.3270	0.9335	1.1719	0.8973	0.8845
	2	0.9985	2.6499	0.9305	2.3890	0.8917	1.8384
	4	0.9980	5.2802	0.9340	4.7409	0.8997	3.6834
	8	0.9979	10.6300	0.9340	9.6618	0.8946	5.6075
	16	0.9978	21.2037	0.9312	19.0954	0.8969	3.8131
(20, 10)	1	0.9983	1.2889	0.9302	0.9115	0.8937	0.7721
	2	0.9990	2.5790	0.9338	1.8189	0.9014	1.5436
	4	0.9982	5.1536	0.9344	3.6573	0.9019	3.1016
	8	0.9985	10.3360	0.9351	7.3068	0.9008	6.1925
	16	0.9975	20.6481	0.9328	14.6065	0.8931	12.3813
(20, 40)	1	0.9996	1.0665	0.9404	0.6254	0.9258	0.5711
	2	0.9996	2.1283	0.9404	1.2428	0.9253	1.1350
	4	0.9997	4.2666	0.9418	2.5008	0.9259	2.2855
	8	0.9995	8.5451	0.9391	5.0129	0.9242	4.5824
	16	0.9997	17.0835	0.9381	10.0070	0.9236	9.1612
(40, 20)	1	0.9998	1.0533	0.9378	0.5829	0.9219	0.5404
	2	0.9999	2.1101	0.9426	1.1648	0.9246	1.0798
	4	0.9998	4.2199	0.9407	2.3383	0.9269	2.1676

	8	0.9999	8.4298	0.9384	4.6579	0.9233	4.3197
	16	0.9998	16.8480	0.9409	9.3221	0.9253	8.6371
(40, 40)	1	0.9999	0.9480	0.9410	0.4688	0.9326	0.4476
	2	1.0000	1.8923	0.9408	0.9336	0.9319	0.8910
	4	0.9999	3.7917	0.9443	1.8774	0.9356	1.7926
	8	0.9999	7.5991	0.9420	3.7573	0.9343	3.5883
	16	0.9998	15.1987	0.9444	7.5116	0.9365	7.1789
(20, 20)	1	0.9990	1.1544	0.9382	0.7225	0.9180	0.6524
	2	0.9995	2.3132	0.9374	1.4475	0.9169	1.3064
	4	0.9997	4.6236	0.9418	2.8911	0.9193	2.6108
	8	0.9991	9.2202	0.9379	5.7490	0.9183	5.1909
	16	0.9990	18.4932	0.9417	11.5339	0.9236	10.4191
(50, 50)	1	1.0000	0.8917	0.9430	0.4129	0.9359	0.3979
	2	1.0000	1.7847	0.9441	0.8260	0.9378	0.7957
	4	1.0000	3.5697	0.9490	1.6555	0.9421	1.5942
	8	1.0000	7.1428	0.9449	3.3072	0.9389	3.1877
	16	1.0000	14.2753	0.9427	6.6108	0.9330	6.3733
(100, 100)	1	1.0000	0.7437	0.9519	0.2874	0.9506	0.2795
	2	1.0000	1.4883	0.9510	0.5685	0.9464	0.5586
	4	1.0000	2.9699	0.9493	1.1360	0.9478	1.1150
	8	1.0000	5.9517	0.9440	2.2778	0.9409	2.2348
	16	1.0000	11.8993	0.9498	4.5506	0.9463	4.4682

4. Conclusion

The confidence intervals for the difference, sum and ratio of coefficients of variation of normal distribution with a known ratio have been studied. The performances of theses confidence intervals were assessed in terms of coverage probabilities and expected lengths through simulation studies. The simulation results indicated that the GCI method produce the coverage probability close to the nominal confidence level, 0.95 for all the sample sizes and values of c. Therefore, the recommendation for the difference, sum and ratio of CVs when the ratio of variances is used to construct the confidence intervals is suggested.

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