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Mathematical exploration: The mathematical aspect of simultaneously liking and disliking works of the same artist

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Abstract
The primary focus of this exploration is to establish a relationship between art and mathematics. An original question that might trouble several artists has been posed and the exploration aims at tackling this abstract concept through mathematical concepts. To understand and establish the desired relation, an understanding of proportions and golden ratio will be required. After an explication of all concepts related to this topic and interpreting their possible impact on beauty, the golden ratio will be searched for in few places. This study will go on and select a few pieces of art created by the author and conduct a survey to gather data about people's perspective of the art. Using the chi square test, the dependence of beauty on ranking will be explored. Furthermore, the paintings selected will be mathematically analysed and compared. The research will aim to see parallel interpretations and dependence between people's perspectives and mathematical data. Lastly, the exploration will be looking into how to the qualitative factors that are not mathematical affect the likability to finally answer the original question and have a concrete and complete understanding of how mathematics affects visual beauty.

Keywords: Beauty, art, golden ratio, likability, proportion, chi square test, rank correlation

1. Introduction

As a student of the sciences with a deep interest in programming and mathematics, my natural instinct has been to quantify the world and reduce everything into a formula or an algorithm. A computer is just a combination of 0s and 1s; it is easy to explain and it is easy to understand. But art is not. Even though the world of numbers and data seems easier to understand for me, I am not completely robotic and do house an interest for art and painting. When I present my paintings in front of my family and friends, I always have a conception about them. Sometimes I love them and sometimes I am just not satisfied enough. If asked why, my only answer is "I don't know." I have never liked having such an answer for anything. Feelings, emotions, art, beauty and general human psychology are all things that have too many conditions and too many exceptions. They very often do not let us provide calculated answers to questions. At least, that is what I have believed my whole life. But Galileo Galilei once said "Measure what can be measured, and make measurable what cannot be measured." This made me think: what if the world has limited itself from developing actual mathematical expressions for everything? What if there is some concept that could help me finally explain why I do not like looking at something I made myself? And what if mathematics could, to a certain extent, nullify emotions and psychological conditions? The world is still, after all, a system that could be broken down into a number. Hence, piece by piece, I'd like to break this world down mathematically. To start with, art and painting, one of my areas of pleasure, is a good topic to quantify. This investigation would focus on using mathematical concepts like the Golden Ratio and Proportion to explain beauty and aesthetics of chosen paintings and apply the understood concepts to all forms of art. In this research paper I will try to establish a mathematical expression for the likability of a given painting. This investigation would aim at finding the factors which have helped make the creative works of some people increasingly popular.

2. Proportion

Proportion is a concept in math which talks about the equality of two ratios.

$$a{:}b :: c{:}d \Rightarrow \frac{a}{b} = \frac{c}{d}$$

It is the mathematical comparison between two numbers and is usually referred to as a part, or share, or a number in the comparative relation to a whole. Proportion is used varyingly in many different concepts, but in every such place it is used to describe the relation between two objects.

Proportion has played an important role in art even without any actual mathematical calculations being involved. Often when drawing figures or objects, the object length to breadth proportion, object area to page area proportion and many other proportions are considered. In every drawing, the different parts must be "proportionate" to each other. This essentially means that if one is drawing a typical human figure, the body cannot have 10 cm long hands and 5 cm long legs. The relative size of each object or figure drawn has to be taken into consideration. Colours are also an important part of art. The world consists of a plethora of colours and they are all said to be made simply by the combinations of different proportions of the three basic colours: red, blue, green. The infinite combinations of colours in varying quantities play a very crucial role in art.

3. Golden Ratio

The golden ratio is an irrational number with the value $\frac{1+\sqrt{5}}{2}$ and it is denoted by the Greek symbol ϕ (phi). Two quantities are said to be in the golden ratio if the ratio of the larger number to the smaller number is equal to the ratio of the sum of the two values to the larger value.

Fig 1: https://mythicalroutes.com/images/Mythical-History/Parthenon-of-Athens.jpg

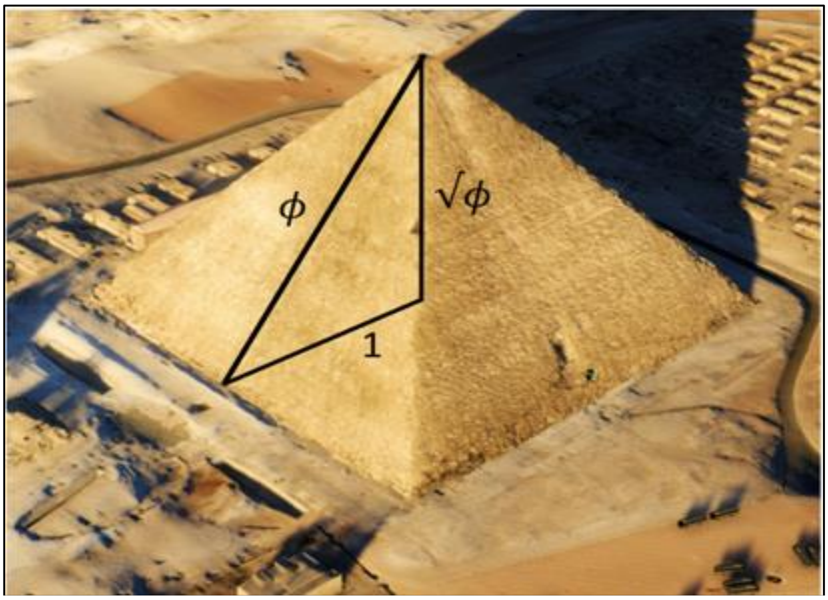

Fig 2: https://d.newsweek.com/en/full/699341/khufus-pyramid-aerial-view.jpg

This value is also called the Divine Proportion. This number is often referred to as "The Most Beautiful Number." It is considered this special because the golden ratio can be found in countless objects around us.

More than once, the existence of the golden ratio has been associated with beauty and aesthetics. According to a study by Professor Adrian Bejan from Duke Pratt School of Engineering, the aesthetic appeal of the golden ratio exists because the eye is able to most efficiently scan the scene in the shape of the golden rectangle. "When you look at what so many people have been drawing and building, you see these proportions everywhere," Bejan said. "It is well known that the eyes take in information more efficiently when they scan side-to-side, as opposed to up and down."

3. A Golden Ratio in Geometric Figures

i. The Golden Rectangle

$$\frac{a+b}{a} = \frac{a}{b} = f$$

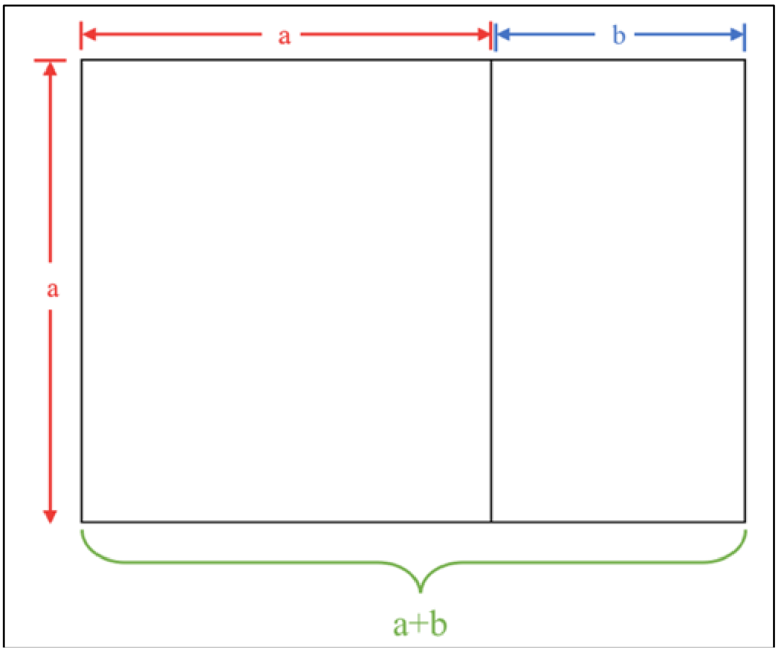

Fig 3: Illustrated by author

Derivation of Golden Ratio through the Golden Rectangle

$we\ have$

$$\frac{a}{b} = \frac{a+b}{a}$$

$$\Rightarrow \frac{a}{b} = \frac{a}{a} + \frac{b}{a}$$

$$\Rightarrow \frac{a}{b} = 1 + \frac{b}{a}$$

$$now\ \frac{a}{b} = \phi\ and\ \frac{b}{a} = \frac{1}{\phi}$$

$$\Rightarrow \phi = 1 + \frac{1}{\phi}$$

$hence\ \phi\ can\ be\ defined\ in\ terms\ of\ itself$

$$\Rightarrow \phi^2 = \phi + 1$$

$$\Rightarrow \phi^2 - \phi - 1 = 0$$

$olving\ the\ quadratic\ equation, we\ get$

$$\phi = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$$

$$\Rightarrow \phi = \frac{1 \pm \sqrt{1+4}}{2}$$

$$\Rightarrow \phi = \frac{1 \pm \sqrt{5}}{2}$$

$(negatic\ value\ is\ neglected)$

$$therefor\ \phi = \frac{1+\sqrt{5}}{2} \approx 1.618.$$

a). Regular Pentagon

$$\frac{y}{x} = \frac{diagonal}{side} = \phi$$

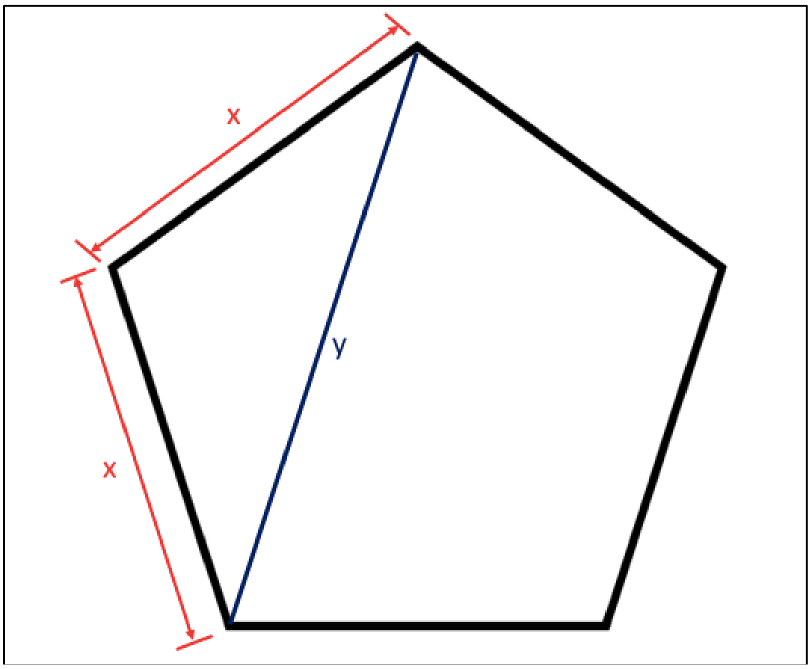

Fig 4: Illustrated by author

b) The Pentacle

$If\ a = 1$
$\Rightarrow\ b = \phi$
$\Rightarrow\ c = \phi^2$
$\Rightarrow\ d = \phi^3$

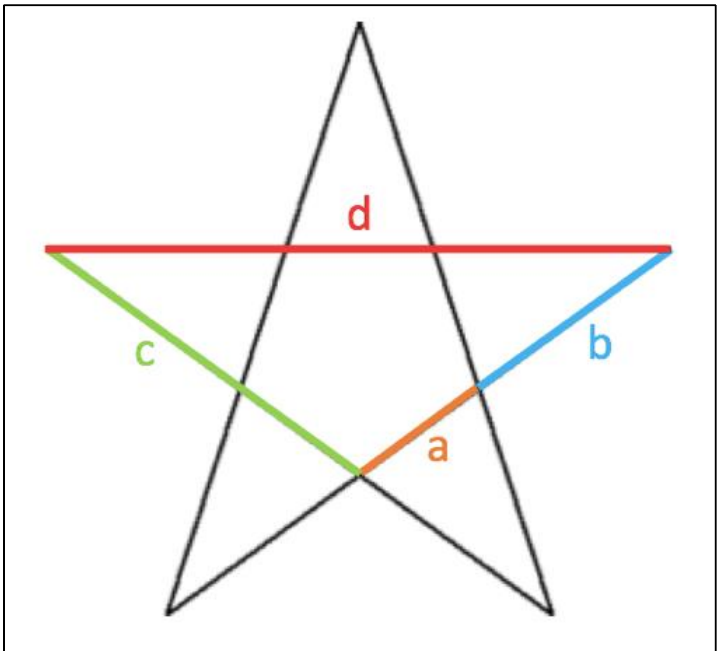

Fig 5: Illustrated by author

3.1 B The Golden Ratio and The Fibonacci Series

The Fibonacci series is a series of numbers formed when each term of the series is a sum of the previous two terms. It is represented by the given formula

$$F_n = F_{n-1} + F_{n-2}$$

Hence, we get the series:
1, 1, 2, 3, 5, 8, 13, 21, 34, 55,….
The Fibonacci sequence is considered an important sequence because of its appearance in nature. The Fibonacci series is used by financial and technical analysts to make predictions and to understand the trading trends. The series is also used to optimize and create many algorithms such as the computational run-time analysis of Euclid’s theorem, number generator and the binary tree. Using the series, a figure called the golden spiral can be created (see **III.C**). This spiral occurs in many forms of nature.
It is observed that when the terms of the series (3 and onwards) are divided by the term preceding them, we get a value close to the golden ratio.

$$\frac{3}{2} \approx \frac{5}{3} \approx \frac{8}{5} \approx \frac{13}{8} \approx \frac{21}{13} \approx \frac{34}{21} \approx \frac{55}{34} \approx 1.618$$

From this series we can get the expression for ϕ:

$$\phi = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cdots}}}}$$

$$\Rightarrow \phi = 1 + \frac{1}{\phi}$$

Fibonacci series is an important concept to note when talking about beauty and art. This series can give us different patterns with an interesting incorporation of symmetry. It is always noticed that symmetrical visuals are easier on the eyes. Hence, Fibonacci series can be applied to make art work more attractive.

Some examples of Fibonacci based patterns are given below:

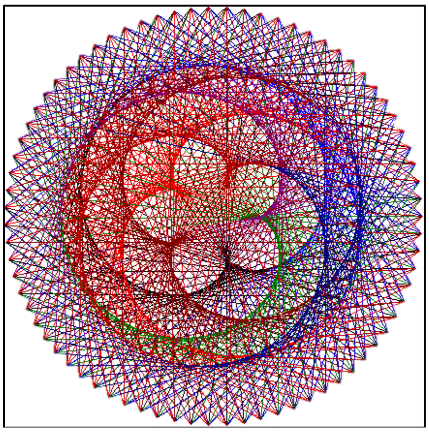

Fig 6: http://media.tumblr.com/3c90ae0196e79f4209ded824fe9b8696/tumblr_inline_mq3kakSZ3a1qz4rgp.gif

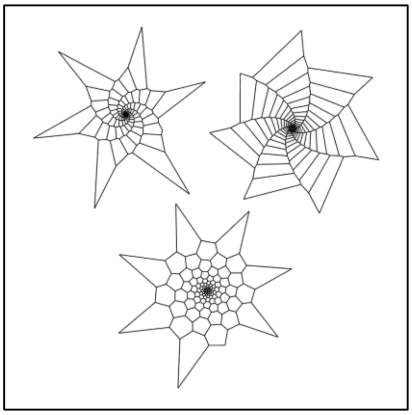

Fig 7: https://www.craftsmanspace.com/sites/default/files/free-patterns/fibonacci_number_2d_patterns.gif

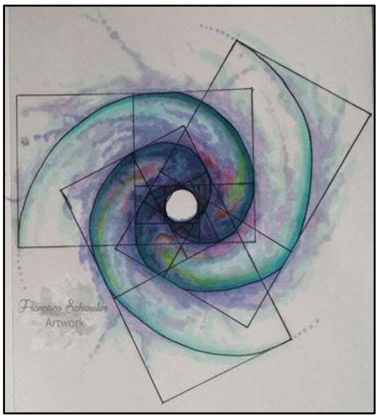

Fig 8: https://i.pinimg.com/474x/d7/54/af/d754afa87b2af028f76af9947f11e0e3.jpg

III. c The Golden Spiral

The Golden Spiral is constructed by adjoining squares where the sides have the values of consecutive Fibonacci numbers. The ratio of the size of squares to each other would be powers of the golden ratio.

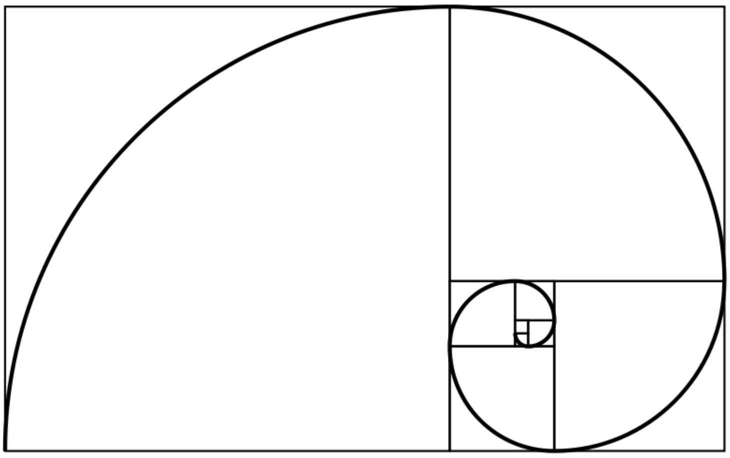

Fig 9: https://i.pinimg.com/originals/0f/a2/a3/0fa2a347e5262b90cd7d13de5e11ec9e.png

The golden spiral is found in the head of a sunflower, in snails shells and nautilus shells, in hurricanes, and in spiral galaxies.

III. d The Golden Triangle

The Golden Triangle is an isosceles triangle in which the ratio of length of the common side to the base is the golden ratio.

$$\frac{a}{b} = f$$

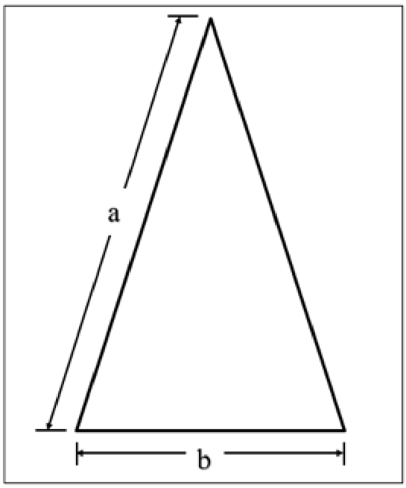

Fig 10: Illustrated by author

III.e The Golden Ratio in the World

As discussed earlier, the golden ratio is observed in several places naturally. Science has taught me that the world is not just a coincident combination of elements. There is a reason the world works the way it does and every component has something to contribute to its functionality. Hence, the repeated appearance of golden ratio suggests that the number must have some significance beyond simple coincidence. To further explore how the golden ratio is omni present and how it could be contributing to beauty, four people were randomly selected and their facial features were measured.

Table: The Golden Ratio in the Rorld

All values are taken in cm	a	b	c	d	e	f
Sameer	22	17	12.5	4.5	5.5	11.5
Aditya	21	17	13	5	6	10
Shikha	20	15.5	14	3.5	5	9
Reeya	20.5	13	14.5	5	6.5	12

Table: The Golden Ratio in the World

	a/b	c/f	f/e	e/d
Sameer	1.2941	1.0869	2.0909	1.2222
Aditya	1.2345	1.7000	1.6667	1.2000
Shikha	1.2809	1.5555	1.8000	1.4285
Reeya	1.5769	1.2083	1.8461	1.3000

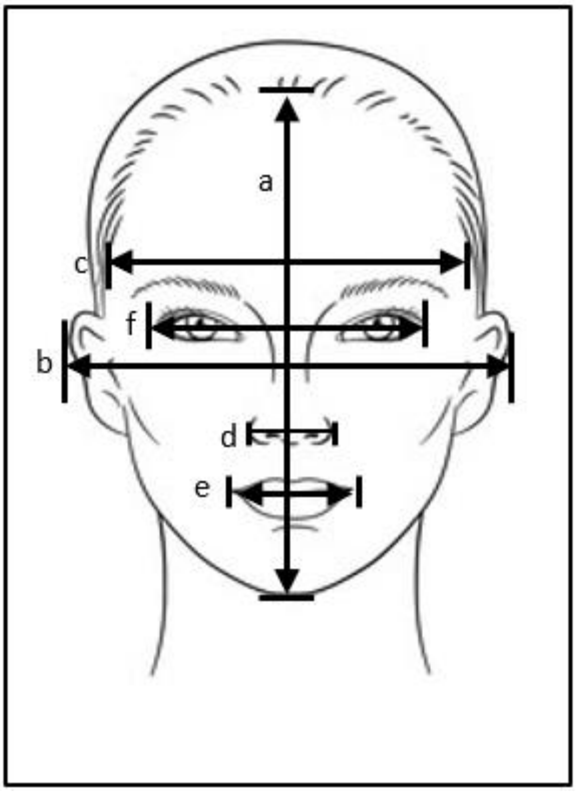

Fig 11: https://www.shutterstock.com/search/women+face+outline+sketch

It is observed that all relative values of the face measurements are oscillating around the golden ratio. Even though none of the values are exactly 1.618, it is seen that they are inching close to the perfect proportion. Hence, the golden ratio is found around us.

IV. Experiment to Understand Relation between Art and Mathematics

Proportion, golden ratio and Fibonacci series seem to have significance in aesthetic and visual beauty. These different mathematical concepts that point to beauty have been discussed in the topics before. To analyse and prove the dependence of visual beauty on mathematics and incorporate people's perspective in the discussion, four paintings made by the same artist are selected:

Fig 12, 13, 14, 15: Painted by the author

To understand popularity of the subject paintings and find the relation between the ranking and beauty of the painting, a survey with 81 participants was conducted. The participants were asked to rate each painting on a scale of 1 to 5; 1 being the lowest and 5 being the highest.

The results have been tabulated below:

Ratings	Moderate (1-3)	High (4-5)	
Painting I	21	60	81
Painting II	21	60	81
Painting III	16	65	81
Painting IV	32	49	81
	90	234	324

IV. A Chi Square Test:

Observed Frequency (O_f)	Expected Frequency (E_f)	$O_f - E_f$	$(O_f - E_f)^2$	χ^2
21	22.5	-1.5	2.25	0.1
60	58.5	1.5	2.25	0.03846154
21	22.5	-1.5	2.25	0.1
60	58.5	1.5	2.25	0.03846154
16	22.5	-6.5	42.25	1.87777778
65	58.5	6.5	42.25	0.72222222
32	22.5	9.5	90.25	4.01111111
49	58.5	-9.5	90.25	1.54273504
				$\Sigma\chi^2$=8.43076923

Null Hypothesis H_0: Beauty of the painting is independent of the ranking given.
Alternate Hypothesis H_1: Beauty of the painting is not independent of the ranking given.
Degree of Freedom:

$= (n_r - 1)(n_c - 1)$
$= (4 - 1)(2 - 1)$
$= 3 \times 1$
$= 3$

Levels of Significance= 10%
χ^2 **Critical Value: 6.251**
$\Sigma\chi^2 > \chi^2_{\text{critical value}}$
Hence, null hypothesis is rejected and alternate hypothesis is accepted. This means that the ranking of the paintings is dependent on beauty.
Now that it is established that the rankings given to the painting by the people in the survey has a relation with its inherit beauty, I would like to mathematically analyse all paintings to see if it gives results parallel with the results of the survey.
For each of the four paintings, I will try to identify special mathematical shapes and calculate general ratios of lengths of different parts with each other.

V. Mathematical Inspection of The Subject paintings
V.A Buddha

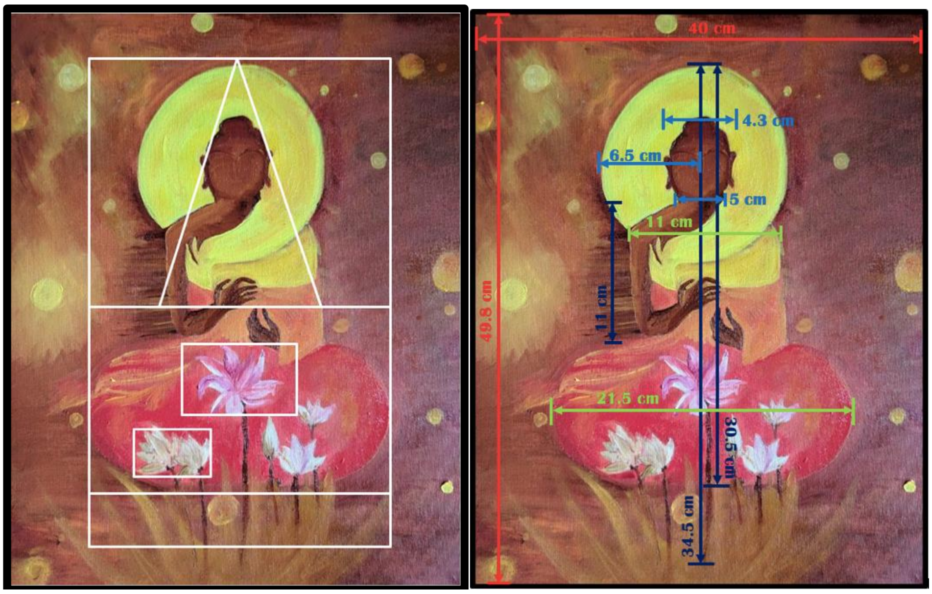

Fig 16 and 17: Made by author

In FIG 16 we observe four golden rectangles that encompass different parts of the painting. Interestingly, the sides of these rectangles are directly related to each other by ϕ. The rectangles that are formed when a golden spiral are made from a smaller rectangle, are the rectangles that have been distributed to fit different objects in the painting. The painting also has the golden triangle (III.D) fit between the upper body of buddha. There are **five** mathematical shapes observed. The different lengths in the painting were measured and noted in Fig. 17. The proportions throughout the paintings are found to be

$$total\ length: total\ breadth = \frac{49.8}{40} = 1.2450$$

$$total\ length: statue\ length = \frac{49.8}{30.5} = 1.6327$$

$$total\ breadth: statue\ breadth = \frac{40}{21} = 1.9047$$

$$statue\ length: statue\ width = \frac{30.5}{21.5} = 1.4186$$

$$yellow\ circle\ radius: neck\ width = \frac{6.5}{4} = 1.6250$$

$$statue\ width: circle\ diametre = \frac{21.5}{13} = 1.6538$$

$$Average\ of\ above\ ratios = 1.5799$$

V. B Walk Through the Village

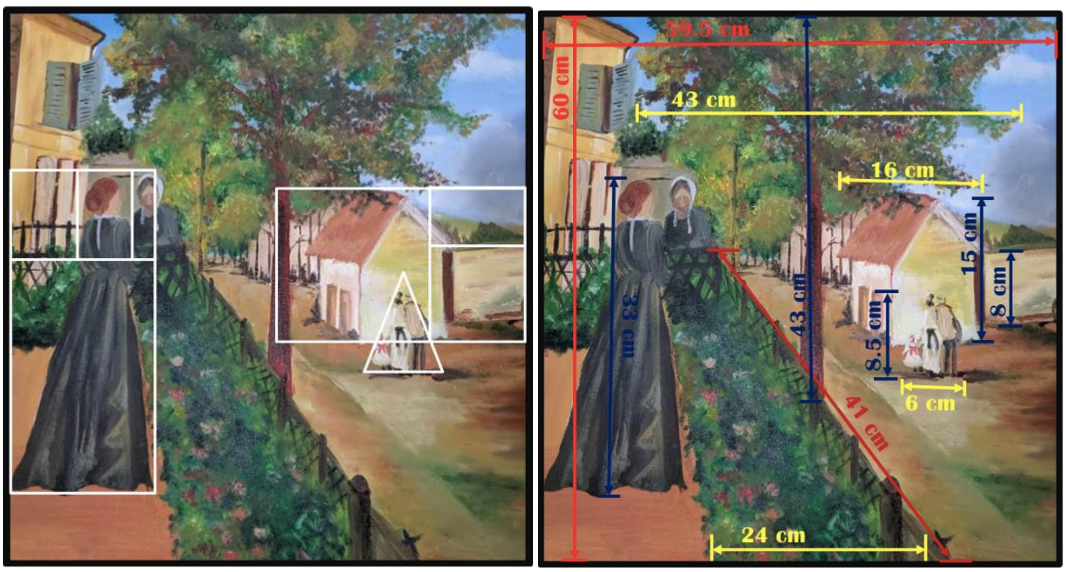

Fig 18 and 19: Made by author

In Fig 18, towards the left we find the first lady to have dimensions fitting with a golden rectangle. We see towards the right side that the small hut has three golden rectangles encompassing definite parts. A golden triangle also seems to be present around the family of three. There are **seven** mathematical shapes observed. The different lengths in the painting were measured and noted in Fig 19. The proportions throughout the paintings are found to be:

$$total\ length: total\ breadth = \frac{60}{59.5} = 1.0084$$

$$total\ length: lady\ height = \frac{60}{33} = 1.8181$$

$$family\ height: family\ width = \frac{8.5}{6} = 1.4166$$

$$tree\ height: tree\ width = \frac{43}{43} = 1.0000$$

$$tree\ heigh: hut\ height = \frac{43}{15} = 2.8666$$

$$garden\ length: garden\ width = \frac{41}{24} = 1.7083$$

$$hut\ height: wall\ length = \frac{15}{8} = 1.8750$$

$$hut\ width: family\ width = \frac{16}{6} = 2.6666$$

$$total\ width: tree\ width = \frac{59.5}{43} = 1.3837$$

$$hut\ width: hut\ height = \frac{16}{15} = 1.0666$$

$$Average\ ratio = 1.6809$$

V. C The Streets of Grindelwald

Fig 20 & Fig 21: Made by author

In Fig 20, towards the right side we observe the golden spiral described in III.C. Towards the left, we again see the golden spiral on both corners of the rectangle. Each rectangle drawn in the painting is a golden rectangle. **Eleven** mathematical symbols are observed in this painting. These figures do not intersect and they are able to encapsulate the most important sections of the painting. The different lengths in the painting were measured and noted in Fig 21. The proportions throughout the paintings are found to be:

$$otal\ length: total\ width = \frac{90}{60} = 1.5000$$

$$total\ length: building\ length = \frac{90}{62} = 1.4516$$

$$building\ length: building\ width = \frac{62}{19} = 3.2631$$

$$building\ width: building\ level\ 1\ height = \frac{19}{17.5} = 1.0857$$

$$house\ length: house\ width = \frac{24}{22} = 1.0909$$

$$mountain\ width: house\ width = \frac{43.5}{22} = 1.9772$$

$$garden\ diagonal: garden\ width = \frac{23}{43.5 - 22} = 1.0697$$

$Average\ ratio = 1.6340$

V.D Vietnamese Women

Fig 22 & Fig 23: Made by author

In Fig 22, we observe **five** golden rectangles. The different lengths in the painting were measured and noted in Fig 23. The proportions throughout the paintings are found to be:

$$total\ length: total\ breadth = \frac{59.5}{44.5} = 1.3370$$

$$total\ length: left\ figure = \frac{59.5}{40} = 1.4875$$

$$total\ length: right\ figure = \frac{59.5}{38.5} = 1.5454$$

$$total\ left\ figure: left\ figure\ skirt = \frac{40}{22} = 1.8181$$

$$total\ right\ figure: right\ figure\ skirt = \frac{38.5}{21.5} = 1.7906$$

$$left\ hat\ width: left\ waist\ width = \frac{9}{5.5} = 1.6363$$

$$right\ hat\ width: right\ waist\ width = \frac{9}{5.9} = 1.5254$$

$$pot\ length: pot\ width = \frac{9.5}{6.5} = 1.4615$$

$$Average\ of\ above\ ratio = 1.5752$$

V.E Analysis

Analysing paintings from the point of view of geometric designs has shown that works of art that are considered beautiful have unintentionally incorporated forms of the golden ratio. While making these paintings, I did not have any focus on incorporating the golden ratio or the special shapes. However, they are present nonetheless. When the average of ratios of different parts were taken with each other, it was found that the general proportion for all paintings came out to be close to the golden ratio ϕ.

From the survey, it is apparent that Painting III (Fig 13) is the most liked by everyone. It is followed by Painting II (Fig 12), then Painting I (Fig 11) and finally Painting IV (Fig 14). Surprisingly, the mathematical analysis has given a result similar to this ranking.

Assume that:

$$likability\ \alpha\ \frac{1}{\Delta x}$$

$$likability\ \alpha\ n$$

$where:$

$-\Delta x\ is\ the\ deviation\ of\ the\ standard\ proportion\ of\ the\ painting\ from\ the\ golden\ ratio$

$-n\ is\ the\ number\ of\ mathematical\ shapes\ present$

Painting III, as the most liked painting, had the least Δx and the highest n. Painting II was the second most liked. However, its Δx was more than Painting I and Painting IV while its n was greater than both the other paintings. This could suggest that presence of mathematical shapes has more impact on likability than general proportion.
The results of the survey are parallel to our assumptions and so it is reasonable to believe that the presence of the golden ratio and geometric shapes enhances the beauty of a painting

VI. Qualitative Analysis

The chi square test and analysis of the paintings has proved that beauty and ranking are dependent on mathematical factors. However, art is an abstract and subjective concept. There are a hundred different techniques in art including crayons, pencil sketching, water painting, acrylic painting, oil painting, knife painting, and much more. Every method and technique has its own range of appeal. Moreover, the appeal may also differ from person to person. Hence, mathematics cannot be the only contributing factor. This leads to the question about the other factors that appeal to the spectators. In order to answer this question, a qualitative analysis using rank correlation will be done.
Painting III, the painting liked the most by the people in the survey, was sent to two artists: Mrs. Vandana Garg and Mr. Siddhant Rai. They were asked to give the painting marks out of 10 on the basis of different qualitative factors.

Table: Qualitative Analysis

	Qualitative Factors	**Marks given by Vandana Garg**	**Marks given by Siddhant Rai**	**Ranking by Vandana Garg(R_1)**	**Ranking by Siddhant Rai (R_2)**	**$d^2 = R_1 - R_2$**	**d^2**
1	Originality	9	2	3	7	-4	16
2	Theme/ Concept	9	10	3	1	2	4
3	Colour Scheme and Tone	8	9	6	3	3	9
4	Creativity	8	6	6	6	0	0
5	Memory Association	10	9	1	3	-2	4
6	Technique	9	8	3	5	-2	4
7	Overall Presentation	8	9	6	3	3	9
							$\sum d^2 = 46$

$let\ rank\ correlation\ be\ R$

$$for\ no\ repeated\ marks, R = 1 - \frac{1 - 6\sum d^2}{n^3 - n}$$

$$for\ repeated\ marks, R = 1 - \frac{6[\sum d^2 + {}^1/_{12}(m^3 - m)}{n^3 - n}$$

$hence$

$$R = 1 - \frac{6[46 + {}^1/_{12}(3^3 - 3) + {}^1/_{12}(3^3 - 3) + {}^1/_{12}(3^3 - 3)]}{7^3 - 7}$$

$$R = 1 - \frac{6[46 + {}^1/_{12} \times 24 + {}^1/_{12} \times 24 + {}^1/_{12} \times 24]}{343 - 7}$$

$$R = 1 - \frac{6[46 + 2 + 2 + 2]}{336}$$

$$R = 1 - \frac{6[52]}{336}$$

$$R = 1 - \frac{6[52]}{336}$$

$$R = 1 - \frac{312}{336}$$

$$R = 1 - 0.9285$$

$$R = 0.0715$$

$since\ |r| < 1, it\ is\ a\ perfect\ correlation$

Since $|r| < 0.5$, the two artists have weak correlation about the qualitative factors of the painting. This means that judgment and critique from all artists is related even though they may have varying perspectives.

VII. Limitations of Research

1. There are various forms of art and different mediums for art. For this research, all paintings selected were oil paintings. Hence, the art of acrylic and other mediums was not taken into consideration. Moreover, only one artist's work was analysed. If there was more diversity in the kind of art selected, the results would be more accurate.
2. All measurements were taken using a scale with least count 1mm. Even so, the measurements would not be completely accurate due to human error.

VIII. Conclusion

The exploration was successful in proving that mathematics has an impact on the likability of paintings. The research was able to identify some key shapes in the paintings that seemed to have been contributing to the beauty of the painting. It was seen that the assumptions made about golden ratio and proportion making the paintings more attractive were true. Hence, the aim of the exploration was achieved.

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