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Effect of hall current on hydromagnetic instability of a couple-stress ferromagnetic fluid in the presence of varying gravitational field through a porous medium

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Abstract
The effect of hall current on hydromagnetic instability of a couple-stress ferromagnetic fluid in the presence of variable gravity field through a porous medium is determined. We have used a linear stability theory and normal mode technique to discover the precise solution for a layer of couple-stress ferromagnetic fluid contained between two free boundaries. For the case of stationary convection, we have found that medium permeability, couple-stress, magnetic field and hall current have both stabilizing and destabilizing effect under certain conditions. Also, magnetization has a stabilizing effect on the system. Graphs in each case have been plotted by giving numerical values to these parameters. In the absence of magnetic field (hence hall current), the principle of exchange of stabilities is found to hold true under certain conditions.

Keywords: hydromagnetic instability, couple-stress fluid, ferromagnetic fluid, magnetic field, hall current, porous medium

1. Introduction

The growing importance of the usage of non-Newtonian fluids in modern technology and industries has led various researchers to strive numerous flow problems related with several non-Newtonian fluids. One such fluid that has attracted research workers during the last four decades is the couple-stress fluid. The theory of couple-stress fluids initiated by Stokes [1] is a generalization of the classical theory of viscous fluids, which allows for the presence of couple-stresses and body couples in the fluid medium. Sunil *et al.* [2] have mentioned the effect of Hall currents on thermosolutal instability of compressible Rivlin-Ericksen fluid. Rani and Tomar [3, 4] have investigated thermal and thermosolutal convection problem of micropolar fluid subjected to Hall current. Kumar [5] have examined the effect of Hall currents on thermal instability of compressible dusty visco-elastic fluid saturated in a porous medium subjected to vertical magnetic field.

There are various stability problems on ferromagnetic fluids. The convective instability, also known as Bénard convection (Chandrasekhar [6]) is one of the instability of ferromagnetic fluid. Rosensweig [7] has given a definite introduction about magnetic liquids in his monograph. Finlayson [8] have studied the convective instability of ferromagnetic fluid for a fluid layer heated from below in the presence of uniform vertical magnetic field. The effect of Hall current on thermal instability has been mentioned by many authors (Raghavachar *et al.* [9], Sharma *et al.* [10], Gupta *et al.* [11, 12]). Aggarwal and Makhija [13] have studied the effect of Hall current on thermal stability of ferromagnetic fluids heated from below in porous medium in the presence of horizontal magnetic field.

In the present study, we have discussed the effect of Hall current on hydromagnetic instability of couple-stress ferromagnetic fluid in the presence of variable gravity field saturating in a porous medium. Recently, some stability problems about couple-stress ferromagnetic fluid have been discussed by Nadian *et al.* [14-16]. In the present problem, we have assumed that the gravity is varying as $g = \lambda g_0$ where g_0 is the value of g at the surface of the Earth, which is always positive and λ can be positive or negative.

2. Mathematical Formulation of the Problem

We consider an infinite, incompressible, electrically non-conducting and thin layer of couple-stress ferromagnetic fluid which is bounded by the planes $z = 0$ and $z = d$ as shown in Fig. 1. The fluid layer is heated from below so that a uniform temperature gradient $\beta = \left|\frac{dT}{dz}\right|$ is maintained within the fluid. The whole system is acted upon by a uniform vertical magnetic field $\boldsymbol{H}(0,0,H)$ and variable gravity field $\boldsymbol{g}(0,0,-g)$, where $g = \lambda g_0$. Furthermore, the couple-stress ferromagnetic fluid layer is assumed to be flowing through an isotropic and homogeneous porous medium of porosity ε.

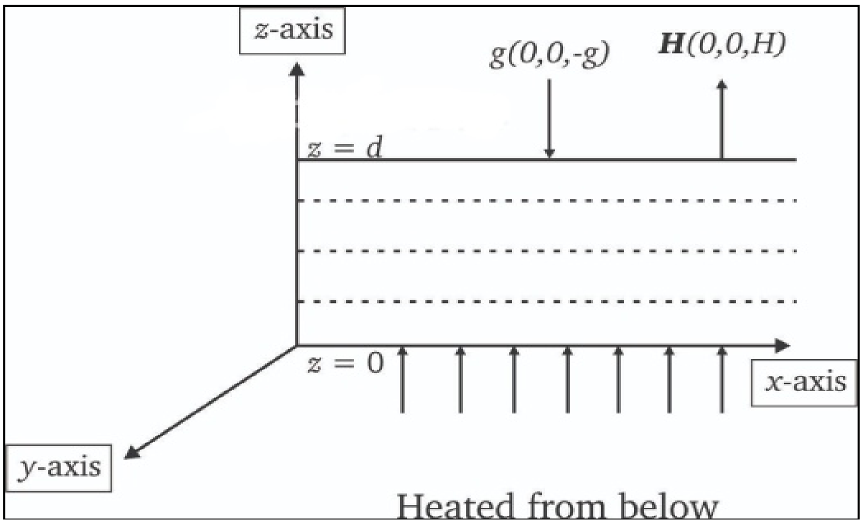

Fig 1: Geometrical Configuration

The equation of conservation of momentum, continuity, temperature and equation of density for the above model are as follows:

$$\frac{1}{\varepsilon}\left[\frac{\partial \boldsymbol{v}}{\partial t}+\frac{1}{\varepsilon}(\boldsymbol{v}.\nabla)\boldsymbol{v}\right] = -\frac{1}{\rho_0}\nabla p + \boldsymbol{g}\left(1+\frac{\delta\rho}{\rho_0}\right) - \frac{1}{k_1}\nu\boldsymbol{v} + \frac{1}{\rho_0}\boldsymbol{M}.\nabla\boldsymbol{H} + \left(\nu - \frac{\mu'}{\rho_0}\nabla^2\right)\nabla^2\boldsymbol{v} + \frac{\mu_e}{4\pi\rho_0}\left[(\nabla\times\boldsymbol{H})\times\boldsymbol{H}\right], \quad (1)$$

$$\nabla.\boldsymbol{v} = 0, \quad (2)$$

$$E\frac{\partial T}{\partial t} + (\boldsymbol{v}.\nabla)T = \kappa\nabla^2 T, \quad (3)$$

$$\rho = \rho_0\left[1-\alpha(T-T_0)\right], \quad (4)$$

where, $\boldsymbol{v}(u_1,u_2,u_3)$ is the fluid velocity, p is the fluid pressure, ρ is the fluid density, ρ_0 is the reference density, T is the temperature, T_0 is the reference temperature, α is the thermal coefficient of expansion, μ_e is the magnetic permeability, μ' is the couple-stress viscosity, ν is the kinematic viscosity, κ is the thermal diffusivity, $E = \varepsilon + (1-\varepsilon)\frac{\rho_s c_s}{\rho_0 c_v}$ (where, ρ_s, c_s, c_v denote the density of the solid (porous) material, heat capacity of the solid (porous) material and heat capacity of the fluid at constant volume), $\boldsymbol{M}$ is the magnetization, $\nabla\boldsymbol{H}$ is the magnetic field gradient.

In presence of Hall current, the Maxwell's equations are given by,

$$\varepsilon\frac{\partial \boldsymbol{H}}{\partial t} = \nabla\times(\boldsymbol{v}\times\boldsymbol{H}) + \varepsilon\eta\nabla^2\boldsymbol{H} - \frac{\varepsilon}{4\pi Ne}\nabla\times\left[(\nabla\times\boldsymbol{H})\times\boldsymbol{H}\right], \quad (5)$$

And $\nabla.\boldsymbol{H} = 0, \quad (6)$

where, $\boldsymbol{H}$ is the magnetic field intensity.

We consider that the magnetization is independent of magnetic field intensity but depend upon the temperature so that $M = M(T)$. As a first approximation, we consider that

$$M = M_0\left[1-\gamma(T-T_0)\right], \quad (7)$$

where, $\gamma = \frac{1}{M_0}\left(\frac{\partial M}{\partial T}\right)_H$ and M_0 is the magnetization at $T = T_0$ with T_0 being the reference temperature.

3. Basic State and Perturbation Equations
The basic state of which we wish to examine the stability is characterized by,

$$\boldsymbol{v} = (0,0,0),\ p = p(z),\ \rho = \rho(z) = \rho_0(1+\alpha\beta z),\ T = T_0 - \beta z,\ \boldsymbol{H} = (0,0,H),\ M = M_0(1+\gamma\beta z),\ \boldsymbol{M} = M(z) \tag{8}$$

Here, β may be either positive or negative.
The perturbed flow may be represented as,

$$\boldsymbol{v} = (0,0,0) + (u_1,u_2,u_3),\ \boldsymbol{h} = (0,0,H) + (h_x,h_y,h_z),\ T = T(z) + \theta,\ \rho = \rho(z) + \delta\rho,\ p = p(z) + \delta p,\ M = M(z) + \delta M. \tag{9}$$

where, $\boldsymbol{v}(u_1,u_2,u_3), \boldsymbol{h}(h_x,h_y,h_z), \theta, \delta\rho, \delta p$ and δM denote respectively the perturbations in fluid velocity, magnetic field, temperature, density, pressure and magnetization.
Linearizing the equations in perturbation and reading the perturbation into normal modes, we anticipate that the perturbation quantities are of the form,

$$(u_3,\theta,\zeta,\xi,h_z) = \left[W(z),\Theta(z),Z(z),X(z),K(z)\right].\exp\left\{i\left(k_x x + k_y y\right) + nt\right\}, \tag{10}$$

where, $k = \left(k_x{}^2 + k_y{}^2\right)^{1/2}$ is the resultant wave number of the disturbance and n is the frequency of any arbitrary disturbance (that is generally a complex constant).

We eliminate the physical quantities using the non-dimensional parameters $a = kd, \sigma = \frac{nd^2}{\nu}, p_1 = \frac{\nu}{\kappa_T}, p_2 = \frac{\nu}{\eta}$, $F = \frac{\mu'}{\rho_0 d^2 \nu}, P_1 = \frac{k_1}{d^2}, D^* = dD.$ Also dropping $(*)$ for convenience, we obtain,

$$\left(D^2-a^2\right)\left[\left(\frac{\sigma}{\varepsilon}+\frac{1}{P_1}\right)+F\left(D^2-a^2\right)^2-\left(D^2-a^2\right)\right]W+\frac{\lambda\alpha a^2 d^2}{\nu}\left(g_0-\frac{\gamma M_0\nabla H}{\rho_0\alpha\lambda}\right)\Theta-\frac{\mu_e H d}{4\pi\rho_0\nu}\left(D^2-a^2\right)DK = 0, \tag{11}$$

$$\left[\left(\frac{\sigma}{\varepsilon}+\frac{1}{P_1}\right)+F\left(D^2-a^2\right)^2-\left(D^2-a^2\right)\right]Z = \frac{\mu_e H d}{4\pi\rho_0\nu}DX, \tag{12}$$

$$\left(D^2-a^2-\sigma p_2\right)X = -\frac{Hd}{\varepsilon\eta}DZ - \frac{H}{4\pi N e\eta d}\left(D^2-a^2\right)DK, \tag{13}$$

$$\left(D^2-a^2-\sigma p_2\right)K = -\frac{Hd}{\varepsilon\eta}DW + \frac{Hd}{4\pi N e\eta}DX, \tag{14}$$

$$\left(D^2-a^2-E\sigma p_1\right)\Theta = -\frac{\beta d^2}{\kappa}\mathrm{W}. \tag{15}$$

Now, eliminating X, Θ, K, Z among Eqs. (11) - (15), we obtain the stability governing equation,

$$\lambda R_f a^2 W = \left(D^2-a^2\right)\left\{\left(\frac{\sigma}{\varepsilon}+\frac{1}{P_1}\right)+F\left(D^2-a^2\right)^2-\left(D^2-a^2\right)\right\}\left(D^2-a^2-E\sigma p_1\right)W$$
$$+\frac{\frac{Q}{\varepsilon}D^2\left(D^2-a^2\right)\left[\frac{Q}{\varepsilon}D^2+\left(D^2-a^2-\sigma p_2\right)\left\{\left(\frac{\sigma}{\varepsilon}+\frac{1}{P_1}\right)+F\left(D^2-a^2\right)^2-\left(D^2-a^2\right)\right\}\right]\left(D^2-a^2-E\sigma p_1\right)W}{\left(D^2-a^2-\sigma p_2\right)\left[\frac{Q}{\varepsilon}D^2+\left(D^2-a^2-\sigma p_2\right)\left\{\left(\frac{\sigma}{\varepsilon}+\frac{1}{P_1}\right)+F\left(D^2-a^2\right)^2-\left(D^2-a^2\right)\right\}\right]+M_h D^2\left(D^2-a^2\right)\left\{\left(\frac{\sigma}{\varepsilon}+\frac{1}{P_1}\right)+F\left(D^2-a^2\right)^2-\left(D^2-a^2\right)\right\}} \tag{16}$$

where, $R_f = \frac{\alpha\beta d^4}{\nu\kappa}\left(g_0 - \frac{\gamma M_0 \nabla H}{\rho_0 \alpha\lambda}\right)$ is the Rayleigh number for ferromagnetic fluid, $Q = \frac{\mu_e H^2 d^2}{4\pi\rho_0 \nu\eta}$ is the Chandrasekhar number and $M_h = \left(\frac{H}{4\pi Ne\eta}\right)^2$ is the Hall current parameter.

Now, the appropriate boundary condition (when we take the case of two free boundaries) are,

$$W = 0, D^2W = 0, D^4W = 0, \Theta = 0, Z = 0, X = 0, DZ = 0, DX = 0, DK = 0 \text{ at } z = 0 \text{ and } z = 1. \quad (17)$$

Here, it is obvious that all even order derivatives of W at the boundaries vanish. Therefore, the solution of Eq. (16) characterizing the lowest mode is,

$$W = W_0 \sin \pi z, \text{ where } W_0 \text{ is a constant.} \quad (18)$$

Now, using Eq. (18) in Eq. (16), we get,

$$R_1 = \frac{(1+x)}{\lambda x}\left\{\left(\frac{i\sigma_1}{\varepsilon} + \frac{1}{P}\right) + F_1(1+x)^2 + (1+x)\right\}(1 + x + Ei\sigma_1 p_1)$$

$$+ \frac{Q_1}{\lambda x\varepsilon} \frac{(1+x)\left[\frac{Q_1}{\varepsilon} + (1 + x + i\sigma_1 p_2)\left\{\left(\frac{i\sigma_1}{\varepsilon} + \frac{1}{P}\right) + F_1(1+x)^2 + (1+x)\right\}\right](1 + x + Ei\sigma_1 p_1)}{(1 + x + i\sigma_1 p_2)\left[\frac{Q_1}{\varepsilon} + (1 + x + i\sigma_1 p_2)\left\{\left(\frac{i\sigma_1}{\varepsilon} + \frac{1}{P}\right) + F_1(1+x)^2 + (1+x)\right\}\right] + M_h(1+x)\left\{\left(\frac{i\sigma_1}{\varepsilon} + \frac{1}{P}\right) + F_1(1+x)^2 + (1+x)\right\}} \quad (19)$$

where, $x = \frac{a^2}{\pi^2}, i\sigma_1 = \frac{\sigma}{\pi^2}, F_1 = \pi^2 F, R_1 = \frac{R_f}{\pi^4}, Q_1 = \frac{Q}{\pi^2}, P = \pi^2 P_1.$

4. Analytical Discussion

- **Stationary Convection**

When stability sets in at stationary convection, the marginal state will be characterized by $\sigma_1 = 0$. So, put $\sigma_1 = 0$ in Eq. (19), we get,

$$R_1 = \frac{(1+x)^2}{\lambda x}\left\{\frac{1}{P} + F_1(1+x)^2 + (1+x)\right\} + \frac{Q_1}{\lambda x\varepsilon} \frac{(1+x)\left\{\frac{Q_1}{\varepsilon} + \frac{(1+x)}{P} + F_1(1+x)^3 + (1+x)^2\right\}}{\left\{\frac{Q_1}{\varepsilon} + \frac{(1+x)}{P} + F_1(1+x)^3 + (1+x)^2\right\} + M_h\left\{\frac{(1+x)}{P} + F_1(1+x)^3 + (1+x)^2\right\}}. \quad (20)$$

Now, to study the effect of medium permeability parameter, couple-stress parameter, magnetic field parameter and Hall current parameter, we examine the behavior of $\frac{dR_1}{dP}, \frac{dR_1}{dF_1}, \frac{dR_1}{dQ_1}$ and $\frac{dR_1}{dM_h}$.

So, by Eq. (20), we have,

$$\frac{dR_1}{dP} = \frac{(1+x)^2}{\lambda x P^2}\left[\frac{M_h Q_1^2}{\varepsilon^2\left[\frac{Q_1}{\varepsilon} + (1+M_h)\left\{\frac{(1+x)}{P} + F_1(1+x)^3 + (1+x)^2\right\}\right]^2} - 1\right], \quad (21)$$

which indicates that medium permeability has a stabilizing effect on the system under the condition,

$$\lambda > 0, M_h Q_1^2 > \varepsilon^2\left[\frac{Q_1}{\varepsilon} + (1+M_h)\left\{\frac{(1+x)}{P} + F_1(1+x)^3 + (1+x)^2\right\}\right]^2$$

and $\lambda < 0, M_h Q_1^2 < \varepsilon^2 \left[\frac{Q_1}{\varepsilon} + (1+M_h)\left\{ \frac{(1+x)}{P} + F_1 (1+x)^3 + (1+x)^2 \right\} \right]^2$.

Also, medium permeability has a destabilizing effect on the system under the condition,

$$\lambda > 0, M_h Q_1^2 < \varepsilon^2 \left[\frac{Q_1}{\varepsilon} + (1+M_h)\left\{ \frac{(1+x)}{P} + F_1 (1+x)^3 + (1+x)^2 \right\} \right]^2$$

and $\lambda < 0, M_h Q_1^2 > \varepsilon^2 \left[\frac{Q_1}{\varepsilon} + (1+M_h)\left\{ \frac{(1+x)}{P} + F_1 (1+x)^3 + (1+x)^2 \right\} \right]^2$.

In the absence of magnetic field, Eq. (21) becomes,

$$\frac{dR_1}{dP} = -\frac{(1+x)^2}{\lambda x P^2},$$

which indicates that medium permeability has a stabilizing effect for $\lambda < 0$ and destabilizing effect for $\lambda > 0$.

$$\frac{dR_1}{dF_1} = \frac{(1+x)^4}{\lambda x} \left[1 - \frac{M_h Q_1^2}{\varepsilon^2 \left[\frac{Q_1}{\varepsilon} + (1+M_h)\left\{ \frac{(1+x)}{P} + F_1 (1+x)^3 + (1+x)^2 \right\} \right]^2} \right], \tag{22}$$

which indicates that couple-stress has a stabilizing effect on the system under the condition,

$$\lambda > 0, M_h Q_1^2 < \varepsilon^2 \left[\frac{Q_1}{\varepsilon} + (1+M_h)\left\{ \frac{(1+x)}{P} + F_1 (1+x)^3 + (1+x)^2 \right\} \right]^2$$

and $\lambda < 0, M_h Q_1^2 > \varepsilon^2 \left[\frac{Q_1}{\varepsilon} + (1+M_h)\left\{ \frac{(1+x)}{P} + F_1 (1+x)^3 + (1+x)^2 \right\} \right]^2$.

Also, couple-stress has a destabilizing effect on the system under the condition,

$$\lambda > 0, M_h Q_1^2 > \varepsilon^2 \left[\frac{Q_1}{\varepsilon} + (1+M_h)\left\{ \frac{(1+x)}{P} + F_1 (1+x)^3 + (1+x)^2 \right\} \right]^2$$

and $\lambda < 0, M_h Q_1^2 < \varepsilon^2 \left[\frac{Q_1}{\varepsilon} + (1+M_h)\left\{ \frac{(1+x)}{P} + F_1 (1+x)^3 + (1+x)^2 \right\} \right]^2$.

In the absence of magnetic field or hall current, Eq. (22) becomes,

$$\frac{dR_1}{dF_1} = \frac{(1+x)^4}{\lambda x},$$

which clearly indicates that couple-stress has a stabilizing effect for $\lambda > 0$ and destabilizing effect for $\lambda < 0$.

$$\frac{dR_1}{dQ_1}=\frac{(1+x)}{\lambda\varepsilon x}\left[\frac{\frac{M_h Q_1}{\varepsilon}\left\{\frac{(1+x)}{P}+F_1(1+x)^3+(1+x)^2\right\}+\left\{\frac{Q_1}{\varepsilon}+\frac{(1+x)}{P}+F_1(1+x)^3+(1+x)^2\right\}.\left[\frac{Q_1}{\varepsilon}+(1+M_h)\left\{\frac{(1+x)}{P}+F_1(1+x)^3+(1+x)^2\right\}\right]}{\left[\frac{Q_1}{\varepsilon}+(1+M_h)\left\{\frac{(1+x)}{P}+F_1(1+x)^3+(1+x)^2\right\}\right]^2}\right],$$

which clearly indicates that magnetic field has a stabilizing effect on the system for $\lambda>0$ and destabilizing effect for $\lambda<0$.

$$\frac{dR_1}{dM_h}=-\frac{Q_1(1+x)}{\lambda\varepsilon x}\left\{\frac{Q_1}{\varepsilon}+\frac{(1+x)}{P}+F_1(1+x)^3+(1+x)^2\right\}.\frac{\left\{\frac{(1+x)}{P}+F_1(1+x)^3+(1+x)^2\right\}}{\left[\frac{Q_1}{\varepsilon}+(1+M_h)\left\{\frac{(1+x)}{P}+F_1(1+x)^3+(1+x)^2\right\}\right]^2},$$

which clearly indicates that Hall current has a stabilizing effect on the system for $\lambda<0$ and destabilizing effect for $\lambda>0$.

To see the effect of magnetization, we examine the effect of $\frac{dR}{dM_0}$ analytically.

$$\frac{dR}{dM_0}=\left[\frac{\pi^4(1+x)^2}{\lambda x}\left\{\frac{1}{P}+F_1(1+x)^2+(1+x)\right\}+\frac{Q_1\pi^4}{\lambda x\varepsilon}\frac{(1+x)\left\{\frac{Q_1}{\varepsilon}+\frac{(1+x)}{P}+F_1(1+x)^3+(1+x)^2\right\}}{\left\{\frac{Q_1}{\varepsilon}+\frac{(1+x)}{P}+F_1(1+x)^3+(1+x)^2\right\}+M_h\left\{\frac{(1+x)}{P}+F_1(1+x)^3+(1+x)^2\right\}}\right].$$

$$\left(1-\frac{\gamma M_0\nabla H}{\rho_0\alpha g_0\lambda}\right)^{-2}.\left(\frac{\gamma\nabla H}{\rho_0\alpha g_0\lambda}\right),$$

which clearly indicates that magnetization has a stabilizing effect on the system for both $\lambda>0$ and $\lambda<0$.

- **Oscillatory convection**

Multiplying Eq. (11) by the conjugate of W i.e. W^* and integrate over the range of z and making use of Eqs. (12) - (15) together with boundary conditions (17), we get,

$$\left(\frac{\sigma}{\varepsilon}+\frac{1}{P_1}\right)I_1+I_2+FI_3+d^2\left\{\left(\frac{\sigma^*}{\varepsilon}+\frac{1}{P_1}\right)I_4+I_5+FI_6+\frac{\mu_e\varepsilon\eta}{4\pi\rho_0\nu}(I_7+p_2\sigma I_8)\right\}+\frac{\mu_e\varepsilon\eta}{4\pi\rho_0\nu}\left(I_9+p_2\sigma^* I_{10}\right)$$

$$-\frac{\lambda\alpha a^2\kappa_T}{\beta\nu}\left(g_0-\frac{\gamma M_0\nabla H}{\rho_0\alpha\lambda}\right)\left(I_{11}+Ep_1\sigma^* I_{12}\right)=0, \quad (23)$$

where, $I_1=\int\left(|DW|^2+a^2|W|^2\right)dz, I_2=\int\left(\left|D^2W\right|^2+2a^2|DW|^2+a^4|W|^2\right)dz,$

$$I_3=\int\left(\left|D^3W\right|^2+3a^2\left|D^2W\right|^2+3a^4|DW|^2+a^6|W|^2\right)dz, I_4=\int\left(|Z|^2\right)dz,$$

$$I_5=\int\left(|DZ|^2+a^2|Z|^2\right)dz, I_6=\int\left(\left|D^2Z\right|^2+2a^2|DZ|^2+a^4|Z|^2\right)dz,$$

$$I_7=\int\left(|DX|^2+a^2|X|^2\right)dz, I_8=\int|X|^2dz,$$

$$I_9=\int\left(\left|D^2K\right|^2+2a^2|DK|^2+a^4|K|^2\right)dz, I_{10}=\int\left(|DK|^2+a^2|K|^2\right)dz,$$

$$I_{11}=\int\left(|D\Theta|^2+a^2|\Theta|^2\right)dz, I_{12}=\int\left(|\Theta|^2\right)dz.$$

All these integrals from I_1 to I_{12} are positive definite.

Now, putting $\sigma = i\sigma_i$ (where σ_i is real) in Eq. (23) and equating the imaginary part in the absence of magnetic field (hence hall current), we get,

$$\sigma_i \left\{ \frac{1}{\varepsilon} I_1 + \frac{\mu_e \varepsilon \eta d^2}{4\pi\rho_0 \nu} p_2 I_8 + \left(\frac{\lambda \alpha a^2 \kappa_T}{\beta \nu} \right) \left(g_0 - \frac{\gamma M_0 \nabla H}{\rho_0 \alpha \lambda} \right) E p_1 I_{12} \right\} = 0. \tag{24}$$

If $\lambda g_0 \geq \dfrac{\gamma M_0 \nabla H}{\rho_0 \alpha}$, then the terms in bracket are positive definite, which implies that $\sigma_i = 0$. Therefore, oscillatory modes are not allowed and the principle of exchange of stabilities is satisfied if $\lambda g_0 \geq \dfrac{\gamma M_0 \nabla H}{\rho_0 \alpha}$.

5. Numerical Discussion

The dispersion relation (20) is analyzed numerically also. The numerical value of thermal Rayleigh number R_1 is decided for numerous values of medium permeability parameter P, couple-stress parameter F_1, magnetic field parameter Q_1, Hall current parameter M_h and magnetization parameter M_0. Also, graphs have been plotted between R_1 and these parameters as shown in Figures (2) - (15),

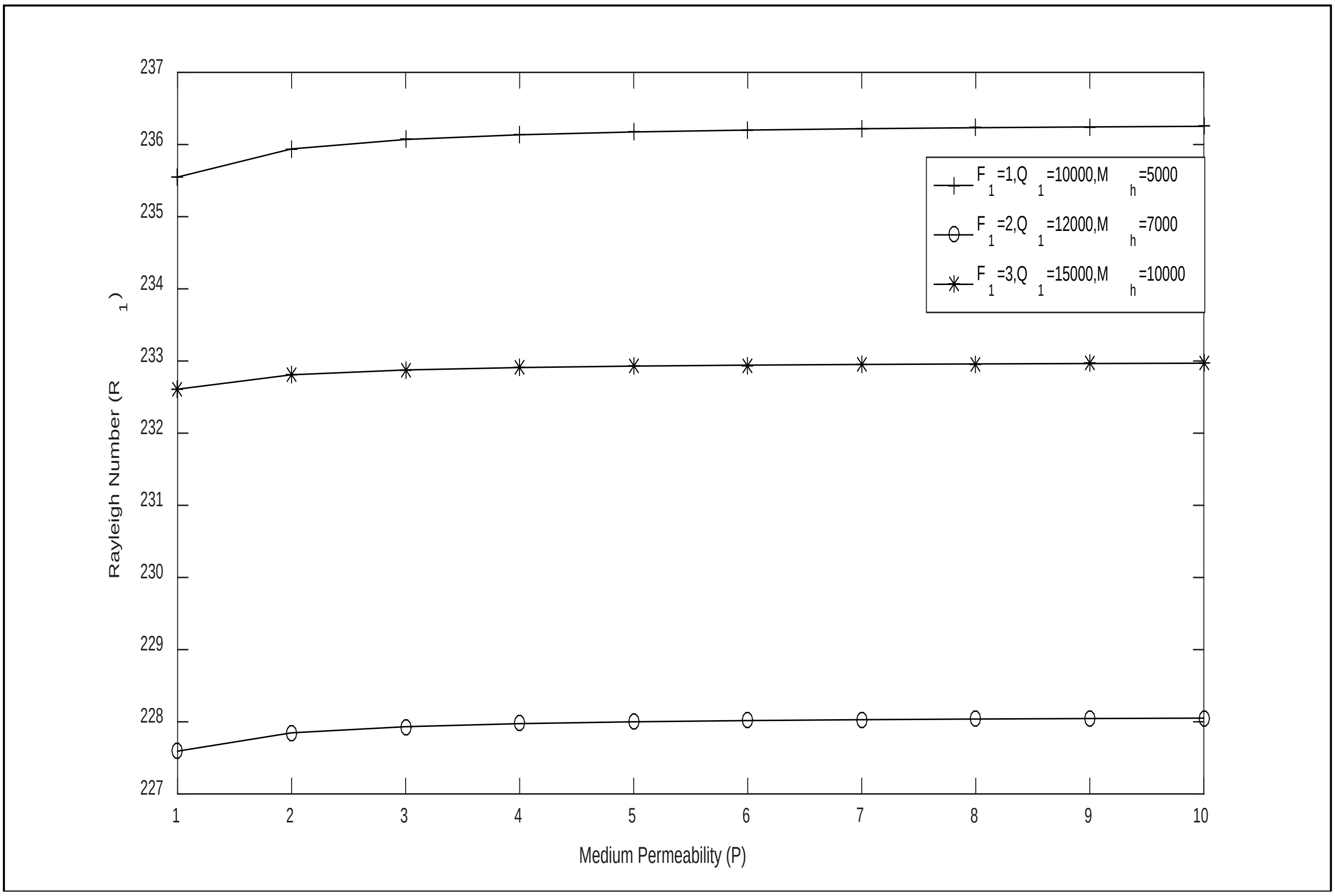

Fig 2: Variation of R_1 with P for $\lambda > 0$.

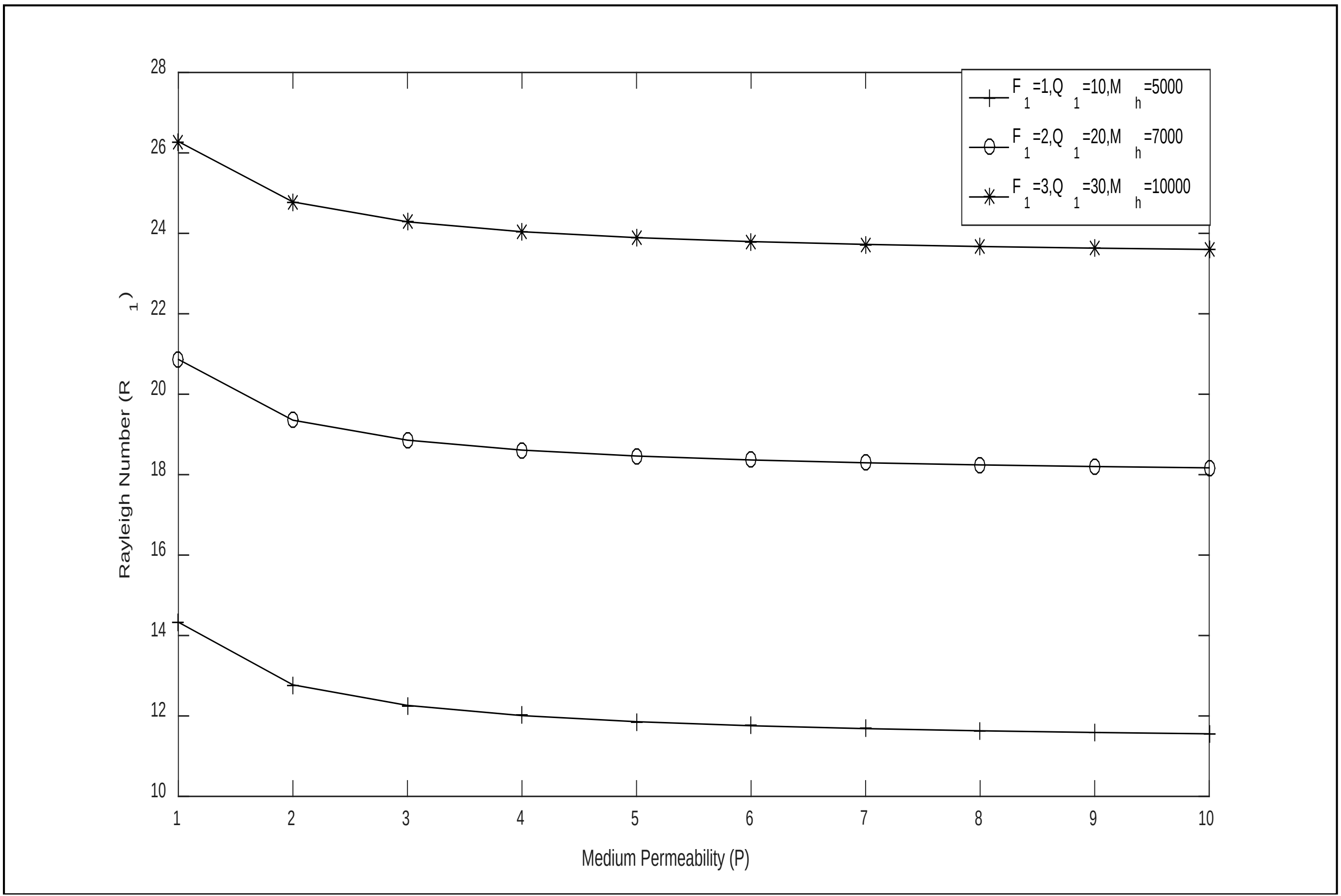

Fig 3: Variation of R_1 with P for $\lambda > 0$.

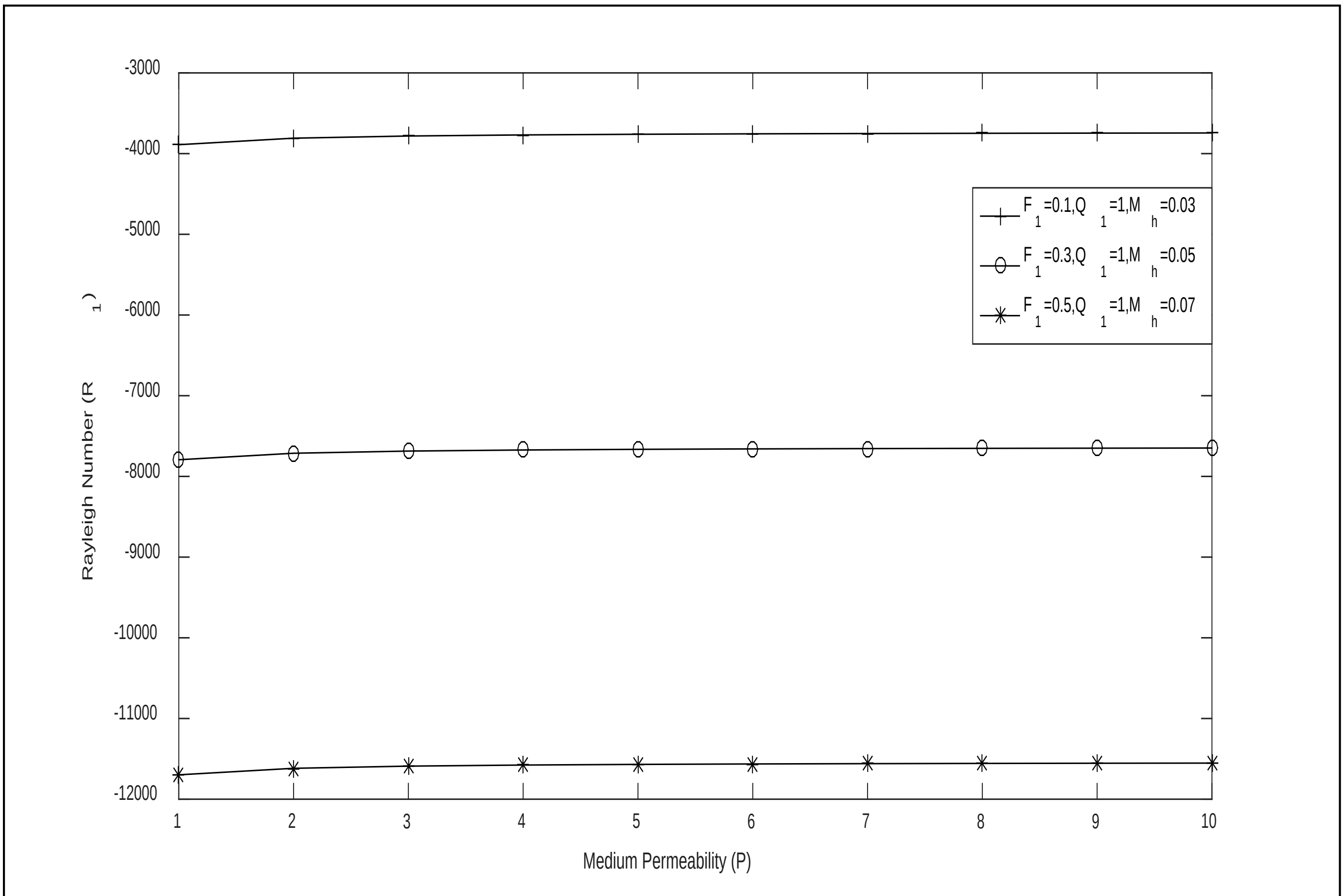

Fig 4: Variation of R_1 with P for $\lambda < 0$.

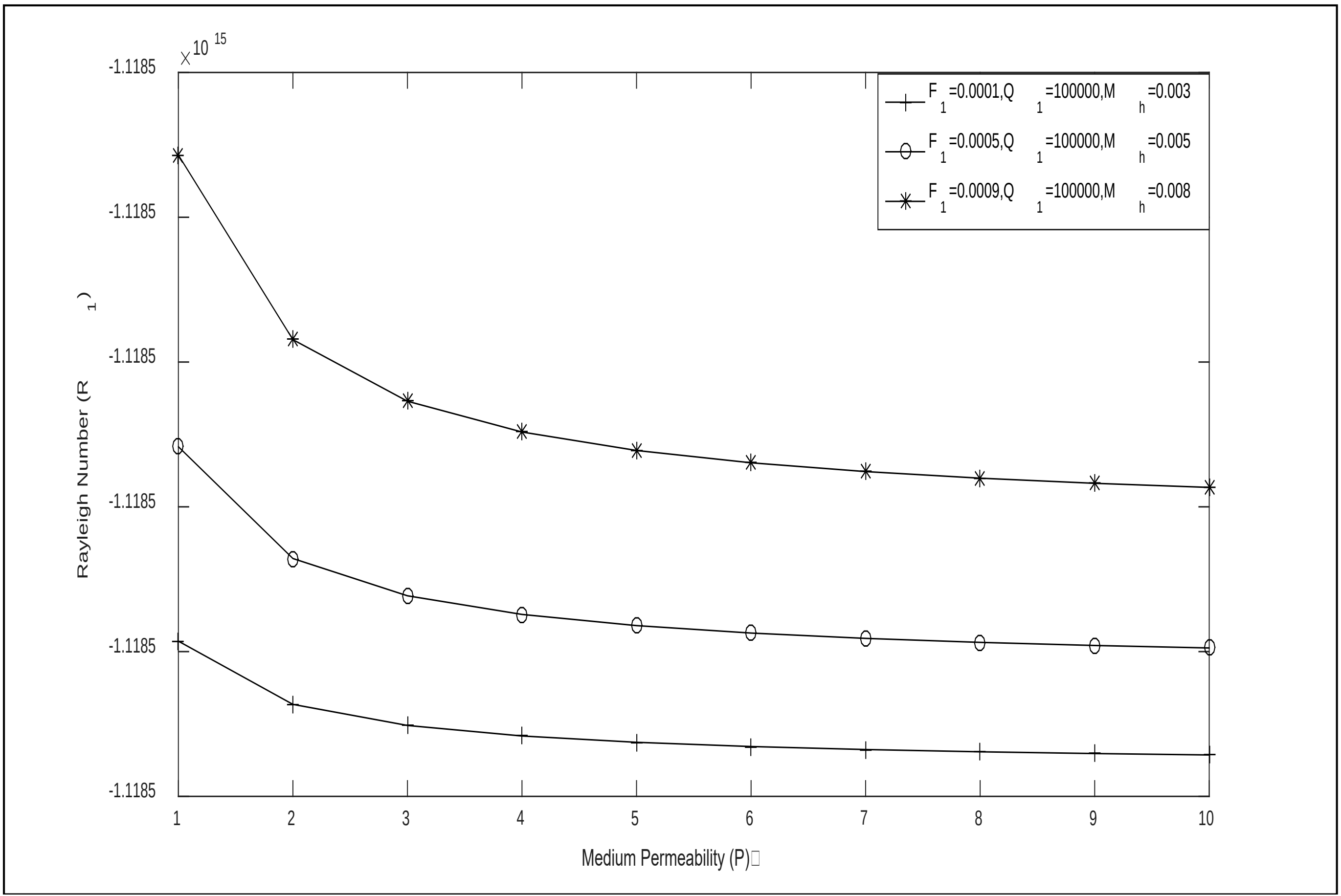

Fig 5: Variation of R_1 with P for $\lambda < 0$.

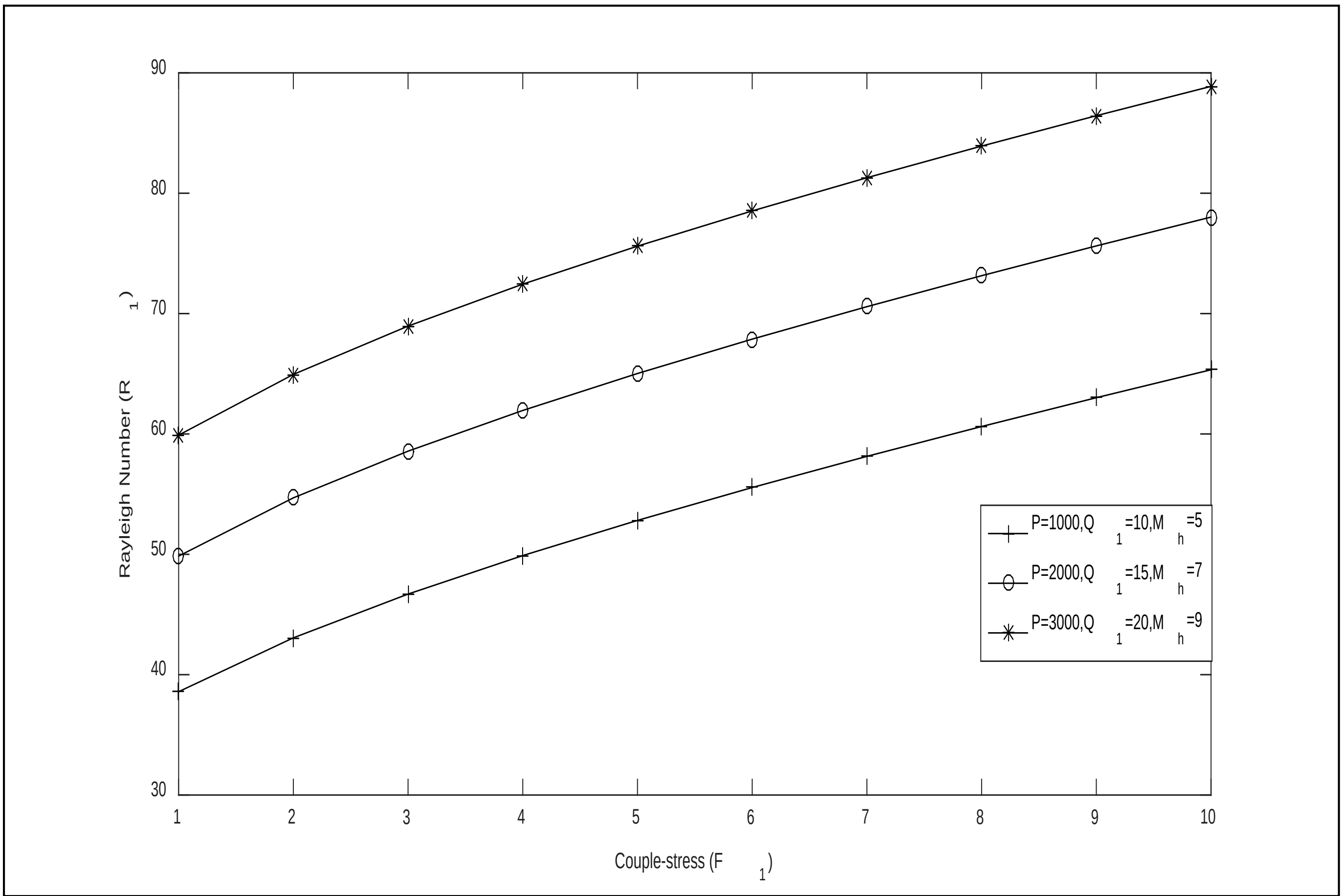

Fig 6: Variation of R_1 with F_1 for $\lambda > 0$.

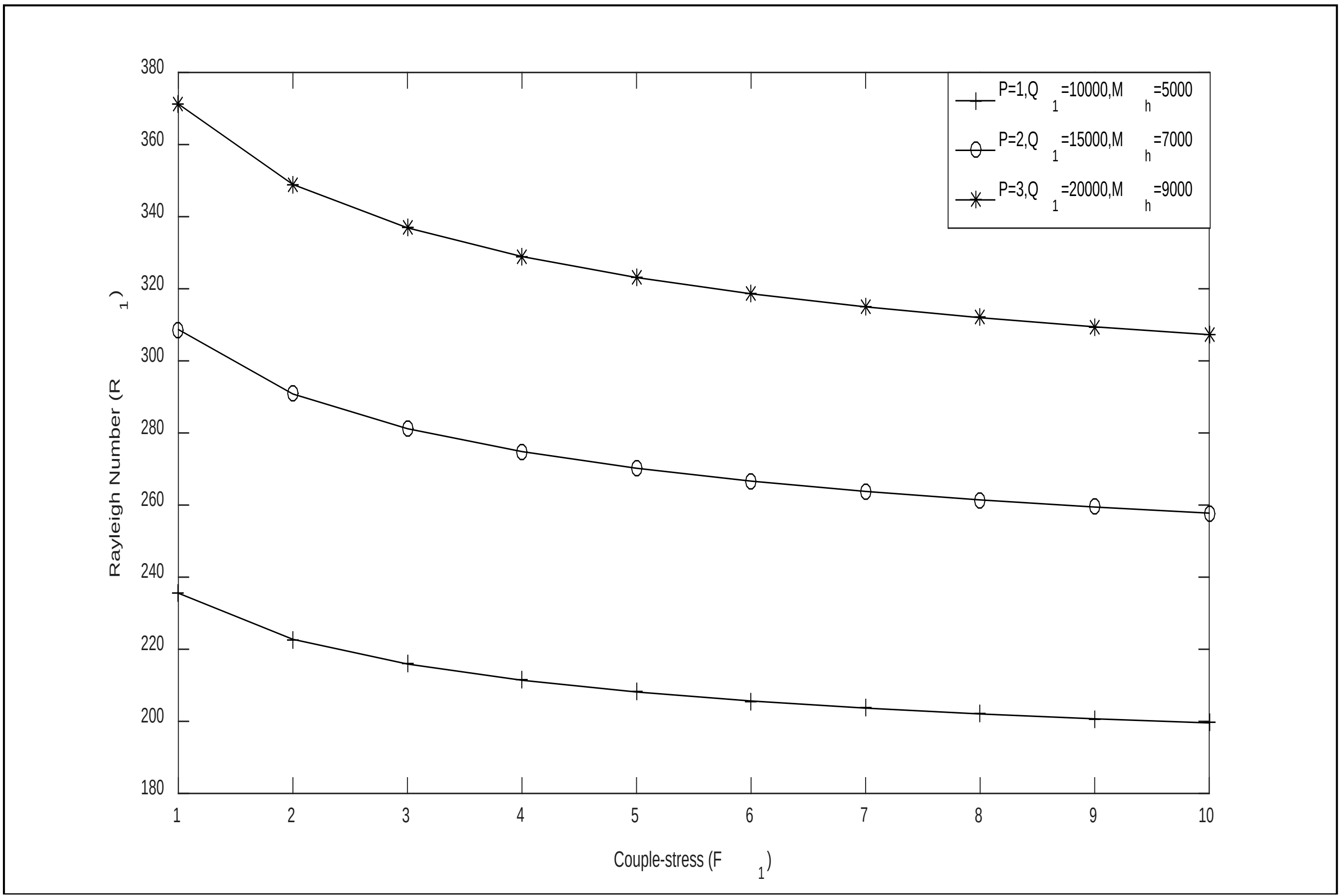

Fig 7: Variation of R_1 with F_1 for $\lambda > 0$.

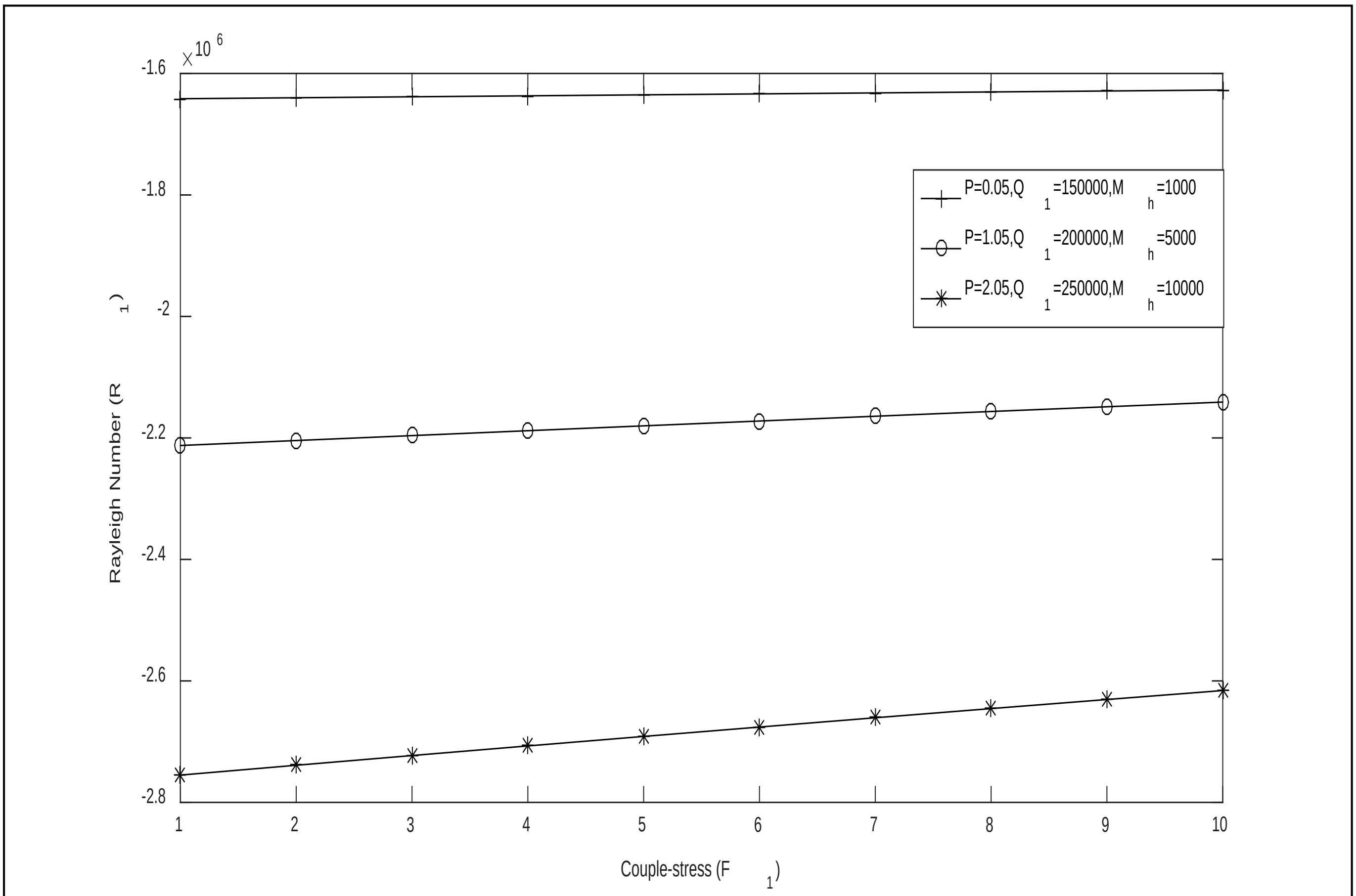

Fig 8: Variation of R_1 with F_1 for $\lambda < 0$.

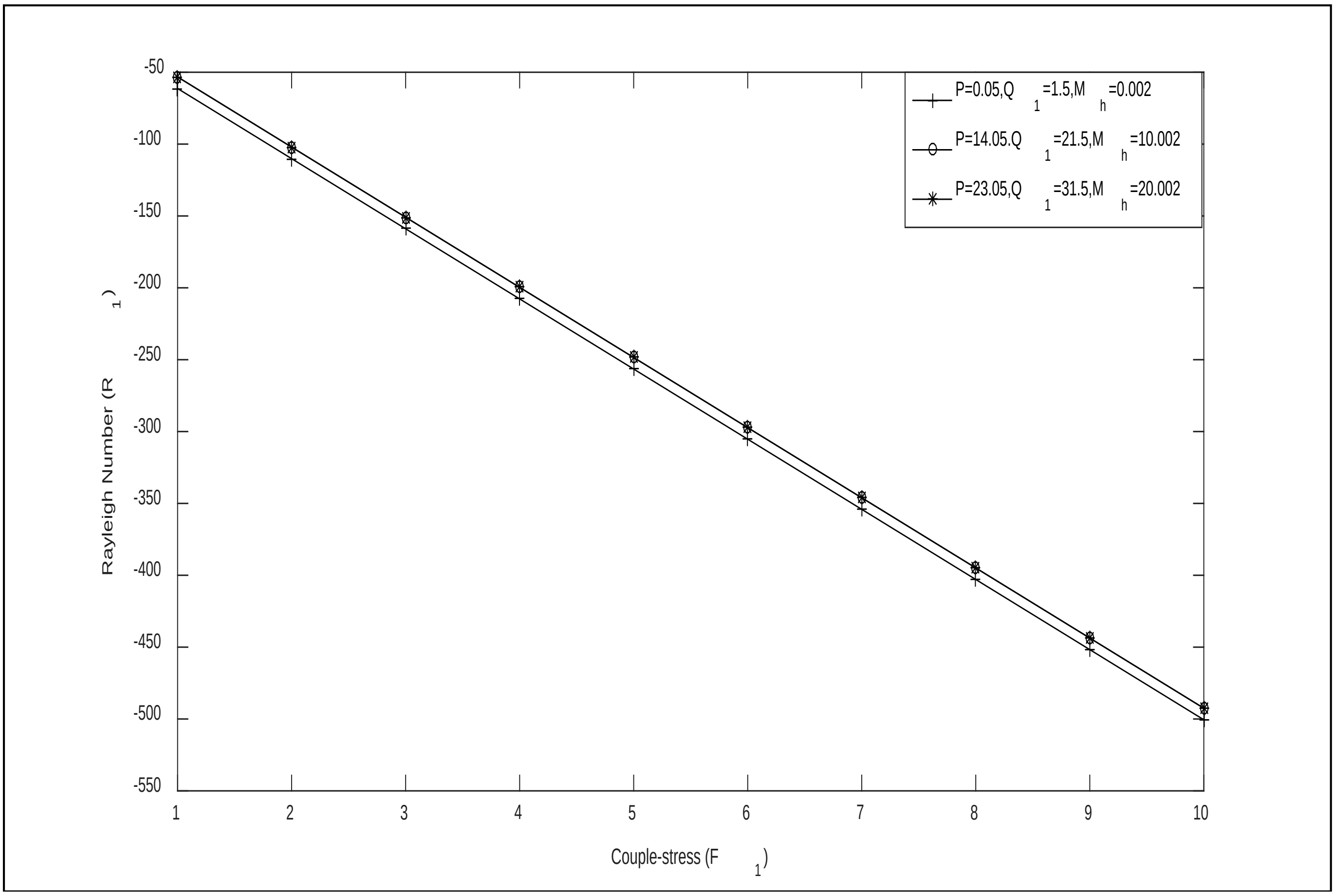

Fig 9: Variation of R_1 with F_1 for $\lambda < 0$.

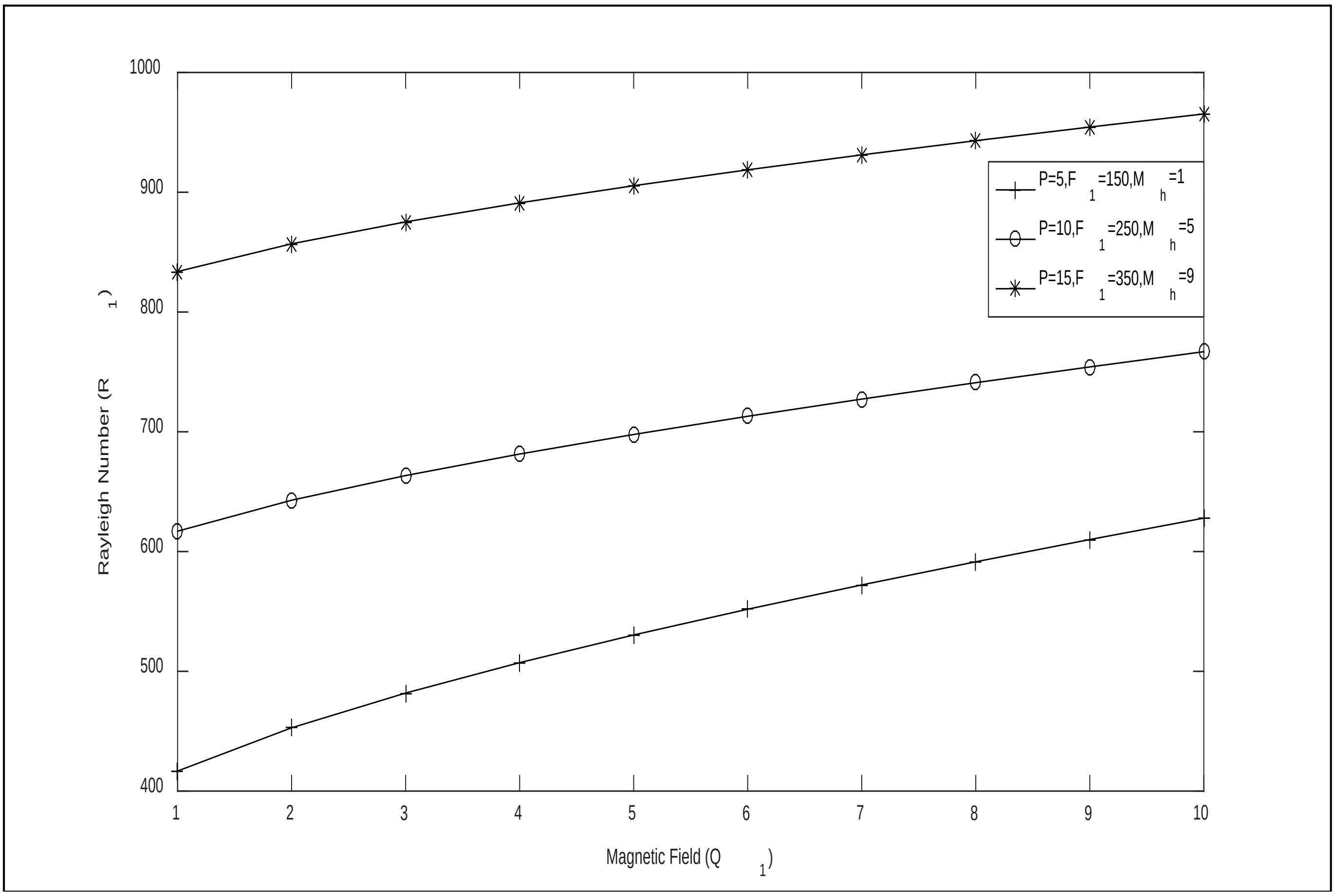

Fig 10: Variation of R_1 with Q_1 for $\lambda > 0$.

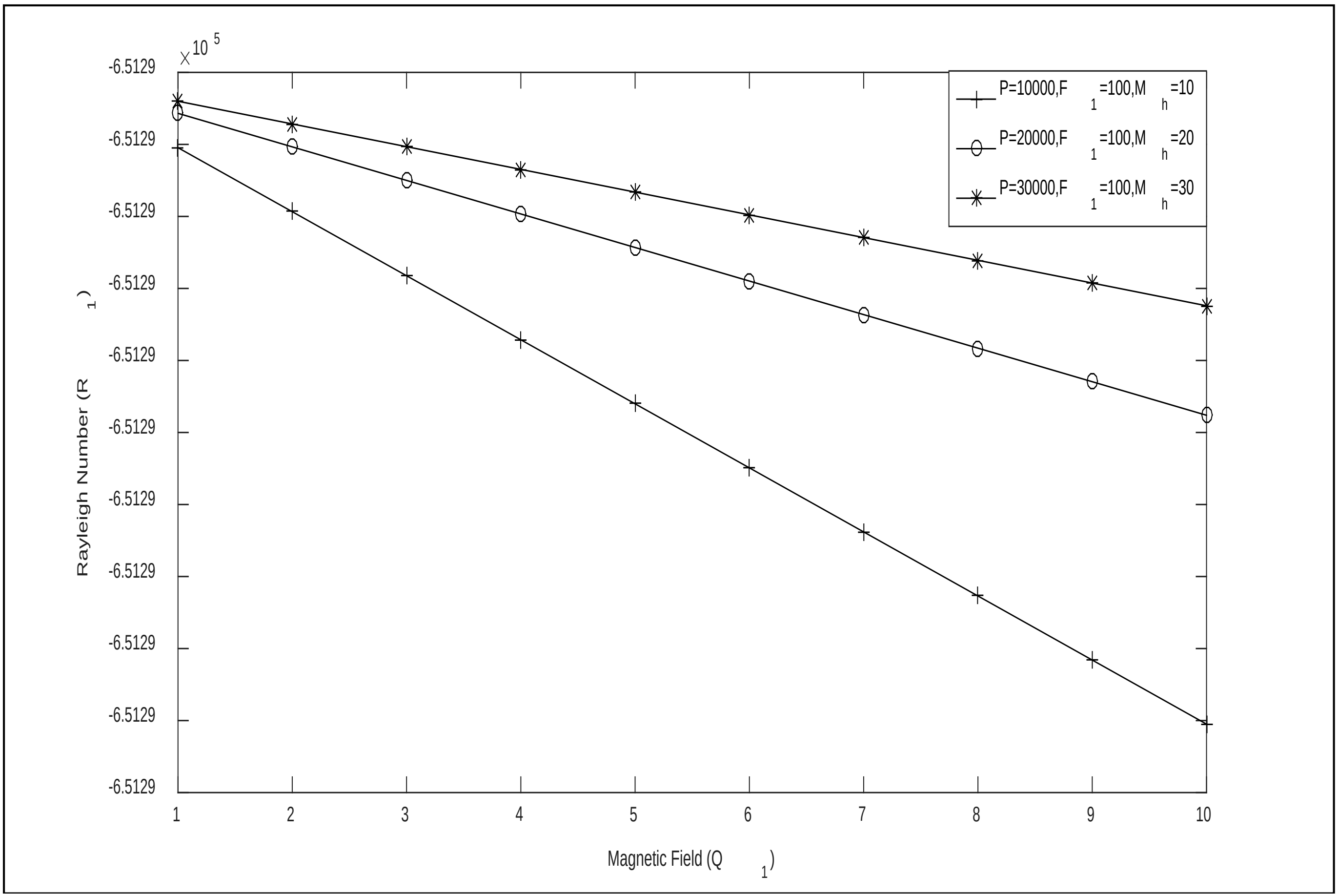

Fig 11: Variation of R_1 with Q_1 for $\lambda < 0$.

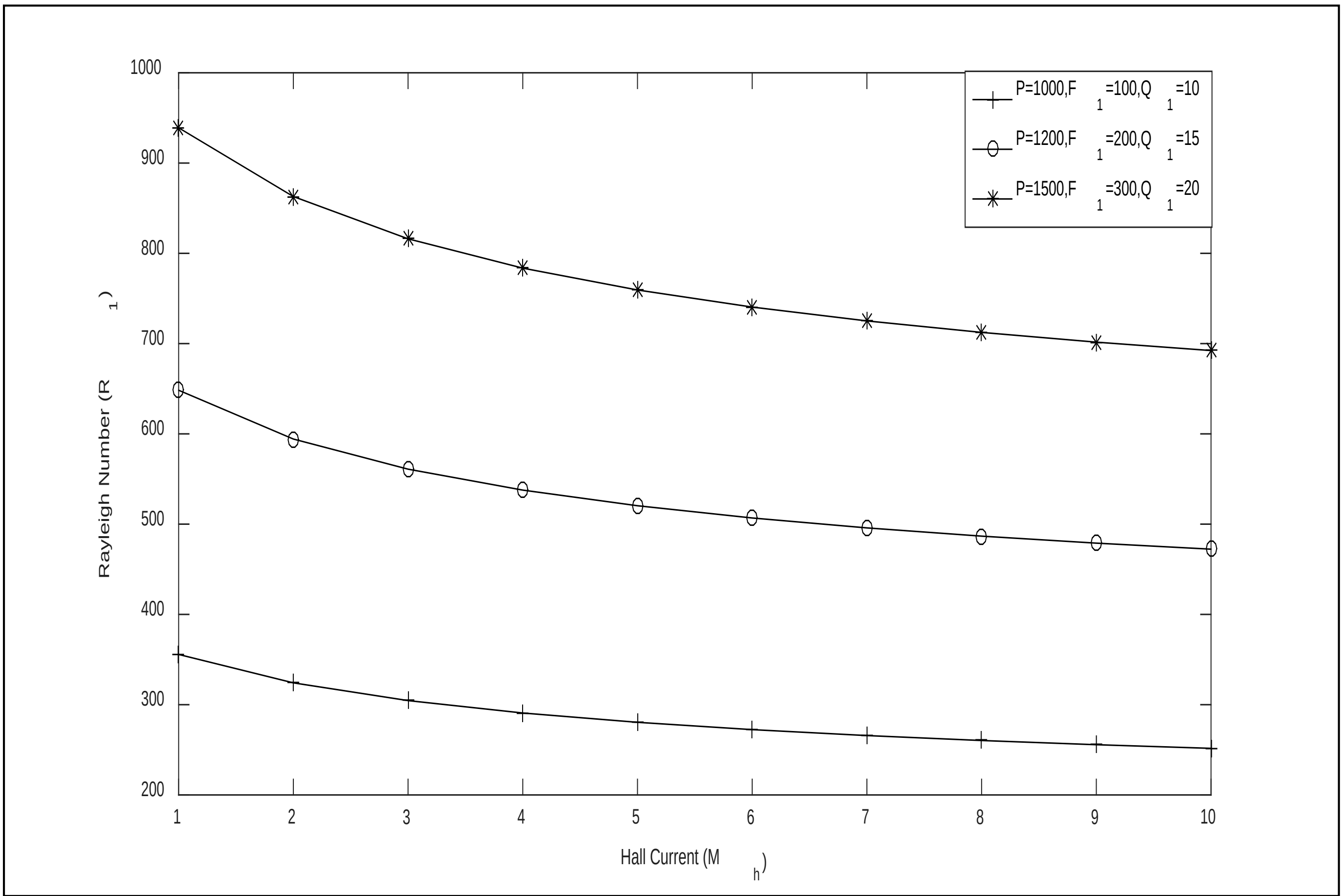

Fig 12: Variation of R_1 with M_h for $\lambda > 0$.

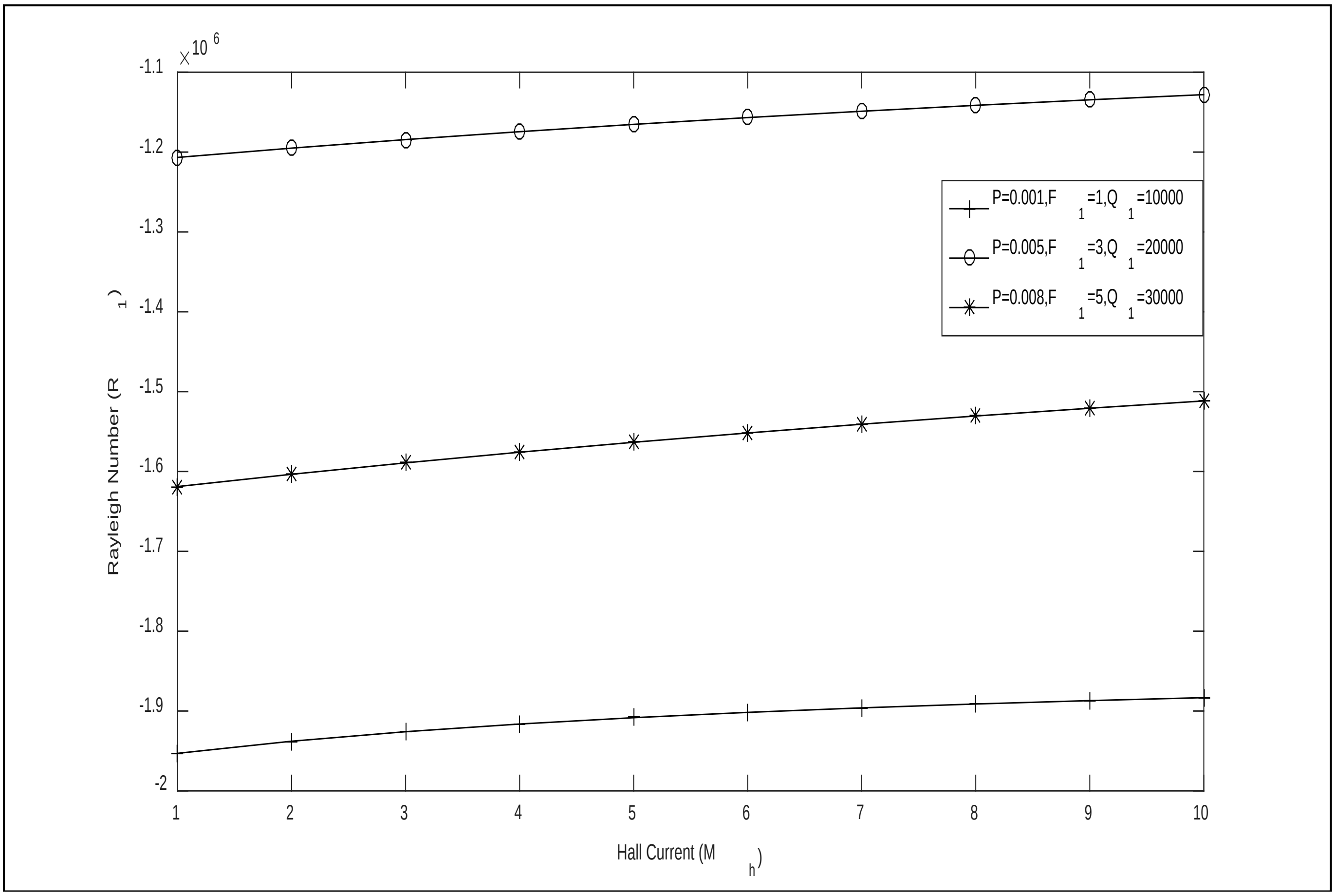

Fig 13: Variation of R_1 with M_h for $\lambda < 0$.

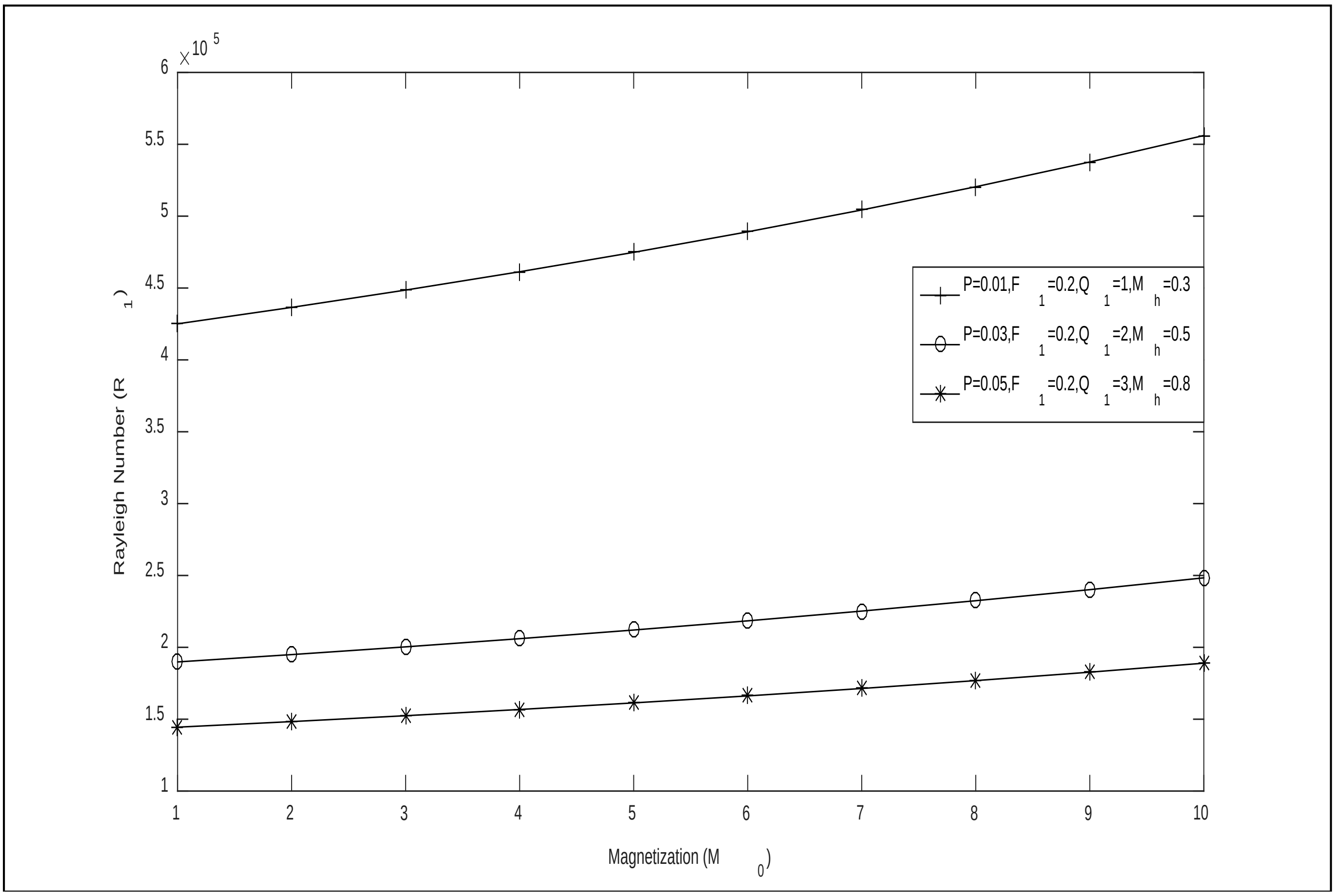

Fig 14: Variation of R_1 with M_0 for $\lambda > 0$.

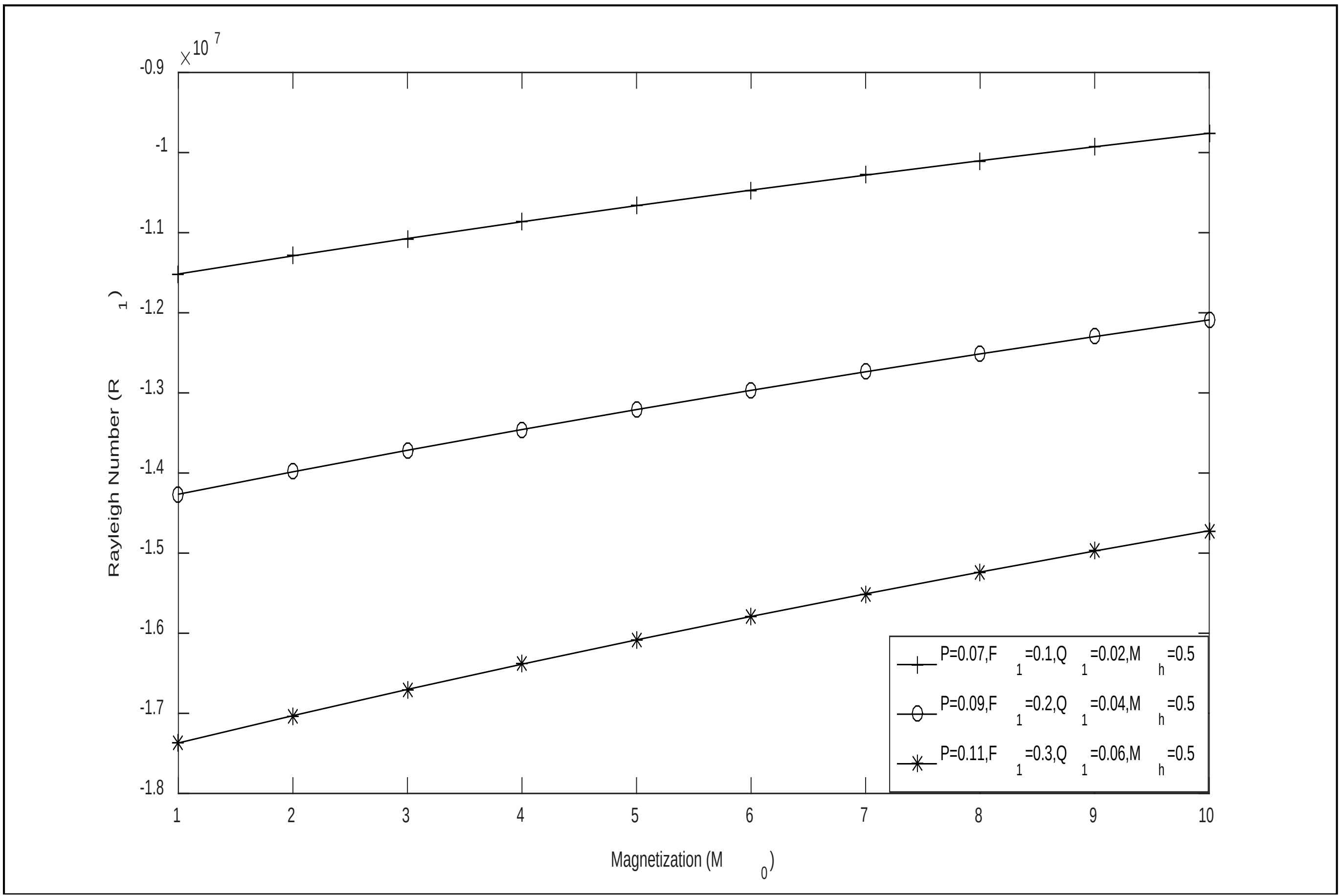

Fig 15: Variation of R_1 with M_0 for $\lambda < 0$.

In fig. 2, critical Rayleigh number R_1 increases with increase in medium permeability parameter P for $\lambda = 50$, which indicates that medium permeability has a stabilizing effect on the system. In fig. 3, critical Rayleigh number R_1 decreases with increase in medium permeability parameter P for $\lambda = 2$, which indicates that medium permeability has a destabilizing effect on the system. In fig. 4, critical Rayleigh number R_1 increases with increase in medium permeability parameter P for $\lambda = -5$, which indicates that medium permeability has a stabilizing effect on the system. In fig. 5, critical Rayleigh number R_1 decreases with increase in medium permeability parameter P for $\lambda = -0.00001$, which indicates that medium permeability has a destabilizing effect on the system.

In fig. 6, critical Rayleigh number R_1 increases with increase in couple-stress parameter F_1 for $\lambda = 5$, which indicates that couple-stress has a stabilizing effect on the system. In fig. 7, critical Rayleigh number R_1 decreases with increase in couple-stress parameter F_1 for $\lambda = 50$, which indicates that couple-stress has a destabilizing effect on the system. In fig. 8, critical Rayleigh number R_1 increases with increase in couple-stress parameter F_1 for $\lambda = -10000$, which indicates that couple-stress has a stabilizing effect on the system. In fig. 9, critical Rayleigh number R_1 decreases with increase in couple-stress parameter F_1 for $\lambda = -2000$, which indicates that couple-stress has a destabilizing effect on the system.

In fig. 10, critical Rayleigh number R_1 increases with increase in magnetic field parameter Q_1 for $\lambda = 3$, which indicates that magnetic field has a stabilizing effect on the system. In fig. 11, critical Rayleigh number R_1 decreases with increase in magnetic field parameter Q_1 for $\lambda = -15$, which indicates that magnetic field has a destabilizing effect on the system.

In fig. 12, critical Rayleigh number R_1 decreases with increase in hall current parameter M_h for $\lambda = 4$, which indicates that magnetic field has a destabilizing effect on the system. In fig. 13, critical Rayleigh number R_1 increases with increase in hall current parameter M_h for $\lambda = -0.5$, which indicates that magnetic field has a stabilizing effect on the system.

In fig. 14, critical Rayleigh number R_1 increases with increase in magnetization parameter M_0 for $\lambda = 0.2$, which indicates that magnetic field has a stabilizing effect on the system. In fig. 15, critical Rayleigh number R_1 increases with increase in magnetization parameter M_0 for $\lambda = -0.25$, which indicates that magnetic field has a stabilizing effect on the system.

6. Conclusions

The main results from the evaluation of the present paper are as below:

- For stationary convection,

1. Medium permeability has both stabilizing and destabilizing effect on the system for $\lambda > 0$ and $\lambda < 0$ under certain conditions. Furthermore, in the absence of magnetic field, medium permeability has a stabilizing effect on the system for $\lambda < 0$ and destabilizing effect for $\lambda > 0$.
2. Couple-stress has both stabilizing and destabilizing effect on the system for $\lambda > 0$ and $\lambda < 0$ under certain conditions. Furthermore, in the absence of magnetic field, couple-stress has a stabilizing effect on the system for $\lambda > 0$ and destabilizing effect for $\lambda < 0$.
3. Magnetic field has a stabilizing effect on the system for $\lambda > 0$ and destabilizing effect for $\lambda < 0$.
4. Hall current has a stabilizing effect on the system for $\lambda < 0$ and destabilizing effect for $\lambda > 0$.
5. Magnetization has a stabilizing effect on the system for both $\lambda > 0$ and $\lambda < 0$.

- The principle of exchange of stabilities is not valid for the present problem under consideration, whereas in the absence of magnetic field (hence hall current), it is valid for the present problem if $\lambda g_0 \geq \frac{\gamma M_0 \nabla H}{\rho_0 \alpha}$.

7. Acknowledgment

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8. References

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