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Mathematical modeling of flow of fluids in an open channel of parabolic cross-section

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Abstract
This study finds turbulent, non-uniform and Newtonian fluid flows with a parabolic cross-section in the open channel. On parabolic channels, we examine the effects of top width, channel slope and energy coefficient on flow velocity. Because of its stability, convergence and precision, the governing equations are solved by finite difference approximation approach. Using MATLAB software, the result is presented graphically. The findings are that; an increase in the channel slope, top width and energy coefficient has been shown to lead to increased velocity of flow. This study helps in floods control, construction of channels such as canals and crop irrigation.

Keywords: rate of metabolism, blood mass stream rate, warm conductivity, warm era, limited component method, Pennes Bio - heat model

1. Introduction

1.1 Background information

Heavy Rainfall in Kenya has been experienced in most parts of the country in several years. Areas most affected by the Heavy rainfall includes; Baringo, Budalangi, TanaRiver, West Pokot, Makueni and Machakos. This amount of water from heavy rainfall causes flooding in the affected areas. Flooding leads to the destruction of houses, bridges and other land structures. It also leads to the death of people and animals. The soil, crops and plantations have been destroyed. In addition, roads made impassable and transport paralyzed. Moreover, flooding water is a breeding ground for mosquitoes that causes malaria.

The handling of such unexpected amounts of water is a challenge for open channel designers in Kenya and around the world. The solution to such disasters is the design of channels of different cross-sections such as circular, triangular, rectangular, trapezoidal, elliptical and parabolic. Currently floods are still a challenge in Kenya and there is a need to design an efficient channel that would transport the maximum amount of water in flood areas to the areas needed for crop irrigation and hydroelectric power generation. The current Government of Kenya focuses on four main agendas: Food security and nutrition, universal health coverage and affordable housing and Manufacturing.

This study cuts across the four agendas. Firstly, food security, irrigation of crops from flood waters. Secondly, the Universal Health Survey, floods lead to health hazards such as cholera. Thirdly, Manufacturing, flooding of water to the hydroelectric power plant for the generation of electricity for the manufacturing industry. Finally, affordable housing, flood control, housing and roads will not be destroyed.

The goal of this study is to investigate parabolic open channel as an efficient channel that would push excess water out of flooded areas and direct the excess water to crop irrigation and hydroelectric power generation for industrial and domestic use.

List of notations

Symbol	meaning
Re	Reynolds number (dimensionless)
g	Acceleration due to gravity (ms^{-2})
q	Uniform inflow (m^2s^{-1})
t	Time (s)
y	Flow depth (m)
z	Co-ordinate direction, bed level relative to adatum
A	Area of cross-section of flow (m^2)
C	Coefficient of the flow (Chézy coefficient)
D	Hydraulic depth (m)
Fr	Froude number (dimensionless)
L	Length of the channel (m)
Q	Discharge (m^3s^{-1})
Re	Reynolds number (dimensionless)
s	Slope of the channel bottom
s_f	Friction slope
x	Distance along the main flow direction (m)
T	Top width of the free surface (m)
v	Mean flow velocity (m/s)
m	Dimension of length
n	The Manning's coefficient of roughness ($sm^{-1/3}$)
r	Radius of the conduit
α	Energy coefficient

2. Literature review

2.1 Related literature review

The study of open channel flow is a common research is with studies carried out on Natural channels like rivers and man-made channels such as irrigation canals. In open channel flows gravity, viscosity, and inertia are the main forces at work, individually playing a key function. For a long time, studies on Open channels has been a subject of discussion with the Chézy equation as one of the oldest uniform flow equations developed for computation of average velocity of a uniform flow. Chézy formula provided unsatisfactory results to the designers of open channels. Henderson (1966). Manning formula has been proven to be the most used formula in the study of open channel; this formula was developed through studies conducted by manning in 1895. The Manning equation makes uses of coefficient of roughness called manning constant in the open channel flow. This has made the equation very reliable and more desired for the design of open channels. The Manning coefficient considers the degree of irregularity of the channel, channel size, bed material and variation in shape and comparative effect of channel obstruction, meandering and growth of vegetation in channel (Chadwick & Morfet, 1993) [1].

Kwanza *et al*., (2007) [6] investigated the effects of slope of channel, width of channel, slope of channel and channel discharge for both trapezoidal and rectangular channels. The findings were noted trapezoidal open channel flows are efficient hydraulically than rectangular cross sectional open channels. They noted that, the volume of flow increases when identified factors are varied upwards.

Tsombe (2011) [13] investigated fluid flow in open channels with circular cross section. He found out that increasing the flow depth, causes a reduction of fluid velocity. Further, increasing the channel slope results to an increase in velocity of flow. Also, increasing the radius of the channel results to a drop of velocity of flow. In addition, the findings were that, a decrease in the slope of the channel results to a drop in the flow velocity because slope and the flow velocity are directly proportional. Furthermore, for a fixed flow area, the flow velocity increases with increasing depth from the channel bottom to the free stream, with the maximum velocity occurring just below the free surface.

Thiong'o (2011) [12] investigated fluid flow in an open rectangular and triangular channels. The findings were that open channels with rectangular cross section are efficient hydraulically than open channels with triangular cross section. Further findings were that for both triangular and rectangular channels an increase in energy coefficient, Top width and slope of the channel results to arise in velocity of flow. Also, the velocity of flow increases as depth increases and becomes maximum slightly below the free surface. Velocity profile for both rectangular and triangular channel indicate that the channel that is rectangular in shape moves more water at a faster rate than an open triangular channel at constant depth and width.

Macharia *et al*., (2014) [7]. They studied the flow of fluids in an open rectangular channel with lateral inflow channels and discovered that increasing the channel's lateral inflow angle does not increase the velocity of fluid in the core channel. The flow speed in the main channel is reduced as the cross sectional area of the lateral inflow is increased. The flow velocity in the open rectangular channel increases as the lateral inflow channel velocity increases, while the velocity in both channels decreases as the lateral inflow channel length increases.

Kinyanjui *et al*., (2014) [6] did an investigation on modeling flow of fluid in open channels with circular cross-section, the findings of the study were that for a static area of flow, the velocity of flow increases as the depth of flow increase from the lowest part of the channel to the free stream and that maximum velocity is attained just below the free surface.

Ojiambo (2014) [11] the study looked into a mathematical model of fluid flow in an open channel with a circular cross-section, and the results showed that decreasing the cross-sectional area of the channel and flow depth results in an increase in flow velocity. The velocity of flow increases as the lateral inflow rate per unit length of the channel decreases.

Jomba *et al*., (2015) investigated mathematical model of fluid flow in open channel with Horseshoe cross-section. From the study the findings were that as the velocity of flow increases the depth increases for a fixed flow area, towards the free stream. Also it was established that an increase in hydraulic radius and roughness coefficient results to a reduction of velocity due to increased shear stresses. A decrease on slope of the channel results to a drop in flow velocity since flow velocity and slope of the channel are directly proportional. Increasing the cross-sectional area of flow leads to a drop in the flow velocity.

Marangu *et al*., (2016) [9] did an investigation on a model of open channel fluid flow with Trapezoidal cross-section and a segment base. The purpose of this research was to look into the relevance of trapezoidal cross-sections with segment bases in drainage system design. The analysis took into account a constant, uniform open channel flow. The finite difference approximation method was used to solve the saint-Venant partial differential equations of continuity and momentum that control free surface flow in open channels. The flow velocity is investigated in relation to the channel radius, cross section area, flow depth, and manning coefficient. The flow parameters are cross section area of flow, channel radius, channel slope, and manning coefficient, and the flow variables are velocity and flow depth. The study discovered that increasing the flow's cross section area causes a decrease in flow velocity. Furthermore, increase in cross section and the channel radius of flow causes a reduction in flow velocity, and increasing the roughness coefficient results to a reduction in flow velocity. The results of the study were that the flow velocity reduces as a result of increasing the radius of the circle forming the segment. The findings were that increase in depth of flow, channel radius and cross-sectional area produces a corresponding decrease in fluid velocity. Also the results were that, an increase the bed slope of the waterway resulted to an increasing flow velocity.

Omari *et al*., (2018) [15] Modeling circular closed channels for Sewer lines. The result showed that increasing the area of cross sectional of Sewer flow results to a decrease in the Sewer depth. It was observed that decreasing the friction slope results to an increasing Sewer flow velocity. Also it was found out that an increase in tunnel angle of inclination results to an increase in Sewer velocity.

Mese *et al*., (2019) investigated mathematical modeling of flow of fluid in an open channel with an Elliptical cross-section. The findings showed an increase hydraulic radius, results to an increasing fluid depth. The depth of fluid flow reduces along the channel due to accumulation of eroded particles which consequently reduces the fluid velocity. Variation of friction slope also affects flow velocity. When friction is raised, flow velocity is reduced. Friction arises from the shear forces on the walls and channel bed which offers resistance to the smooth flow of water.

Although a lot of research has been done in the last two decades on open channels with different cross-sectional area no research has been made on parabolic channels. The problem of flooding still persists in the current years and a channel that has the ability to convey maximum discharge on flooded areas into irrigational land has to be designed, and this is what this research strives to explore.

3. Research methodology

3.1 Assumptions and Approximations

In this study, the following assumptions will be utilized.

1. Unsteady flow is considered.
2. The fluid is flowing in one-direction.
3. Newtonian fluid is considered.
4. Forces due to gravity cause the fluid to flow.
5. Incompressible flow is considered.

3.2 Governing equations

The study of flow of open channels considers the main equations as; momentum and continuity. These governing equations are used in the study of one dimensional flow and used to solve partial differential equations in this research. The Navier-stoke'sequation derives the continuity equation while the Newton's second Law of motion derives the equation of momentum.

3.2.1 Continuity equation

In the study of uniform flow, the Continuity equation is regarded as one the important principles used. The principle is derived from the concept that mass is conserved always in fluid systems flowing in any direction and flow complexity.

The discharge Q is obtained as;

$$Q = AV$$

For a given pair of regions, the discharge Q is expressed as;

$$Q = A_1V_1 = A_2V_2 \tag{3.1}$$

Where Q = discharge
A = area of cross-section fluid flow
V = meanrate

The continuity equation governing unsteady flow in open channel of arbitrary shape is

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = q \tag{3.2}$$

If we Substitute equation (3.1) into equation (3.2) above, then we differentiate partially with respect to x gives:

$$V\frac{\partial A}{\partial x} + \frac{\partial A}{\partial t} + A\frac{\partial V}{\partial X} - q = 0 \tag{3.3}$$

Expressing the derivatives of A as a function of y since the area flow is assumed to be a depth function.

$$\frac{\partial A}{\partial x} = \frac{dA}{dy}\frac{\partial y}{\partial x} = T\frac{\partial y}{\partial x} \tag{3.4}$$

$$\frac{\partial A}{\partial t} = \frac{dA}{dy}\frac{\partial y}{\partial t} = T\frac{\partial y}{\partial t} \tag{3.5}$$

For this research $T = \frac{dA}{dy}$

If we Substitute equation (3.4) and (3.5) in equation (3.3) yields:

$$VT\frac{\partial y}{\partial x} + A\frac{\partial V}{\partial x} + T\frac{\partial y}{\partial t} - q \tag{3.6}$$

Dividing equation (3.6) by T we obtain;

$$\frac{\partial y}{\partial t} + \frac{A}{T}\frac{\partial V}{\partial x} + V\frac{\partial y}{\partial x} - \frac{q}{T} = 0 \tag{3.7}$$

3.2.2 Momentum equation

The Newton's second law of motion derives the Momentum equations.

The momentum equation governing unsteady flow in open channels of arbitrary shape is:

$$\frac{\partial V}{\partial t} + \alpha V\frac{\partial V}{\partial x} + g\frac{\partial y}{\partial x} = g(s_o - s_f) \tag{3.8}$$

The channel bottom slope s_o can be conveniently expressed as:

$$s_o = -\frac{dz}{dx} \tag{3.9}$$

Where z is the bed level or channel bottom elevation relative to a datum. The term $\frac{dz}{dx}$ is the change of elevation of the bottom of the channel with respect to distance or the bottom slope.

The friction slope, S_f also known as the friction term due to bed's roughness is expressed as:

$$s_f = -\frac{dH}{dx} \tag{3.10}$$

Where H is the total energy at any cross section of the channel. The term $\frac{dH}{dx}$ is the change of energy with longitudinal distance or the friction slope.

Rearranging equation (3.8) yields:

$$\frac{\partial V}{\partial t} + \alpha V \frac{\partial V}{\partial x} + g \frac{\partial y}{\partial x} - g(s_0 - s_f) = 0 \quad (3.11)$$

Equation (3.7) and (3.11) are first order partial differential equation which are non-linear type and would be solved by use of finite difference method and MATHLAB programme.

3.2.3 Chezy equation

Chézy equation was developed by the Antoine Chézy a French Engineer in 1768 while designing open canal for the supply of water.

$$V = c\sqrt{R_h S_f} \quad (3.12)$$

3.2.4 Manning equation

$$V = \frac{1}{n} R^{2/3} S_o{}^{1/2} \quad (3.13)$$

Discharge is given by;

$$Q = AV \quad (3.14)$$

Substituting equation (3.14) into equation (3.13) we obtain,

$$Q = \frac{1}{n} A R^{2/3} S_o{}^{1/2} \quad (3.15)$$

Finite difference equations for parabolic channel are;
Discretization of derivatives

$$\frac{\partial y}{\partial t} = \frac{y(i,j+1) - y(i,j)}{\Delta t}$$

$$\frac{\partial v}{dt} = \frac{v(i,j+1) - v(i,j)}{\Delta t}$$

$$\frac{\partial y}{\partial x} = \frac{y(i+1,j) - y(i-1,j)}{2\Delta x}$$

$$\frac{\partial v}{\partial x} = \frac{v(i+1,j) - v(i-1,j)}{2\Delta x}$$

From equation 3.7

$$\frac{y(i,j+1) - y(i,j)}{\Delta t} + \frac{A}{T}\left(\frac{v(i+1,j) - v(i-1,j)}{2\Delta x}\right)$$

$$+ v(i,j)\left(\frac{y(i+1,j) - y(i-1,j)}{2\Delta x}\right) = \frac{q}{T}$$

$$y(i,j+1) =$$

$$\Delta t \begin{bmatrix} -\frac{A}{T}\left(\frac{v(i+1,j)-v(i-1,j)}{2\Delta x}\right) \\ - v(i,j)\left(\frac{y(i+1,j)-y(i-1,j)}{2\Delta x}\right) + \frac{q}{T} \end{bmatrix} + y(i,j) \quad (3.12)$$

From equation 3.11

$$\frac{v(i,j+1) - v(i,j)}{\Delta t} + \alpha\, v(i,j)\left(\frac{v(i+1,j) - v(i-1,j)}{2\Delta x}\right)$$

$$+g\left(\frac{y(i+1,j) - y(i-1,j)}{2\Delta x}\right) = g(s_o - s_f$$

$$v(i,j+1) = \Delta t\left[-\alpha\, v(i,j)\left(\frac{v(i+1,j)-v(i-1,j)}{2\Delta x}\right) - g\left(\frac{y(i+1,j)-y(i-1,j)}{2\Delta x}\right) + g\left(s_o - s_f\right)\right] + v(i,j) \quad (3.13)$$

Equations 3.12 and 3.13 are the momentum and continuity equations of an open parabolic channel in finite difference form.

3.2.5 Conditions of flow for parabolic channel in finite difference form

The initial conditions as per the program in finite difference form are;

v (0,t) = 10, y(i,o) = 15

The boundary conditions as per the program in finite difference forms are;

v (x_o, j) = 20, y(x_o,j) = 15

v (x_n, j) =20 y(x_n, j) = 30

Where i denotes the distance along the channel, j denotes time. x_o *and* x_n denotes the entry point and the exit point respectively of the section of the channel.
A uniform mesh in which Δx = 0.1 and Δt = 0.0012 have been considered.
The convergence is achieved by the condition;

$\frac{\Delta y}{\Delta t^2}$< 0.5.

4. Results and Discussion

4.1 Results

The equations 3.12 and 3.13 are solved using MATLAB program. The effects of energy coefficient, Top width and channel slope on the flow velocity are represented graphically as shown in fig 1 – 3.

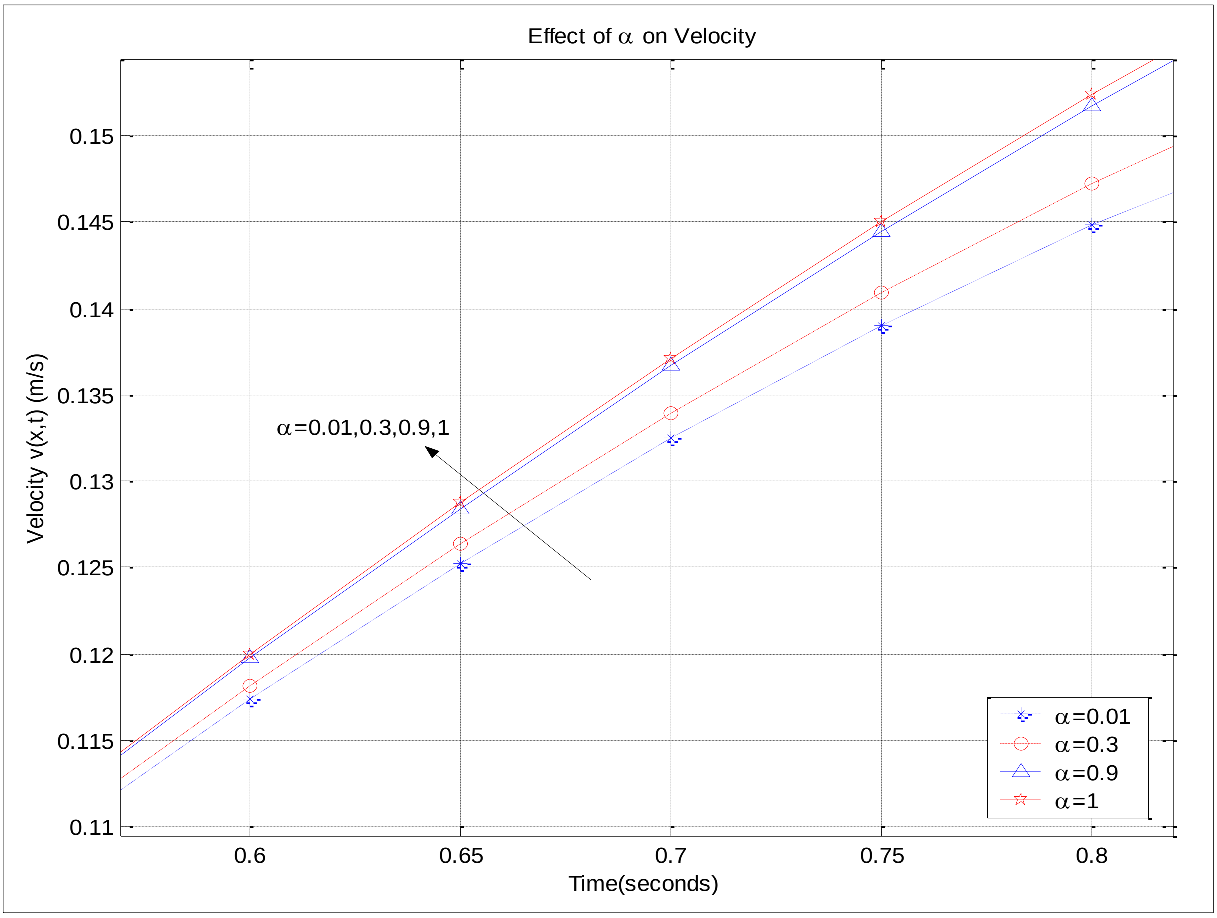

Fig 1: Effects of varying energy coefficient (α) on velocity of flow

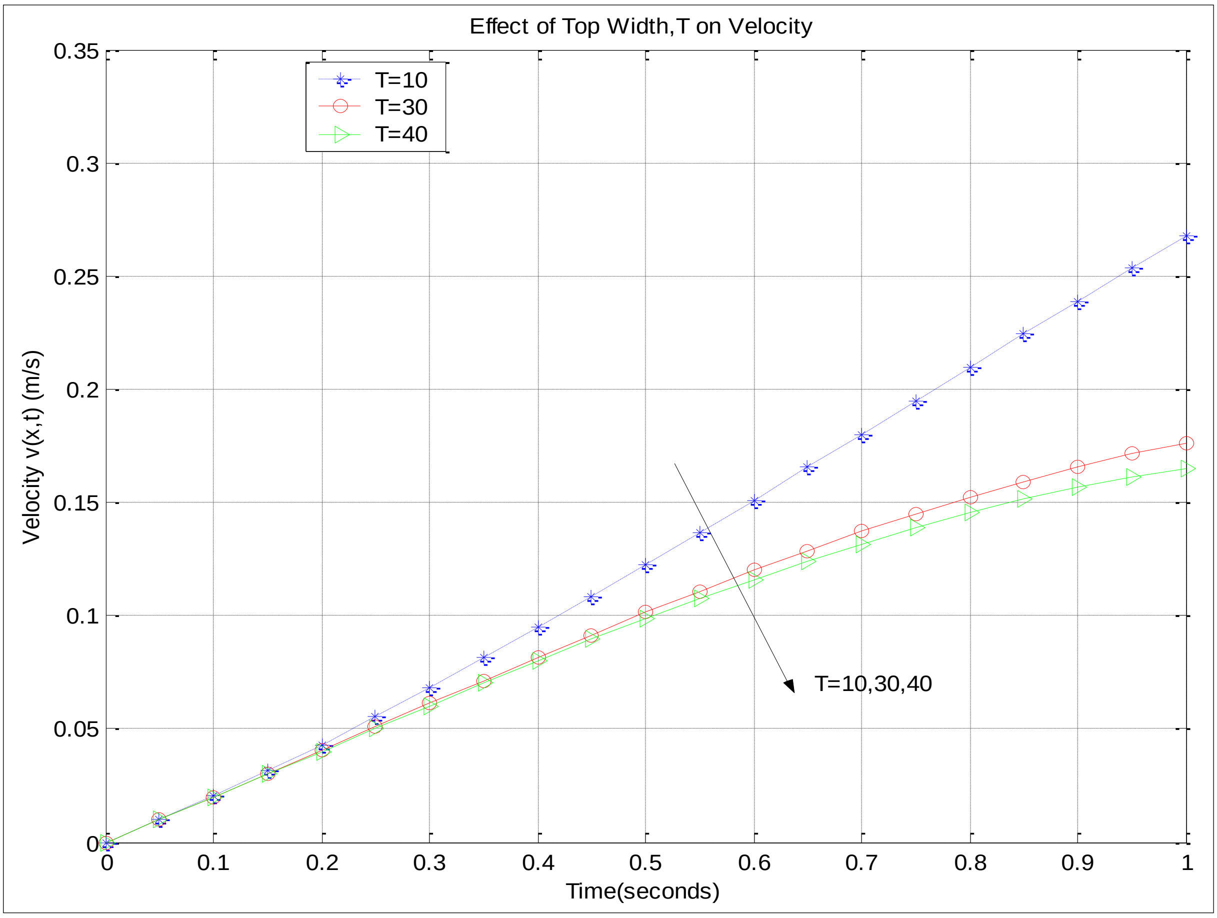

Fig 2: Effect of top-width, T on velocity

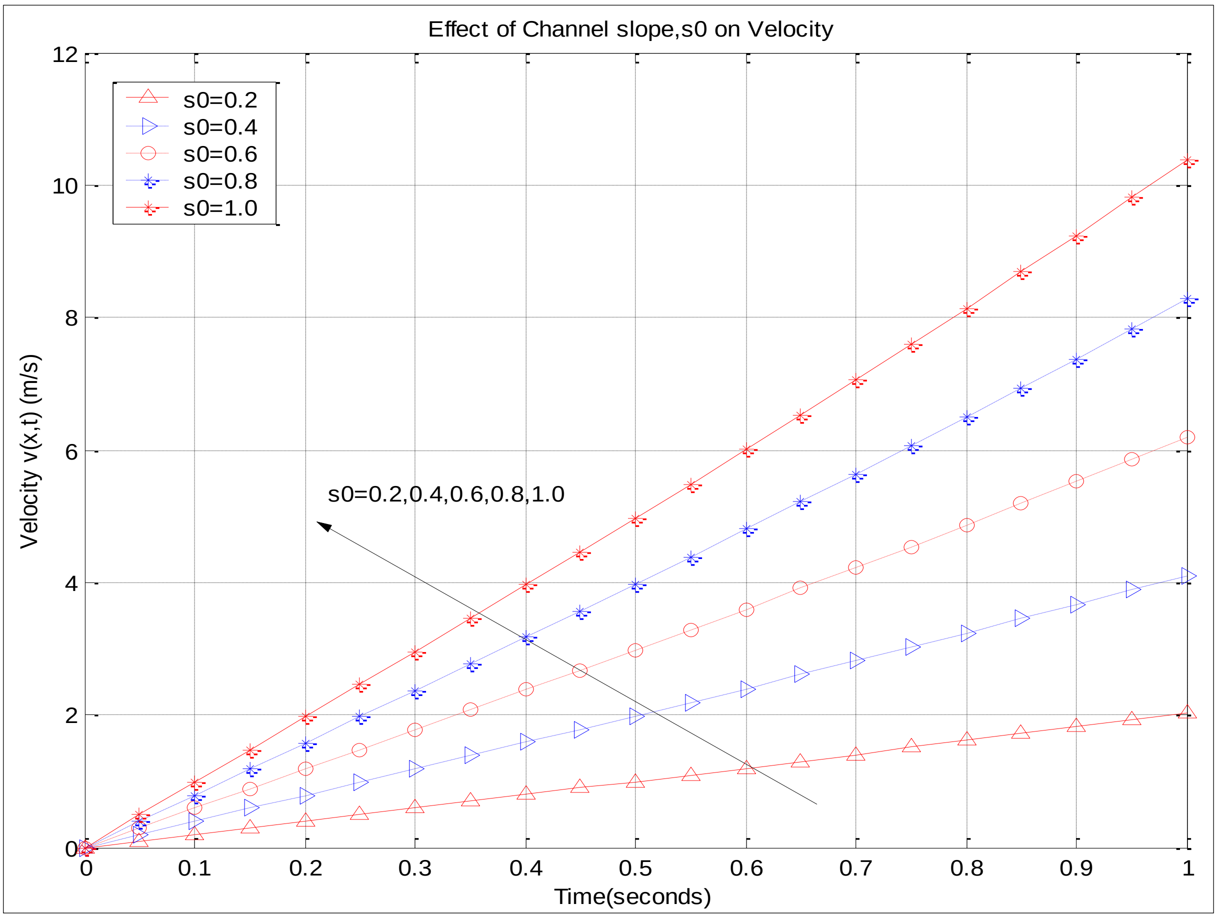

Fig 3: Effect of channel slope, s_o on velocity

4.2 Discussion

From fig. 1 we observed that as the energy coefficient increases from 0.01 to 1.0 the velocity increases, hence when energy coefficient increases, the velocity increases. According to Kinetic theory of matter fluid molecules possesses kinetic energy (energy in motion) which is reflected clearly from the graph. Increasing the energy of the flow leads to kinetic energy increasing of the particles hence particles moves faster.

From fig. 2 it is noted that reduction in Top width increases the velocity of fluid flow in parabolic channels. The Top width of 10m yields a higher velocity as compared to the higher values up to 100m.

From fig. 3 for the same depth decreasing the channel slope from 1.0 to 0.2 decreases the velocity of flow in parabolic channel. Hence, the velocity value when the channel slope is 0.2 is lower than when the channel slope is 1.0

According to Manning's velocity formula, velocity is directly proportional to the slope and therefore as the slope increases, velocity also increases.

5. Conclusion and Recommendation

5.1 Conclusion

The study was conducted over the effects of energy coefficient, channel slope and Top width on fluid flow velocity of an open channel with parabolic cross section. From the analysis it is clear that the energy coefficient, Top width and channel slope for parabolic open channel affect the velocity of flow of fluid. The following conclusions were made from the results obtained:

a. Increasing the energy coefficient increases the velocity of flow; this is as a result to an increasing energy of the fluid which increases molecular energy of the fluid leading to a random motion.
b. Reduction in the channel Top width, results to an increasing velocity of channel.
c. Increasing the channel slope of flow, leads to an increase in the velocity of flow in parabolic channel since flow velocity is directly proportional.

The parabolic open channel cross section has an advantage over other channels in that it has the ability to maintain a greater velocity at low volume of fluid which reduces the tendency to deposit sediments on the channel bed. Furthermore, discharge with lower velocity can carry floating debris easily than flat-bottomed channel.

5.2 Recommendations

We recommend that future research should be done on;

a. In this research, the fluid flow was 1-dimensional the same research can be carried out by considering 2-D, 3-D flows
b. Further research should be carried out by keeping other parameters constant other than depth.
c. Comparison of fluid flow in elliptical and parabolic channel.

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