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Variable control charts based on percentiles of the new Rayleigh-Pareto distribution

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Abstract
In this paper, we consider the New Rayleigh-Pareto distribution as a life time model. Based on the evaluated percentiles of sample estimates like sample mean, median, midrange, range and standard deviation, the control limits for the respective control charts are developed. The admissibility and power of the control limits are assessed in comparison with those on the popular Shewhart control limits.

Keywords: most probable, Pdf, Cdf, Equi-tailed, Percentiles, NRPD

1. Introduction

The well-known Shewhart control charts are developed under the assumption that the quality characteristic follows a normal distribution. If $x_1, x_2, \ldots\ldots, x_n$ is a collection of observations of size n on a variable quality characteristic of a product and if $t(x) = t_n$ a statistic is based on this sample, the control limits of Shewharts variable control chart are $E(t_n) \pm 3S.E(t_n)$. In quality control studies data is always in small samples only. Therefore if the population is not normal there is a need to develop separate procedure for the construction of control limits. In this paper we assume that the quality variate follows the new Rayleigh-Pareto model and develop control limits for such data on par with the presently available control limits. If a process quality characteristic is assumed to follow the new Rayleigh-Pareto distribution the online process of such a quality can be controlled through the theory of the new Rayleigh-Pareto distribution. In the absence of any such specification of the population model we generally use the normal distribution and the associated constants available in all standard text books of statistical quality control. However, normality is only an assumption that is rarely verified and found to be true. Unless the sample is very large in size this assumption may not be taken for granted without proper goodness of fit test procedure. At the same time central limit theorem cannot be made use of, because central limit theorem gives only asymptotic normality for any statistic. Therefore, if a distribution other than normal is a suitable model for a quality variate, separate procedures are to be developed. We present the construction of quality control charts when the process variate is assumed to follow the new Rayleigh-Pareto distribution. Let X be a random variable from a Pareto distribution with its cumulative distribution function (cdf) for $x \geq \alpha$ given by

$$F_1(x, \alpha, \beta) = 1 - \left(\frac{\alpha}{x}\right)^{\beta} \tag{1.1}$$

Where $\alpha \succ 0$ is a scale parameter and $\beta \succ 0$ is the shape parameter.

The probability density function (pdf) corresponding to (1.1) is

$$f_1(x,\alpha,\beta) = \frac{\beta\alpha^{\beta}}{x^{\beta+1}} \tag{1.2}$$

The NRPD has a cdf of the form

$$G(x) = \int_0^{\frac{1}{R(x)}} f_2(x)dx \tag{1.3}$$

Where $R(x)$ the survival is function of the Pareto distribution and is given by

$R(x) = 1 - F_1(x,\alpha,\beta)$ While $f_2(x)$ is the pdf of a Rayleigh distribution and is given by

$$f_2(x) = \frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}}, x \succ 0, \sigma \succ 0 \tag{1.4}$$

Using (1.3) and (1.4), and given that

$$R(x) = \left(\frac{\alpha}{x}\right)^{\beta}$$

$$\frac{1}{R(x)} = \left(\frac{x}{\alpha}\right)^{\beta}$$

The cdf of the NRPD is given by

$$G(x) = \int_0^{\left(\frac{x}{\alpha}\right)^{\beta}} \frac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}} dx \tag{1.5}$$

$$G(x) = 1 - e^{-\frac{1}{2\sigma^2}\left(\frac{x}{\alpha}\right)^{2\beta}} \tag{1.6}$$

If we take $2\beta = \lambda$ then

$$G(x) = 1 - e^{-\frac{1}{2\sigma^2}\left(\frac{x}{\alpha}\right)^{\lambda}} \tag{1.7}$$

The probability density function (pdf) is given by

$$g(x) = \frac{\lambda}{2\sigma^2\alpha^{\lambda}} x^{\lambda-1} e^{-\frac{1}{2\sigma^2}\left(\frac{x}{\alpha}\right)^{\lambda}} \tag{1.8}$$

Where $0 < x < \infty, \lambda > 0, \alpha > 0, \sigma > 0$

The hazard function is given by

$$h(x) = \frac{\lambda}{2\sigma^2\alpha^{\lambda}} x^{\lambda-1} \tag{1.9}$$

From the hazard function the following can be observed:

1. if $\lambda = 1,$ the failure rate is constant, which makes the NRPD suitable for modeling systems or components with failure rate
2. If $\lambda > 1,$ the hazard is an increasing function, which makes the NRPD suitable for modeling components that wears faster with time.
3. If $\lambda < 1,$ the hazard is a decreasing function, which makes the NRPD suitable for modeling components that wears slower with time. The distributional properties are:

$$Mean = E(x) = \alpha\left(2\sigma^2\right)^{\frac{1}{\lambda}} \Gamma\left(\frac{\lambda+1}{\lambda}\right) \tag{1.10}$$

$$Median = \alpha\left(2\sigma^2\right)^{\frac{1}{\lambda}} \left[\ln(2\ )\right]^{\frac{1}{\lambda}} \tag{1.11}$$

$$Variance = \alpha^2\left(2\sigma^2\right)^{\frac{2}{\lambda}} \Gamma\left(\frac{\lambda+2}{\lambda}\right) - \left[\alpha\left(2\sigma^2\right)^{\frac{1}{\lambda}} \Gamma\left(\frac{\lambda+1}{\lambda}\right)\right]^2 \tag{1.12}$$

The pdf of largest order statistics $x_{(n)}$ is given by

$$a_{(n)} = \frac{1}{2\sigma^2}\frac{n\lambda}{\alpha}\left(\frac{x}{\alpha}\right)^{\lambda-1} e^{-\frac{1}{2\sigma^2}\left(\frac{x}{\alpha}\right)^{\lambda}}\left(1 - e^{-\frac{1}{2\sigma^2}\left(\frac{x}{\alpha}\right)^{\lambda}}\right)^{n-1} \tag{1.13}$$

The pdf of the smallest order statistic $X_{(1)}$ is given by

$$a_{(1)} = \frac{1}{2\sigma^2}\frac{n\lambda}{\alpha}\left(\frac{x}{\alpha}\right)^{\lambda-1} e^{-\frac{1}{2\sigma^2}\left(\frac{x}{\alpha}\right)^{\lambda}}\left(e^{-\frac{1}{2\sigma^2}\left(\frac{x}{\alpha}\right)^{\lambda}}\right)^{n-1} \tag{1.14}$$

The other distributional properties are thoroughly discussed by Nasiru and Luguterah (2015) [8]. Skewed distributions to develop statistical quality control methods are attempted by many authors. Some of them are Edge-man (1989) [3]- Inverse Gaussian Distribution, Gonzalez and Viles (2000) [4]- Gamma Distribution, Kantam and Sriram (2011) [5] – Gamma Distribution, Chan and Cui (2003) [2] have developed control chart constants for skewed distributions where the constants are dependent on the coefficient of skewness of the distributions, Kantam *et al* (2006) [6] – Log logistic Distribution, Betul and Yaziki (2006) [1] – Burr Distribution, Subba Rao and Kantam (2008) [11] –Double exponential distribution, Kantam and Rao (2010) [7] – control charts for process variate, Rao and Sarath Babu (2012) [9] - Linear failure rate distribution, Rao and Kantam (2012) [10] - Half logistic distribution, K. Rosaiah, R.R.L. Kantam, B. Srinivas Rao (2012) [12] Variable control charts for Half logistic distribution, Srinivas Rao Boyapati, Suleman Nasiru, K.N.V.R. Lakshmi (2015) [13] – Variable Control Charts Based on Percentiles of the new Weibull-Pareto Distribution and references there in.

NRPD is another situation of skewed distribution which is paid much attention with respect to development of control charts in the present study. If $\lambda = 0.5, \alpha = 2.5, \sigma = 1.5$

Then the hazard function indicates a decreasing failure rate function (shown in the graph), which makes the $NRPD$ suitable for modeling components that wears slower with time. At the same time it is one of the probability models applicable for life testing and reliability studies also. Accordingly, if a lifetime data is considered as a quality data, development of control charts for the same is desirable for the use by practitioners. Since $NRPD$ is a skewed distribution, this paper makes an attempt to study in a comparative manner. An attempt is made in this paper to address this problem and solve it to the extent possible. The rest of the paper is organized follows. The basic theory and the development of control charts for the statistics – average, median, midrange, range and standard deviation are presented in Section 2. The comparative study to the developed control limits in relation to the Shewarts limits is given in Section 3. Summary and conclusions are given in Section 4.

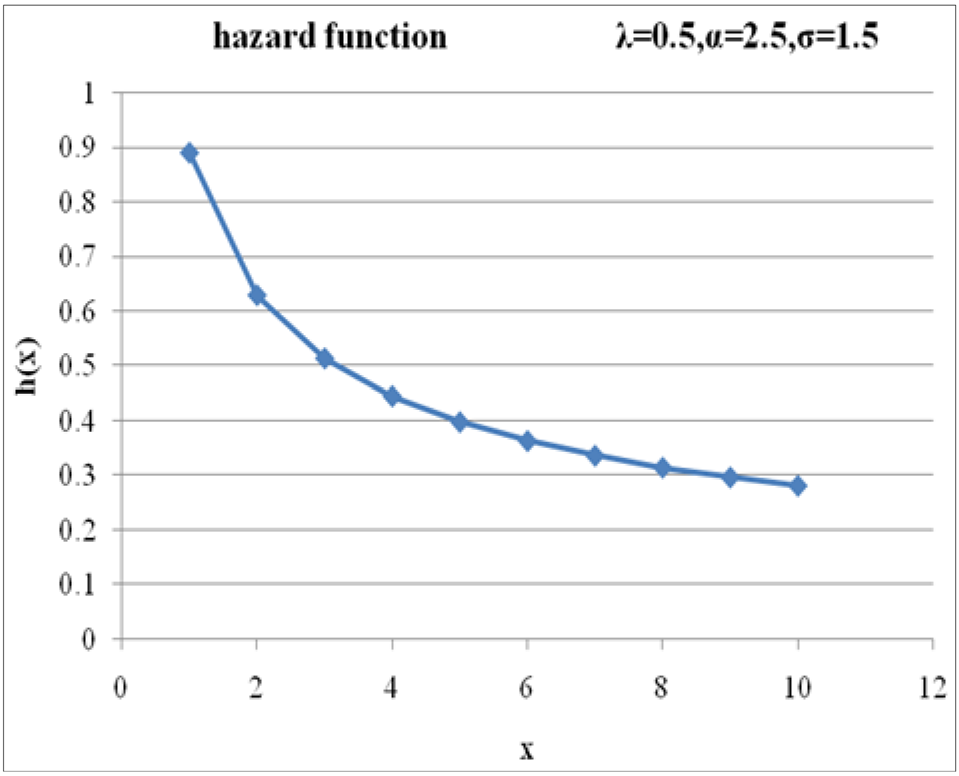

Fig 1: Hazard function of the New Rayleigh-Pareto distribution

2. Control chart constants through percentiles

2.1 Mean- chart

Let $x_1, x_2, x_3, \ldots\ldots, x_n$ be a random sample of size n supposed to have been drawn from $NRPD$ with $\lambda = 0.5, \alpha = 2.5, \sigma = 1.5$ this is considered as a subgroup of an industrial process data with a targeted population average, under repeated sampling the statistic $\bar{x}$ gives whether the process average is around the targeted mean or not. Statistically speaking, we have to find the 'most probable' limits with in which $\bar{x}$ fails. Here the phrase 'most probable' is a relative concept which is to be considered in the population sense. As the existing procedures are for normal distribution only, the concept of 3σ control limits is taken as the 'most probable' limits. It is well known that 3σ limits of normal distribution include 99.73% of probability. Hence, we have to search two limits of the sampling distribution of sample mean in $NRPD$ such that the probability content of those limits is 0.9973. Symbolically we have to find Lower control limit and Upper control limit (L and U) such that

$$P(L \le \bar{x} \le U) = 0.9973$$

Where $\bar{x}$ is the mean of the sample size n. taking the equi-tailed concept L, U are respectively 0.00135 and 0.99865 percentiles of the sampling distribution of $\bar{x}$. We resorted to the empirical sampling distribution of $\bar{x}$ through simulation there by computing its percentiles. These are given in Table 1.

Table 1: Percentiles of Mean in NRPD $\lambda = 0.5, \alpha = 2.5, \sigma = 1.5$

n	0.99865	0.9950	0.99	0.975	0.95	0.05	0.025	0.01	0.005	0.00135
2	937.0283	603.3082	421.3133	259.4023	169.0804	1.3431	1.0594	0.8373	0.7396	0.6516
3	782.0261	511.9547	372.9478	242.7141	168.1190	1.7809	1.4531	1.1678	1.0266	0.8415
4	589.4225	384.7379	322.5086	224.1744	149.9244	2.2151	1.8453	1.4825	1.2855	0.9882
5	537,5703	378.1364	287.9457	203.8294	142.3157	2.4629	2.0727	1.7376	1.5179	1.2191
6	471.1949	356.6365	287.1515	201.9097	145.4508	2.7202	2.2763	1.8833	1.7130	1.3786
7	387.7605	292.0844	251.6923	175.7043	132.4924	2.8974	2.5385	2.1651	1.9053	1.6153
8	370.1256	286,4856	242.0783	174.9430	129.3269	3.0790	2.6537	2.2783	2.0502	1.6440
9	360.4601	279.7247	227.6512	163.3620	124.6324	3.3575	2.9208	2.4770	2.2820	1.9559
10	322,0967	248.1250	209.1331	154.1960	118.2283	3.4778	3.0259	2.5991	2.3885	2.0633

The percentiles in the above table are used in the following manner to get the control limits for sample mean. From the distribution of $\bar{x}$, consider

$$P(Z_{0.00135} \le \bar{x} \le Z_{0.99865}) = 0.9973 \qquad (2.2)$$

But $\bar{x}$ of sampling distribution when $\lambda = 0.5, \alpha = 2.5, \sigma = 1.5$ is 101.25 for NRPD. From equation (2.2) over repeated sampling for the i^{th} subgroup mean we can have

$$P(Z_{0.00135} \frac{\bar{\bar{x}}}{101.25} \le \overline{x_i} \le Z_{0.99865} \le \frac{\bar{\bar{x}}}{101.25}) = 0.9973 \qquad (2.3)$$

This can be written as

$$P(A^{*}_{2p} \bar{\bar{x}} \le \overline{x_i} \le A^{**}_{2p} \bar{\bar{x}}) = 0.9973 \qquad (2.4)$$

Where $\bar{\bar{x}}$ is grand mean, $\overline{x_i}$ is i^{th} subgroup mean, $A^{*}_{2p} = \frac{Z_{0.00135}}{101.25}, A^{**}_{2p} = \frac{Z_{0.99865}}{101.25}$, Thus A^{*}_{2p}, A^{**}_{2p} are the percentile constants of $\bar{x}$ chart for $NRPD$ are given in Table 2.

Table 2: Percentile constants of Mean-chart

n	A^{*}_{2p}	A^{**}_{2p}
2	9.2546	0.0064
3	7.7237	0.0083
4	5.8215	0.0098
5	5.3093	0.0120
6	4.6538	0.0136
7	3.8297	0.0160
8	3.6556	0.0162
9	3.5601	0.0193
10	3.1812	0.0204

2.2 Median –chart

We have to search two limits of the sampling distribution of sample median in NRPD such that the probability content of these limits is 0.9973. Symbolically, we have to find L, U such that

$$P(L \le m \le U) = 0.9973 \qquad (2.5)$$

Where m is the median of sample size n. Through simulation, the percentiles observed are given Table 3.

Table 3: Percentiles of Median in $NRPD$ $\lambda = 0.5, \alpha = 2.5, \sigma = 1.5$

n	0.99865	0.9950	0.99	0.975	0.95	0.05	0.025	0.01	0.005	0.00135
2	937.0293	603.3082	421.3133	259.4023	169.0804	1.3431	1.0594	0.8373	0.7396	0.6516
3	367.4579	207.8956	153.9977	83.1014	43.7745	1.1373	0.9408	0.7855	0.7096	0.6229
4	274.8942	170.9696	121.0930	71.0598	42.0982	1.5643	1.2796	1.0370	0.9183	0.8173
5	156.6554	99.6844	71.3429	36.1589	17.9403	1.4213	1.1746	0.9687	0.8789	0.7636
6	163.9502	86.8408	64.6141	35.0793	20.5185	1.7022	1.4442	1.1941	1.043	0.9102
7	87.8772	59.3705	38.7942	17.8748	7.9261	1.6333	1.3762	1.1350	1.0303	0.8777
8	87.6519	48.6291	36.5253	19.9822	11.2651	1.8928	1.6400	1.3435	1.2058	1.0439
9	65.2243	37.0454	24.6618	11.832	7.5873	1.8467	1.5664	1.3577	1.2217	1.0183
10	45.6535	29.6765	21.4937	12.0055	7.7147	2.0226	1.7529	1.4800	1.3324	1.1516

The percentiles in the above table are used in the following manner to get the control limits for median. From the distribution of m, consider

$$P(Z_{0.00135} \le m \le Z_{0.99865}) = 0.9973 \qquad (2.6)$$

But median of sampling distribution when $\lambda = 0.5, \alpha = 2.5, \sigma = 1.5$ is 24.3229 for $NRPD$.
From equation (2.6) over repeated sampling for the i^{th} subgroup median we can have

$$P\left(Z_{0.00135}\frac{\overline{m}}{24.3229} \leq m_i \leq Z_{0.99865}\frac{\overline{m}}{24.3229} \right) \quad (2.7)$$

This can be written as

$$P(A^{*}_{7p}\overline{m} \leq \overline{m_i} \leq A^{**}_{7p}\overline{m}) = 0.9973 \quad (2.8)$$

Where $\overline{m}$ is mean of subgroup medians. Thus $A^{*}_{7p} = \frac{Z_{0.00135}}{24.3229}, A^{**}_{7p} = \frac{Z_{0.99865}}{24.3229}$ are the percentile constants of median chart and are given in Table 4.

Table 4: Percentile constants of Median-chart $\lambda = 0.5, \alpha = 2.5, \sigma = 1.5$

n	A^{*}_{7p}	A^{**}_{7p}
2	38.5245	0.0268
3	15.1075	0.0256
4	11.3019	0.0336
5	6.4407	0.0314
6	6.7406	0.0374
7	3.6129	0.0361
8	3.6037	0.0429
9	2.6816	0.0419
10	1.8770	0.0473

2.3 Midrange-chart

We have to search two limits of the sampling distribution of sample midrange in NRPD such that the probability content of these limits is 0.9973. Symbolically, we have to find L, U such that

$$P(L \leq M \leq U) = 0.9973 \quad (2.9)$$

Where M is the midrange of sample size n. Through simulation, the percentiles observed are given Table 5.

Table 5: Percentiles of Midrange in $NRPD$ $\lambda = 0.5, \alpha = 2.5, \sigma = 1.5$

n	0.99865	0.9950	0.99	0.975	0.95	0.05	0.025	0.01	0.005	0.00135
2	937.0283	603.3082	421.3133	259.4023	169.0804	1.3431	1.0594	0.8373	0.7396	0.6516
3	1093.3837	700.5959	523.8824	345.0097	228.9893	1.8471	1.4812	1.2030	1.0363	0.8673
4	1145.4553	735.9828	578.766	388.8706	264.0914	2.4392	2.0184	1.6170	1.3666	1.0200
5	1245,2709	826.7126	638.4606	438.5595	301.3386	2.8231	2.3703	1.9615	1.7037	1.3707
6	1288.8665	921.8350	712.011	519.3283	360.6267	3.1737	2.7137	2.1964	1.9047	1.4727
7	1208.1923	897.4734	728.7917	519.4619	369.1217	3.4814	3.0548	2.6007	2.2749	1.8832
8	1336.1149	979.2936	801.7606	554.2663	403.3460	3.6719	3.2437	2.7591	2.44506	1.9841
9	1462.7422	1049.3754	809.8843	576.9062	420.6878	3.9988	3.5604	3.0587	2.7659	2.3615
10	1462.7408	1029.7532	835.6108	584.3670	435.0516	4.1520	3.7401	3.2437	2.9949	2.6273

The percentiles in the above table are used in the following manner to get the control limits for midrange. From the distribution of M, consider

$$P(Z_{0.00135} \leq M \leq Z_{0.99865}) = 0.9973 \quad (2.10)$$

The midrange value of $NRPD$ calculated by using $\alpha_{(1)}$ and $\alpha_{(n)}$.

From equation (2.10) for i^{th} subgroup midrange we can have,

$$P\left(Z_{0.00135}\frac{\overline{M}}{\frac{\alpha_{(1)}+\alpha_{(n)}}{2}} \leq M_i \leq Z_{0.99865}\frac{\overline{M}}{\frac{\alpha_{(1)}+\alpha_{(n)}}{2}}\right) \quad (2.11)$$

This can be written as

$$P(A^{*}_{4p}\overline{M} \le M_i \le A^{**}_{4p}\overline{M}) = 0.9973 \qquad (2.12)$$

Where $\overline{M}$ is mean of midranges. Thus $A^{*}_{4p} = \dfrac{2Z_{0.00135}}{\alpha_{(1)}+\alpha_{(n)}}, A^{**}_{4p} = \dfrac{2Z_{0.99865}}{\alpha_{(1)}+\alpha_{(n)}}$ are the percentile constants of midrange chart for $NRPD$ process data given in Table 6.

Table 6: Percentile constants of Midrange-chart $\lambda = 0.5, \alpha = 2.5, \sigma = 1.5$

n	A^{*}_{4p}	A^{**}_{4p}
2	1778288.5000	7.9099
3	177489.9844	8.9634
4	41951.5195	8.5190
5	39074.0117	10.2136
6	35994.1680	10.5302
7	34319.5859	12.5940
8	30761.6406	11.9830
9	31877.4219	15.3548
10	34790.0664	14.5335

2.4 R-chart

We have to search two limits of the sampling distribution of sample Range in NRPD such that the probability content of these limits is 0.9973. Symbolically, We have to find L, U such that

$$P(L \le R \le U) = 0.9973 \qquad (2.13)$$

Where R is the range of sample of size n. Through simulation, the percentiles observed are given in Table 7.

Table 7: Percentiles of Range in $NRPD$ $\lambda = 0.5, \alpha = 2.5, \sigma = 1.5$

n	0.99865	0.9950	0.99	0.975	0.95	0.05	0.025	0.01	0.005	0.00135
2	1792.5670	1097.8423	787.0557	478.6137	298.3763	0.2700	0.1380	0.0561	0.0281	0.0099
3	2185.3918	1396.7192	1037.4709	681.5677	451.0629	1.2602	0.8466	0.5897	0.4103	0.2232
4	2288.7515	1465.2440	1138.2272	773.5310	524.2709	2.4422	1.8469	1.2687	1.0216	0.5725
5	2488.5742	1651.6689	1274.4357	875.2571	598.8456	3.3560	2.6465	1.9599	1.5663	1.0496
6	2571.8535	1840.6733	1420.4451	1036.4128	719.2916	4.2502	3.4574	2.5311	2.1478	1.2769
7	2414.5769	1792.2539	1453.9717	1036.4434	736.4628	4.9057	4.1473	3.2646	2.8039	2.1148
8	2670.9036	1954.6270	1601.9783	1106.9457	805.3129	5.4860	4.6852	3.7484	3.3060	2.4507
9	2923.9709	2094.4844	1618.1107	1150.1475	839.5650	6.1736	5.3418	4.3476	3.8054	3.1797
10	2923.1133	2058.1680	1669.25	1167.3319	867.7080	6.5655	5.7265	4.7892	4.3487	3.5724

The percentiles in the above table are used in the following manner to get the control limits for sample range. From the distribution of R, consider

$$P(Z_{0.00135} \le R \le Z_{0.99865}) = 0.9973 \qquad (2.14)$$

From equation (2.14), for the i^{th} subgroup range we can have

$$P\left(Z_{0.00135}\frac{\overline{R}}{\alpha_{(n)}-\alpha_{(1)}} \le R_i \le Z_{0.99865}\frac{\overline{R}}{\alpha_{(n)}-\alpha_{(1)}}\right) = 0.9973 \qquad (2.15)$$

This can be written as

$$P(D^{*}_{3p}\overline{R} \le R_i \le D^{*}_{4p}\overline{R}) = 0.9973 \qquad (2.16)$$

Where $\overline{R}$ mean of is ranges, R_i is i^{th} subgroup range. Thus

$D^*_{3p} = \dfrac{Z_{0.00135}}{\alpha_{(n)} - \alpha_{(1)}}, D^*_{4p} = \dfrac{Z_{0.99865}}{\alpha_{(n)} - \alpha_{(1)}}$ Are the percentile constants of R chart for NRPD process data and are given in Table 8.

Table 8: Percentile constants of Range -chart

n	D^*_{3p}	D^*_{4p}
2	4.533	0.0006
3	6.2356	0.0018
4	7.8456	0.003
5	9.2323	0.0043
6	10.1235	0.0055
7	11.3546	0.0071
8	11.9542	0.0084
9	12.0135	0.0092
10	12.8594	0.0109

2.5 σ - chart

We have to search two limits of the sampling distribution of sample standard deviation in NRPD such that the probability content of these limits is 0.9973. Symbolically, we have to find L,U such that

$$P(L \le s \le U) = 0.9973 \quad (2.17)$$

Where s is the standard deviation of sample of size n. Through simulation the percentiles observed are given in Table 9.

Table 9: Percentiles of Standard deviation in $NRPD\ \lambda = 0.5, \alpha = 2.5, \sigma = 1.5$

C	0.99865	0.9950	0.99	0.975	0.95	0.05	0.025	0.01	0.005	0.00135
2	896.2835	548.9211	393.5278	239.3069	149.1881	0.1350	0.0690	0.0280	0.0141	0.0049
3	1008.4962	642.0943	487.2334	317.4920	208.3269	0.5404	0.3700	0.2486	0.1728	0.0974
4	958.7794	633.1101	491.8904	329.3040	222.4091	0.9626	0.7231	0.4923	0.3872	0.2259
5	983.8661	667.9033	503.5619	347.8533	238.0112	1.2378	0.9797	0.7185	0.5739	0.4011
6	957.1638	673.3813	532.8484	381.5732	267.2060	1.5022	1.2169	0.9015	0.7542	0.5053
7	829.3943	618.4649	507.6351	358.9405	256.3640	1.6861	1.4176	1.1099	0.9706	0.7127
8	874.1238	650.5952	525.6599	368.1484	268.6326	1.8449	1.5787	1.2554	1.0622	0.7916
9	917.2201	652.5015	509.1854	367.4867	266.3594	2.0318	1.7293	1.4351	1.2685	1.0009
10	866.6826	625.0745	498.1116	352.2227	267.3921	2.1569	1.8633	1.5572	1.3773	1.1283

The percentiles in the above table are used in the following manner to get the control limits for sample standard deviation. From the distribution of s, consider

$$P(Z_{0.00135} \le s \le Z_{0.99865}) = 0.9973 \quad (2.18)$$

But standard deviation of sampling distribution when $\lambda = 0.5, \alpha = 2.5, \sigma = 1.5$ is 226.4018 for $NRPD$ .From equation (2.18), for the ith subgroup standard deviation we can have

$$P(Z_{0.00135}\frac{\overline{S}}{226.4018} \le S_i \le Z_{0.99865}\frac{\overline{S}}{226.4018}) = 0.9973$$

This can be written as

$$P(B^*_{3p}\overline{S} \le S_i \le B^*_{4p}\overline{S}) = 0.9973 \quad (2.20)$$

Where $\overline{S}$ is mean of standard deviation, s_i is ith subgroup standard deviation. Thus $B^*_{3p} = \dfrac{Z_{0.00135}}{226.4018}, B^*_{4p} = \dfrac{Z_{0.99865}}{226.4018}$ are the constants of standard deviation chart for $NRPD$ process data given in Table 10.

Table 10: Percentile constants of SD-chart $\lambda = 0.5, \alpha = 2.5, \sigma = 1.5$

n	B^*_{3p}	B^*_{4p}
2	3.9588	0.0000
3	4.4545	0.0004
4	4,2349	0.0010
5	4.3457	0.0018
6	4.2277	0.0022
7	3.6634	0.0031
8	3.8609	0.0035
9	4.0513	0.0044
10	3.8281	0.0050

3. Comparative study

The control chart constants for the statistics mean, median, midrange, range and standard deviation developed in section 2 are based on the population described by NRPD. In order to use this for a data, the data is confirmed to follow NRPD. Therefore the power of the control limits can be accessed through their application for a true NRPD data in relation to the application for Shewhart limits. With this back drop we have made this comparative study simulating random samples of size n= 2,3,…,10 from NRPD and calculated the control limits using the constants of section 2 for mean, median, midrange, range and standard deviation in succession. The number of statistic values that have fallen within the respective control limits is evaluated and is named as NRPD coverage probability. Similar count for control limits using Shewhart constants available in quality control manuals are also calculated. These are named as Shewhart coverage probability. The coverage probabilities under the two schemes namely true NRPD, Shewhart limits are presented in the following Tables 11,12,13,14 and 15.

Table 11: Coverage Probabilities of Mean-chart $\lambda = 0.5, \alpha = 2.5, \sigma = 1.5$

	Shewhart limits			Percentile limits		
n	$\bar{\bar{x}} - A_2 \bar{R}$	$\bar{\bar{x}} + A_2 \bar{R}$	Coverage Probability	$A^*_{2p} \times \bar{\bar{x}}$	$A^{**}_{2p} \times \bar{\bar{x}}$	Coverage probability
2	0	34.47385805	0.7937	0.2180096	315.2486944	0.9821
3	0	36.18465349	0.7676	0.297804	277.126356	0.9815
4	0	35.23091126	0.7394	0.3428138	203.6418915	0.9705
5	0	36.33082484	0.7141	0.432972	191.5648533	0.9726
6	0	37.32961052	0.7048	0.5043696	172.5908268	0.9653
7	0	35.85376296	0.6979	0.56984	136.3947655	0.9532
8	0	36.7135126	0.6894	0.5911866	133.4038108	0.9526
9	0	37.07659153	0.6765	0.7109541	131.1434037	0.9558
10	0	36.4656906	0.6702	0.7392552	115.2803256	0.9467

Table 12: Coverage Probabilities of Median-chart $\lambda = 0.5, \alpha = 2.5, \sigma = 1.5$

	Shewhart limits			Percentile limits		
n	$\bar{m} - A_7 \bar{R}$	$\bar{m} + A_7 \bar{R}$	Coverage Probability	$A^*_{7p} \times \bar{m}$	$A^*_{7p} \times \bar{m}$	Coverage probability
2	0	35.78028058	0.7989	0.9129152	1312.298568	0.9847
3	0	11.22331583	0.8826	0.27967744	165.0479268	0.9913
4	0	10.83113933	0.8368	0.3544464	119.2237431	0.9896
5	0	6.746066988	0.8326	0.20752574	42.56723037	0.9804
6	0	6.964670026	0.8495	0.25494832	45.94932208	0.9833
7	0	5.376148463	0.7196	0.19049609	19.06491201	0.9765
8	0	5.424272671	0.7477	0.22803495	19.15546735	0.9737
9	0	4.929939564	0.6616	0.20279181	12.97867584	0.9777
10	0	4.840161532	0.6581	0.22449053	8.9084297	0.9594

Table 13: Coverage Probabilities of Mid- range-chart $\lambda = 0.5, \alpha = 2.5, \sigma = 1.5$

	Shewhart limits			Percentile limits		
n	$\bar{M} - A_4 \bar{R}$	$\bar{M} + A_4 \bar{R}$	Coverage Probability	$A^*_{4p} \times \bar{M}$	$A^{**}_{4p} \times \bar{M}$	Coverage probability
2	0	34.1472967	1.0000	0.0374704	60575619.46	1.0000
3	0	48.49931028	1.0000	0.11122455	8583149.198	1.0000
4	0	59.5801592	1.0000	0.20200386	2492461.433	1.0000
5	0	73.48328431	1.0000	0.34421954	2861710.284	1.0000
6	0	88.95753649	0.9834	0.5142744	3191530.888	1.0000
7	0	94.0693444	0.9714	0.5626572	3218360.351	1.0000

8	13.0620838	106.5169429	0.1625	0.78564986	3265929.545	0.9875
9	19.62511	117.6464977	0.1782	0.98475636	3737082.614	0.8563
10	32.7650385	124.64736	0.1900	1.20474388	4320940.175	0.8900

Table 14: Coverage probabilities of Range-chart $\lambda = 0.5, \alpha = 2.5, \sigma = 1.5$

	Shewhart limits			Percentile limits		
n	$D_3 \overline{R}$	$D_4 \overline{R}$	Coverage Probability	$D^*_{3p} \times \overline{R}$	$D^*_{4p} \times \overline{R}$	Coverage probability
2	0	188.894673	0.9168	0.0346914	262.1108727	0.9360
3	0	237.0339	0.8911	0.1656936	573.9994512	0.9656
4	0	263.324544	0.8758	0.346176	905.3194752	0.9813
5	0	303.62094	0.8611	0.6172908	1325.352059	0.9910
6	0	350.17395	0.8594	0.96105625	1768.955081	0.9936
7	14.0745236	356.3076764	0.6237	1.31485681	2102.770864	0.9974
8	28.5835824	391.7632176	0.5917	1.76545656	2512.454858	0.9978
9	42.761508	422.037492	0.5618	2.1380754	2791.931393	0.9983
10	54.9597995	437.9532005	0.5561	2.68637585	3169.282716	0.9991

Table 15: Coverage probabilities of SD-chart $\lambda = 0.5, \alpha = 2.5, \sigma = 1.5$

	Shewhart limits			Percentile limits		
n	$B_3 \overline{S}$	$B_4 \overline{S}$	Coverage Probability	$B^*_{3p} \times \overline{S}$	$B^*_{4p} \times \overline{S}$	Coverage probability
2	0	94.4476632	0.9168	0	114.4473245	0.9309
3	0	108.467184	0.8919	0.0168952	188.149171	0.9419
4	0	110.7047502	0.8749	0.0488547	206.894769	0.9434
5	0	118.2670638	0.8602	0.10190556	246.0283289	0.9529
6	1.940373	127.417827	0.7482	0.14229402	273.4438311	0.9522
7	7.6414794	121.8751206	0.5609	0.20075073	237.2355562	0.9419
8	12.975234	127.297566	0.5319	0.2454774	270.7896268	0.9504
9	17.703686	130.444314	0.5017	0.3259256	300.0959962	0.9615
10	21.4590968	129.6613032	0.5016	0.377801	289.2520016	0.9581

4. Summary and Conclusions

The Tables 11, 12, 13, 14 and 15 show that for a true NRPD if the Shewhart limits are used in a mechanical way it would result in less confidence coefficient about the decision of process variation for mean, median, midrange, range and standard deviation charts. Hence if a data is confirmed to follow NRPD, the usage of Shewhart constants in all the above charts is not advisable and exclusive evaluation and application of NRPD constants is preferable in statistical quality control.

5. References

1. Betul Kan, Berna Yaziki. The Individual Control Charts for BURR distributed data in proceedings of the nindth WSEAS International Conference on Applied Mathematics, Istanbul, Trukey 2006, 645-649.
2. Chan. Lai K, Heng J. Cui Skewness Correction $\overline{x}$ and R-chars for skewed distributions Naval Research Logistics 2003;50;1-19.
3. Edgeman RL. Inverse Gaussian control charts Australian Journal of Statistics 1989;31(10):435-446.
1. Gonzalez I, Viles I, Semi-Economic design of Mean control charts assuming Gamma Distribution Economic Quality Control 2000;15:109-118.
2. Kantam RRL, Sriram B, Variable control charts based on Gamma distribution IAPQR Transactions 2001;26(2):63-67.
3. Kantam RRL, Vasudeva Rao A, Srinivasa Rao G. Control Charts for Log-logistic Distribution Economic Quality Control 2006;21(1):77-86.
4. Kantam RRL, Srinivasa Rao B. An improved dispersion control charts for process variate International journal of Mathematics and Applied statistics 2010;1(1):19-24.
5. Nasiru S, Luguterah A, The new Weibull-Pareto distribution Pakistan Journal of Statistics and Operations Research 2015;11(1):103-114.
6. Srinivasa Rao B, Sarath babu G. Variable control charts based on Linear Failure Rate Model International Journal of Statistics and Systems 2012;7(3):331-341.
7. Srinivasa Rao B, Kantam RRL. Mean and range charts for skewed distributions-A comparison based on half logistic distribution Pakistan Journal of Statistics 2012;28(4):43d7-444.
8. Subba Rao R, Kantam RRL. Variable control charts for process mean with reference to double exponential distribution Acta Cinica Indica 2008;34(4):1925-1930.
9. Srinivasa Rao B, Nasiru S, Lakshmi KNVR. Variable Control Charts based on Percentiles of the New Weibull-Pareto Distribution Pakistan Journal of Statistics and Operations Research 2015;9(4):631-643.