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Study of mathematical modeling of solar air collector with absorber plate

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Abstract
In this Paper, we study the mathematical modeling and simulation of the solar air heater systems using perforate absorbers. It was observed that the efficiency of the collector increases with increase in both the collector length and width and the system attains its optimum values.

Keywords: mathematical modeling, perforate absorbers, simulation, collector increases, system attains, optimum values

Introduction
Many studies have been done both experimental and theoretical to improve the performance of solar heaters systems [1-5]. Modeling and simulation are considered as fast and cheap analytical tools by engineers in developing optimal solar energy systems for a given application prior to their construction.
This study involves modeling of a solar air heater system with trapezoidal corrugate absorber plate. The model is validated using the values obtained from a prototype model built.

Mathematical Modelling
The collector under consideration consists of a glass cover and absorber plate with a well insulated parallel bottom plate, forming a rectangular duct profile through which the air to be heated flows. The corrugation of the absorber plate is trapezoidal in shape and the air is made to flow along the corrugation. The theoretical solutions of the thermal performance of the SAH system involve the formulation of the energy balance equations that describe the heat transfer mechanisms at each component of the solar air collector. The heat distribution through the air heater is as shown in Fig. 1. The energy balance equations obtained are as follows:

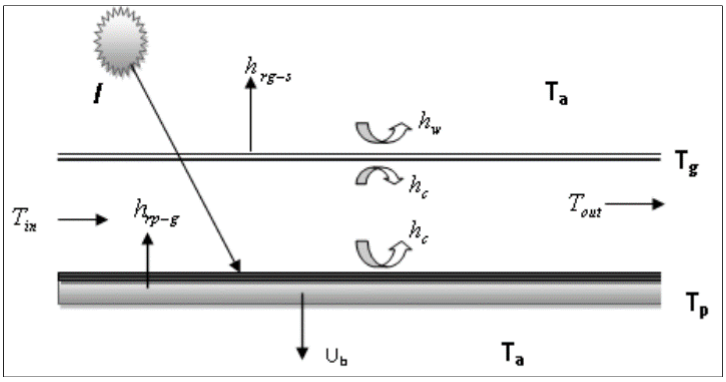

Fig 1: Flat plate solar collector with heat transfer parameters

On the glass cover

$$h_{rp-g}(T_p - T_g) + h_c(T_g - T_f) = U_t(T_g - T_a) \tag{1}$$

On the absorber plate

$$(\tau\alpha)AI = Ah_c(T_p - T_f) + Ah_{r,p-g}(T_p - T_g) + AU_b(T_p - T_a) \tag{2}$$

The air flow

$$\dot{m}C_p(T_{out} - T_{in}) = Ah_c(T_g - T_f) + Ah_c(T_p - T_f) \tag{3}$$

The equations (1) to (3) are used to derive the solutions for the collector components‟ temperatures, i.e. T_p, T_g and Tout as follows:

$$T_p = \frac{(\tau\alpha)AI + h_{rpg}T_g + h_cT_f + U_bT_a}{h_{rpg} + h_c + U_b} \tag{4}$$

$$T_g = \frac{h_{rpg}T_p + U_tT_a + h_cT_f}{U_t + h_c + h_{rpg}} \tag{5}$$

$$T_{out} = \frac{A_ph_c}{\dot{m}C_p}\left[T_p + T_g - 2T_f\right] + T_{in} \tag{6}$$

It is assumed that there is linear temperature rise in the channel, hence T_f is evaluated as the mean of the inlet and the outlet temperatures:

$$T_f = \frac{T_{out} + T_{in}}{2} \tag{7}$$

Heat Transfer Coefficients

The top heat transfer coefficient U_t and wind heat transfer coefficient can be obtained by the expressions:

$$U_t = h_w + h_{rp-g} \tag{8}$$

Where, h_w is calculated from the following empirical correlation suggested by McAdams [6]:

$$h_w = 5.7 + 3.8V \tag{9}$$

The radiation heat transfer coefficient, ($h_{r,g-s}$), from the glass cover to the sky is obtained as [7]:

$$h_{r,g-s} = \sigma\varepsilon_g(T_g^2 + T_s^2)(T_g - T_s)\frac{(T_g - T_s)}{T_g - T_a} \tag{10}$$

Where the sky temperature T_s is estimated by Swinbank [8]:

$$T_s = 0.0552T_a^{1.5} \tag{11}$$

The radiation heat coefficient between the absorber plate and glass cover $h_{r,p-g}$ is given

$$h_{r,p-g} = \frac{\sigma(T_p^2 + T_g^2)(T_p - T_g)}{\left(\frac{1}{\varepsilon_p} + \frac{1}{\varepsilon_g} - 1\right)} \tag{12}$$

The back heat loss coefficient U_b by conduction through the back insulation is determined by the expression;

$$U_b = \frac{k_i}{d} \tag{13}$$

The natural convection heat transfer coefficient between both the glass cover and the absorber plate and the airflow in the duct is calculated as:

$$h_c = Nu\frac{k}{D_k} \tag{14}$$

The hydraulic diameter is given by:

$$D_h = \frac{2WL}{(W+L)} \tag{15}$$

The Nusselt number can be approximated by the following correlation given by Hollands *et al.* [9]

$$Nu = 1.1.44\left[1-\frac{1708(\sin 1.8\theta)^{1.6}}{RaCos\theta}\right]\left[1-\frac{1708}{Ra\,Cos\,\theta}\right]^{+} + \left[\left(\frac{Ra\cos\theta}{5830}\right)^{1/3} - 1\right]^{+} \tag{16}$$

Where θ is the angle of inclination of the collector and Ra is the Rayleigh number

$$Ra = \frac{g\beta'\Delta TL^3}{v\alpha} \tag{17}$$

The notation $[\ ]^{+}$ in eqn (16) is used to denote that if the quantity in the bracket is negative, it should be set equal to zero. Also, the correlation is valid for $0^0 \le \theta \le 75^0$.

Result and Discussion

The validation of the model was done by comparing the predicted and the measured values of the system temperatures. The experimental results were measured on a prototype model built and tested outdoors at the department of physics. Fig. 2 shows the validation results, where the theoretical and experimental output air temperatures are compared for a selected representative day. The graph shows a plot of temperature against time of the day when the measurements were done. It is observed that the theoretical, $T_{o,cal}$ and experimental, $T_{o,exp}$, output air temperatures are almost equal with a small deviation of less than 2.5°C. This gives a good correlation between $T_{o,cal}$ and $T_{o,exp}$.

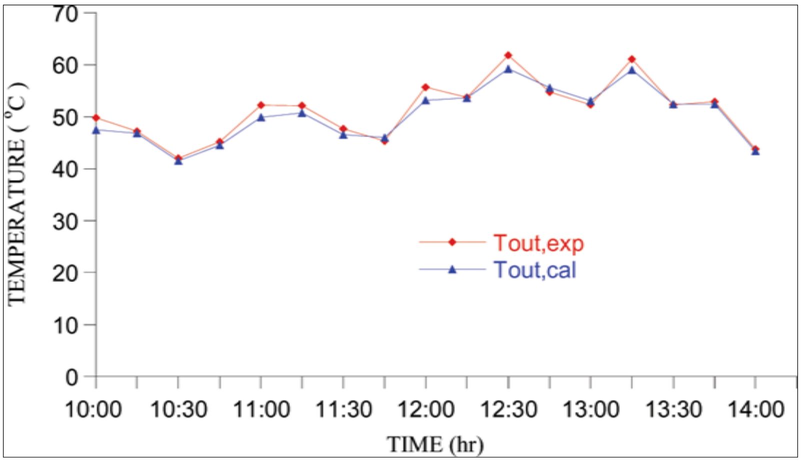

Fig 2: Correlation between the theoretical and experimental output temperature

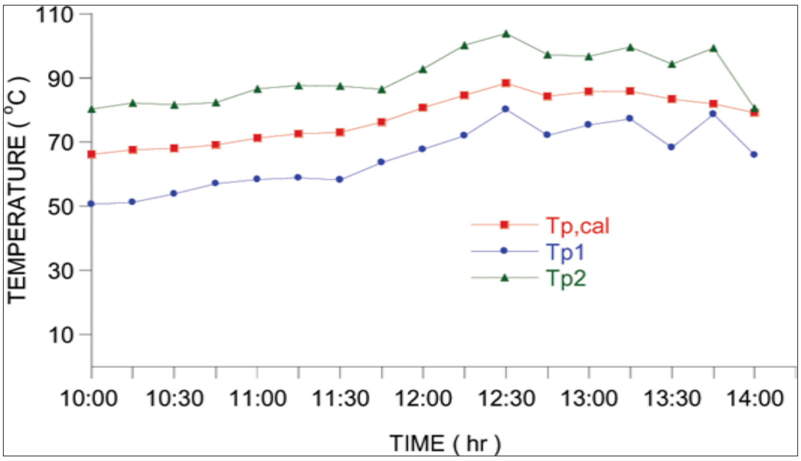

Fig 3: Comparison between the experimental and theoretical plate temperatures

Fig. 3 shows the correlation between the theoretical and experimental values of the plate temperature against time. In order to measure the plate temperature, it is recommended to mount temperature sensors at several points on the absorber plate area to enable the average plate temperature to be obtained. This will give the average plate temperature more accurately but such process is cumbersome.

Thus, the plate temperature is usually measured at two locations on the plate. One sensor is placed at ⅓ and the other at ⅔ from the inlet of the collector and their average value is used as the plate temperature.

Conclusion

The validation results showed a good agreement between experimental and modelling results hence the model could be used with confidence for analytical analysis.

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