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Analysis of the variational interaction method for solving nonlinear singular boundary value problems

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Abstract
The main objective in this paper, the variational interaction method (VIM) is used to study a nonlinear singular boundary value problems arising in various physical equations. The VIM overcomes the singularity issue at the origin x–u. The variation interaction method is tested for its efficiency.

Keywords: variational interaction, various physical, VIM, variation interaction, nonlinear singular, problems arising

Introduction
A lot of attention has been devoted to the study of VIM [1-3] since 1933 to investigate various models, singular and nonsingular, linear and nonlinear, and ODEs and PDEs as well. The variational iteration method was first used by Schunk [1] in 1933 to calculate the bending of cylindrical panels. However, his work passed unnoticed, and the method was rediscovered in the sixties by Zhukov [2], who used it in the calculation of thin rectangular slabs. In 1981, Kirichenko and Krys'ko substantiate the VIM method for a class of equations described by positive definite operators.
The VIM has since been employed by many investigators in the solution of theoretical shell and plate problems, differential and integral equations, linear and nonlinear. The VIM accurately computes the solution in a series solution that converges to the exact solution if such a solution exists. In the existing literature, many authors call this method the He's variational iteration method. It is used widely in a variety of works by many authors such as in [4-11] and many of the references therein. The so-called He's variational iteration method is now used for handling linear and nonlinear equations in a straightforward manner.

Formulation
We will present the essential steps for using the variational iteration method and the determination of the Lagrange multipliers for various values of a. Consider the differential equation

$$Ly + Ny = g(t), \quad (1)$$

Where L and N are linear and nonlinear operators, respectively, and g(t) is the source inhomogeneous term. To use the VIM, a correction functional for Eq. (1) should be used in the form

$$y_{n+1}(t) = y_n(t) + \int_0^t \lambda(Ly_n(\xi) + N\tilde{y}_n(\xi) - g(\xi))d\xi, \quad (2)$$

Where λ is a general Lagrange's multiplier, which can be identified optimally via the variational theory, and $\tilde{y}_n$ as a restricted variation which means $\delta\tilde{y}_n = 0$.
It is obvious now that the main steps of the variational iteration method require first the determination of the Lagrange multiplier λ that will be identified optimally. For Eq. (1), the correction functional reads

$$y_{n+1}(x) = y_n(x) + \int_0^x (\lambda(\xi))_{\xi\xi} + \frac{\alpha}{\xi}(y_n(\xi))_\xi + f(\xi)\tilde{g}(y_n(\xi))d\xi \tag{3}$$

Where $\delta(\tilde{g}(y_n(\xi))) = 0$.

To determine the optimal value of $\lambda(\xi)$, we take the variation for both sides with respect to $y_n(x)$ to obtain

$$\delta y_{n+1}(x) = \delta y_n(x) + \delta\int_0^x \lambda(\xi)\left((\lambda_n(\xi))_{\xi\xi} + \frac{\alpha}{\xi}(y_n(\xi))_\xi + f(\xi)\tilde{g}(y_n(\xi))\right)d\xi \tag{4}$$

or equivalently

$$\delta y_{n+1}(x) = \delta y_n(x) + \delta\int_0^x \lambda(\xi)\left((\lambda_n(\xi))_{\xi\xi} + \frac{\alpha}{\xi}(y_n(\xi))_\xi\right)d\xi \tag{5}$$

Where we used $\delta(\tilde{g}(y_n(\xi))) = 0$.
Integrating the integral at the right side by parts yields

$$\delta y_{n+1}(x) = \delta y_n(x)\left(1 - \lambda'(x) + \frac{\alpha}{x}\lambda(x)\right) + \delta\lambda(x)(y_n)_\xi(x) + \delta\int_0^x y_n\left(\lambda''(\xi) - \alpha\frac{\xi\lambda'(\xi) - \lambda(\xi)}{\xi^2}\right)d\xi \tag{6}$$

This in turn gives the stationary conditions

$$\begin{aligned} &\lambda(\xi = x) = 0, \\ &\lambda'|_{\xi=x} = 1, \\ &\lambda'' - \alpha\frac{\xi\lambda' - \lambda}{x^2} = 0 \end{aligned} \tag{7}$$

To determine λ, four important cases will be examined:

1. For $\alpha = 0$, and by solving (7) we obtain

$$\lambda(\xi) = \xi - x. \tag{8}$$

2. For the cylindrical problems, we have $\alpha = 1$. In this case, solving (7) gives

$$\lambda(\xi)\xi\ln\left(\frac{\xi}{x}\right). \tag{9}$$

3. For the spherical problems, we have $\alpha = 2$ which is the case for the standard Emden–Fowler singular equation. In this case, solving (7) gives the Lagrange multiplier λ by

$$\lambda(\xi) = \frac{\xi(\xi - x)}{x}. \tag{10}$$

4. (iv) For the general case where $\alpha > 1$, solving (7) yields

$$\lambda(xi) = \frac{\xi(\xi^{\alpha-1} - x^{\alpha-1})}{(\alpha-1)x^{\alpha-1}}. \tag{11}$$

The successive approximations y_{n+1}, $n \geqslant 0$ of the solution $y(x)$ will be readily obtained upon using any selective function $y_0(x)$. Consequently, the solution

$$y(x) = \lim_{n\to\infty} y_n(x). \tag{12}$$

Conclusion

In this work we also demonstrate that this method can be well suited to attain numerical solutions with a high degree of accuracy. The challenge due to the singularity at $x = 0$ can be easily overcome here. The Lagrange multipliers for all cases of the parameter α were obtained. We examined in this work the model that arises in various physical applications such as physiology. The results we obtained show that the initial value at $x = 0$ increases with the increase of the parameter α. Moreover, the minimum value of the obtained solution is obtained when $\alpha = 0$ and the maximum value is achieved when $\alpha = 3$.

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