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Application to various fuzzy sets

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Abstract
In this paper, we discuss the concepts of decision in a fuzzy environment. We briefly review some basic development in the field of decision various fuzzy sets. We present various existing classical approaches as well as fuzzy approaches to the formulation of their solutions. In principle, a classical decision problem is extended to a fuzzy decision problem and then a fuzzy decision problem is transformed into some non-fuzzy problem which in turn, can be solved by using some existing traditional techniques and widely available commercial software packages.

Keywords: Fuzzy environment, fuzzy statistical decision theory, fuzzy decision making under constraints

Introduction
This paper comprising with decision making problems in fuzzy environment. Decision making is of central concern in the areas such as engineering design, regional policy, logistics and many other disciplines. Decision theory is often considered to be identical with optimization theory where the search for optimal solution(s) is (are) one of the prime goals. The methods and models which have been used over the past decades in these areas have primarily been "crisp" or "hard", i.e. the solutions were considered to be either feasible or unfeasible, either above a certain aspiration level or below. This dichotomous structure of models or methods very often forced the decision maker (DM) or modeller to approximate real problem situations of the more or less type by yes or no type models, the solutions of which might turn out not to be the solutions to the real problems.
This is particularly true if the problem under consideration includes vaguely defined relationships, human evaluations, uncertainty due to inconsistent or incomplete evidence, if natural language has to be modelled or if state variables can be described approximately.
Thus, one has become aware of the fact that uncertainties concerning the occurrence as well as concerning the description of events out to be modelled in a much more differentiated way. Classical mathematics, however, has not coped with the excessive complexity or too complicated problems involving natural language whose main property is vagueness of its semantics and its capacity of working with vague notions. New concepts and theories have been developed to do this. The concepts and theory of fuzzy sets have been advanced to a stage of remarkable maturity and have already been applied successfully in numerous cases and many areas. This paper aims at providing the systematic technique of the application of fuzzy set theory to decision making problems. The choice of the subject matter was influenced by the interests of the author, by her opinion on its usefulness, and also by her endeavour to present as systematic and explanation as possible.
In section 2.2 we will start by presenting a classical individual model of decision making problem. This deterministic model will then be extended to a fuzzy version. In section 2.3, we will discuss the probabilistic version of the classical model which is known as statistical decision theory. Then we will show how this can be extended to fuzzy model. In section 2.4 we will present the formulation of fuzzy decision making under constraints

1. Preliminaries
1.1 Definition
A fuzzy subset A in a set X is a function A: X → [0, 1]. A fuzzy subset in X is empty iff its membership function is identically 0 on X and is denoted by 0 or μ_{ϕ}.

The set X can be considered as a fuzzy subset of X whose membership function is identically 1 on X and is denoted by μ_X or I_x. In fact every subset of X is a fuzzy subset of X but not conversely. Hence the concept of a fuzzy subset is a generalization of the concept of a subset.

1.2 Definition
A fuzzy set on X is 'Crisp 'if it takes only the values 0 and 1 on X.

1.3 Definition
Let X be a set and be a family of fuzzy subsets of (X, τ) is called a fuzzy topology on X iff τ satisfies the following conditions.

1. μ_ϕ; $\mu_X \in \tau$: That is 0 and $1 \in \tau$
2. If $G_i \in \tau$ for $i \in I$ then $\vee G_i \in \tau$

   $i \in I$
3. If G, H $\in \tau$ then $G \wedge H \in \tau$

The pair (X, τ) is called a fuzzy topological space. The members of τ are called fuzzy open sets and a fuzzy set A in X is said to be closed iff $1 - A$ is an fuzzy open set in X.

1.4 Remark
Every topological space is a fuzzy topological space but not conversely.

1.5 Definition
If A and B are any two fuzzy subsets of a set X, then A is said to be included in B or A is contained in B iff $A(x) \leq B(x)$ for all x in X. Equivalently, $A \leq B$ iff $A(x) \leq B(x)$ for all x in X.

1.6 Definition
Two fuzzy subsets A and B are said to be equal if A(x) = B(x) for every x in X. Equivalently

A = B if A(x) = B(x) for every x in X.

1.7 Definition
The complement of a fuzzy subset A in a set X, denoted by A′ or $1 - A$, is the fuzzy subset of X defined by $A'(x) = 1 - A(x)$ for all x in X. Note that $(A')' = A$.

1.8 Definition
The union of two fuzzy subsets A and B in X, denoted by $A \vee B$, is a fuzzy subset in X defined by

$(A \vee B)(x) = \mathrm{Max}\{\mu_A(x), \mu_B(X)\}$ for all x in X.

1.9 Definition
The intersection of two fuzzy subsets A and B in X, denoted by $A \wedge B$, is a fuzzy subset in X defined by

$(A \wedge B)(x) = \mathrm{Min}\{A(x), B(x)\}$ for all.

2.2. Classical individual decision making and its extension to fuzzy version
The classical individual decision making situation can be characterized as follows
Given a set of n alternative states,

$$X = \{x_1, x_2 \ldots x_n\} \quad (2.21)$$

-a set of m alternative actions, $A = \{a_1, a_2, a_3, \ldots a_m\}$ (2.22)

$$\text{a relation } R \subseteq A \times X \quad (2.23)$$

The relation indicates that if $(a_i, x_j) \in R$, then x_j will be the effect of action a_i.
Then, define a preference ordering, O, over all possible outcomes:

$$O \subseteq X \times X \quad (2.24)$$

This preference ordering can be represented by a value function $V(x_j)$. The decision- maker will decide to take that action a_i, which gives him the highest outcome value $V(x_j)$ $(a_i, x_j) \in R$
Now, the value function, $V(x_j)$ plays an essential role in the model. In order to actually be able to take decisions, there are some necessary requirements to be made to the preference ordering over the alternative states. This has led to an axiomatic treatment resulting in the utility function.
The axiomatic treatment of the preference ordering has the following requirements.

1. All alternatives have to be comparable with each other (preference or indifference)
2. Preference and indifference are transitive relations
3. If a lottery has a lottery as an alternative, it can be decomposed into basic alternatives by means of probability calculus
4. If two lotteries are indifferent, they can be exchanged in a composed lottery
5. If two lotteries have the same two alternatives, the lottery in which the preferred alternative has the highest chance will be preferred
6. If x_s is preferred to x_j and x_j to x_k, there exists lottery with x_s *and* x_k which is indifferent to x_j.

Conventionally, a lottery is defined as a chance mechanism with probabilities

$P1(x_1), P2(x_2), P3(x_3) \ldots.. Pm(x_m)$ that $x_1, x_2, x_3 \ldots.. x_m$

will occur where

$\sum_{i=1}^{m} p_i$

The above axiomatic system defines a real - valued function $\emptyset$ on X such that

1. x_s is preferred to x_j if and only if $\emptyset(x_s) > \emptyset(x_j)$
2. If x_s occurs with probability α, and x_j with probability $1-\alpha$, then

$$\emptyset(\alpha, x_s; 1-\alpha, x_j) = \alpha\emptyset(x_s) + 1-\alpha\,\emptyset(x_j) \quad (2.25)$$

3. If two functions $\emptyset$ and φ on X satisfy these requirements, then

$$\varphi(x_s) = b(x_s) + c \quad (2.26)$$

This function is known as the utility function. The utility function is unique and it is a part from zero and unity, that is, the function has interval-scale property.

There exists a lot of criticism of this utility function, but a discussion of those criticisms falls outside the scope of this chapter. It is also noted that the organizational scientist H.A. Simon does not criticise the utility function but doubts whether all information about alternatives and actions is available to the decision- maker and whether the decision-maker really looks for optimal solution.
Next, we consider the extension of the above model in a fuzzy environment.

In this case, an $m \times n$ utility matrix, U is defined on A $\times$ X*:*

$$U = \left\{ \begin{matrix} U_{11}, U_{12}, \ldots\ldots U1n \\ U_{21}, U_{22}, \ldots\ldots U2n \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ U_{m1}, U_{m2}, \ldots\ldots Umn \end{matrix} \right\}$$

Where u_{ij} is the utility of choosing alternative action a_i when the state of this system is x_j.
This decision problem is then simply that of finding the alternative action $a_i \epsilon A$ which gives the highest utility for the give state x_j of the system.
Now, choose a_{io} such that max u_{ij} Suppose that the knowledge about the state of the system is fuzzy due to the state of the system are not known exactly. Then the state is a fuzzy set X with membership function μ_x (x_j), $x_j \in X$.
The fuzziness of the state of alternative implies that the utility associated with each alternative $a_i \in A$ can no longer remain crisp (exact) and that it becomes fuzzy. The fuzzy utility is a fuzzy set U_i with the membership function: μv_i (U_{ij}). Now the original utility matrix may become a matrix of fuzzy sets, U = {U_{ij}} where each fuzzy utility is a fuzzy set U_{ij} with membership function:, a procedure is outlined on how to obtain the fuzzy set of optimal alternatives and on how to deduce the final alternative action from that fuzzy set.
We define

$$\sum_{i=1}^{m} \mu\, v_{ij} (U_{ij}),\ j = 1, 2, 3 \ldots.\ n \qquad (2.2.10)$$

Then $x_s \leq x_j$ if $r_s \leq r$. Comparing in this way one can select the optimal alternative.
In the following we discuss statistical decision theory.

2.3 Statistical decision theory

Decision making can be viewed as a mapping from measurement space X to the decision space D. The mapping $f: X \longrightarrow D$ is called the decision function and each element $d \in D$ is called a decision. In order to determine a preference for a certain decision above other decisions, a loss function, L, is introduced. This loss function depends on the decision $d \in D$ and on the probability distribution F v on the space X, where v parameterizes the class of distributions.

$$Fv\,(x) = P_v\,(X < x)$$

Thus the loss function becomes $L(v, d)$where $d \in D$ and v is a parameter. The expected value of the loss function will be given by: $E(L(v, d)) = E(L(v, f(x)) = R(v, f)$ $R(v, f)$ is called the risk function.
Now, $d^1 \epsilon D$ is uniformly better than d_2 if

$R(v, f_1) \leq R(v, f)$ for all v
$R(v, f) \leq R(v, f)$ for at least one v,

Where

$d_1 f_1 (x) = d_2 f_2 (x)$

A class D of decision is known as complete if for every $d \in D$, there exists a $d^1 \in D$ such that d^1 is uniformly better than d. The minimizing of $R(v, f)$ will also depend on the parameter v which determines the distribution $F\ v\ (x)$. For all possible v, it is not always be possible to find $f(x)$ which minimizes $R(v, f)$. We now introduce a distribution $\rho(v)$ for the parameter v. The expected value of the risk of a decision function f becomes

$$E^1 (\rho, \mathrm{f}) = E_V \{L(v, f)\, \rho(v)\, dv = \{ R(v, f)\, \rho(v)\, dv$$

Thus, by definition the smaller $E^1 (\rho, f)$, the better f.
Then, by definition, the optimal f^* is given by

$$E^1 (\rho, \mathrm{f}^*) = E^1 (\rho, f)_f$$

This f^* is called a Bayes solution to the decision problem, and which is known as the model of decision making under risk. If the prior information about the distribution $\rho(v)$ is missing then we consider the supremum over v of the risk function $E^1(v, f)$. The decision function is then defined by inf Sup

$$E^1 (\rho, f)\ \mathrm{f\ v}$$

This is called a minimax procedure.

2.3.1 Extension to a fuzzy statistical decision theory

Extension of the statistical decision theory to a fuzzy statistical decision theory is done through the concepts of fuzzy event and the probability of a fuzzy event.
Let X and Y be sets of events {x_1, x_2, $x_3 \ldots x_n$} and {y_1, y_2, $y_3 \ldots y_n$} with Probabilities p (x_i) and p (y_i) respectively. The fuzzy events A and B are fuzzy sets on X and Y characterized by their membership functions

$$\mu A: X \longrightarrow [0, 1] \text{ and } \mu B: X \longrightarrow [0, 1] \text{ respectively.}$$

The probability of a fuzzy event A is defined by

$$P\,(A) = \sum_{i=1}^{m} \mu(A)\, P(x_i).$$

Let p (x_i, y_i) be the joint probability of x_i, and y_i. Then the joint probability of fuzzy events A and B is defined by

$$P\,(A, B) = \sum_{i}^{m} \mu(A) \sum_{j}^{n} \mu(B)\, p\,(x_i, y_i).$$

The extension of the probabilistic decision problem into a fuzzy probabilistic one is done by replacing events with fuzzy events. The decision problem with fuzzy events is defined as a quadruple < X, A, P, u> where {X_1, X_2, $X_3, \ldots X_r$} is a set of fuzzy states which are fuzzy events on a probabilistic space {x_1, x_2, $x_3, \ldots x_r$} {A_1, A_2, $A_3, \ldots A_q$} is a set of fuzzy actions which are fuzzy events on the actions space {a_1, a_2, $a_3, \ldots a_q$} and U is the utility function on A $\times$ X.
The expected utility fuzzy action A_i is defined by:

$$U(Ai) = U\,(Ai, Xj)\, P(Xj)$$

An optimal solution is defined by: $U(A^*) = \max U(A_i)$

2.4 Fuzzy decision making under constraints

A particular kind of decision - making may be characterized with the following situations:

1. A set of decision variables
2. A set of constraints on these decision variables
3. Objective function (s) which orders the alternatives according to their desirability

The problem is to find the optimum solution. When the situation is deterministic, then the problem can be solved by using the existing classical optimization technique(s) (e.g. mathematical programming). When the decision maker is no longer able to specify exactly the constraints and the objective, either because that is just impossible or because he wants to allow himself some leeway, and these quantities can only be expressed in natural language as 'much bigger', 'near to', etc., neither the notion of determinism nor of probability is satisfactory. In such situations the theory of fuzzy sets might be helpful.

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