International Journal of Statistics and Applied Mathematics 2021; 6(4): 49-53

International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
Maths 2021; 6(4): 49-53

www.mathsjournal.com
Received: 03-05-2021
Accepted: 07-06-2021

Sanjay Kumar Suman
Research Scholar, Department of Mathematics, J.P. Univ. Chapra, Bihar, India

Dr. Ashok Kumar
Associate Professor, Department of Mathematics, DAV PG College, Siwan, Bihar, India

Corresponding Author:
Sanjay Kumar Suman
Research Scholar, Department of Mathematics, J.P. Univ. Chapra, Bihar, India

Study on differential equation models for solar heating

Sanjay Kumar Suman and Dr. Ashok Kumar

Abstract
In this study, improved, validated ordinary differential equation (ODE) models (an extended linear and a nonlinear ones) are proposed for a wide sort of solar heating systems with a solar collector, a heat exchanger, a storage and pipes. The applicability and the generalizability of the models are also discussed.

Keywords: validated ordinary, differential equation, applicability

Introduction
There is no doubt on the importance to investigate and develop solar heating systems. Mathematical modeling is the most widely used and theoretically established tool for the purpose. In this study, a wide sort of solar heating systems are investigated having a solar collector, a heat exchanger and a storage as the main working components. The neighbouring components are connected with pipes, through which a pump circulates some heat transfer fluid as shown in Fig. 1.

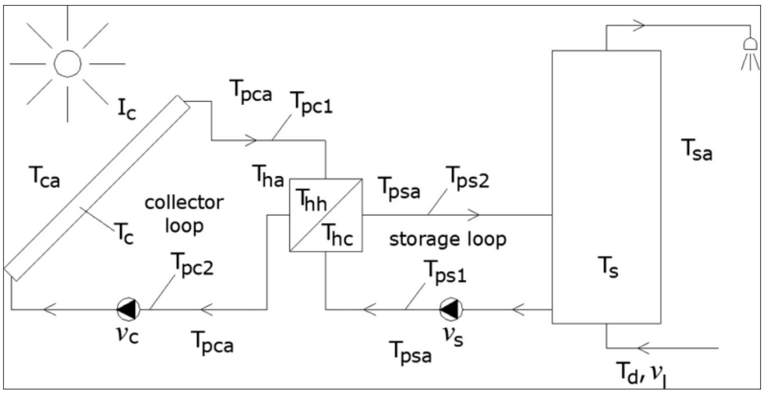

Fig 1: The investigated solar heating system

In many works, the thermal (heat capacity, heat loss) and delaying effects of the pipes are neglected. This is the case, when solar heating systems are modeled with conventionally used ordinary differential equations (ODEs). In [1, 2], collector-storage systems without a heat exchanger are modelled in such a way.
In [2], the collector and the storage are divided into several layers, which are characterized with homogeneous temperatures forming a system of ODEs. In [3, 4], collector-heat exchanger-storage systems are modelled with a linear ODE, the nonlinear version of which can be found in [5].

Basic linear model
The basic linear model has been worked out in [1, 3] and is the following system of four coupled ODEs of first order:

$$\dot{T}_c = \frac{A_c \eta_0}{\rho_c C_c V_c} I_c + \frac{U_{Lb} A_c}{\rho_c C_c V_c}(T_{ca} - T_c) + \frac{v_c}{V_c}(T_{hh} - T_c) \tag{1a}$$

$$\dot{T}_{hh} = \frac{\rho_c C_c v_c}{\frac{C_h m_h}{2} + \rho_c C_c \frac{V_h}{2}}(T_c - T_{hh}) + \frac{\varepsilon k_h A_h}{\frac{C_h m_h}{2} + \rho_c C_c \frac{V_h}{2}}(T_{hc} - T_{hh}) + \frac{\frac{A_a k_a}{2}}{\frac{C_h m_h}{2} + \rho_c C_c \frac{V_h}{2}}(T_{hc} - T_{hh}) \tag{1b}$$

$$\dot{T}_{hh} = \frac{\rho_s C_s v_s}{\frac{C_h m_h}{2} + \rho_s C_s \frac{V_h}{2}}(T_s - T_{hh}) + \frac{\varepsilon k_h A_h}{\frac{C_h m_h}{2} + \rho_s C_s \frac{V_h}{2}}(T_{hc} - T_{hh}) + \frac{\frac{A_a k_a}{2}}{\frac{C_h m_h}{2} + \rho_s C_s \frac{V_h}{2}}(T_{hc} - T_{hh}) \tag{1c}$$

$$\dot{T}_s = \frac{v_1}{V_s}(T_d - T_s) + \frac{v_s}{V_s}(T_{hc} - T_s) + \frac{A_s k_s}{\rho_s C_s V_s}(T_{sa} - T_s), \tag{1d}$$

Basic nonlinear model

The basic nonlinear model (also a system of four coupled ODEs of first order) has been worked out in [5], nevertheless, this is the first time that its derivation is detailed in literature. The standard nonlinear collector model of [6] is according to Eq. (2). (The wind speed dependence and the long-wave irradiance dependence of the heat loss coefficient can be neglected [7].)

$$\dot{T}_c = \frac{A_c \eta_0}{\rho_c C_c V_c} I_c + \frac{k_1 A_c}{\rho_c C_c V_c}(T_{ca} - T_c) - \frac{k_2 A_c}{\rho_c C_c V_c}(T_{ca} - T_c)^2 + \frac{v_c}{V_c}(T_{hh} - T_c) \tag{2}$$

Where

k_1: linear heat loss coefficient of the solar collector, W/(m^2 K);

k_2: second order heat loss coefficient of the solar collector, W/(m^2 K^2).

Let us assume that the collector pump is off ($v_c = 0$) and the solar irradiance intensity is zero ($I_c = 0$). This can be attained e.g. by suitable shading of the collector. In this case only the effect of $T_{ca} - T_c$ (that is the second and third members) plays a role in the r.h.s. of (2) and the derivative of the collector temperature is a function of the mentioned temperature difference according to curve (2) in Fig. 2.

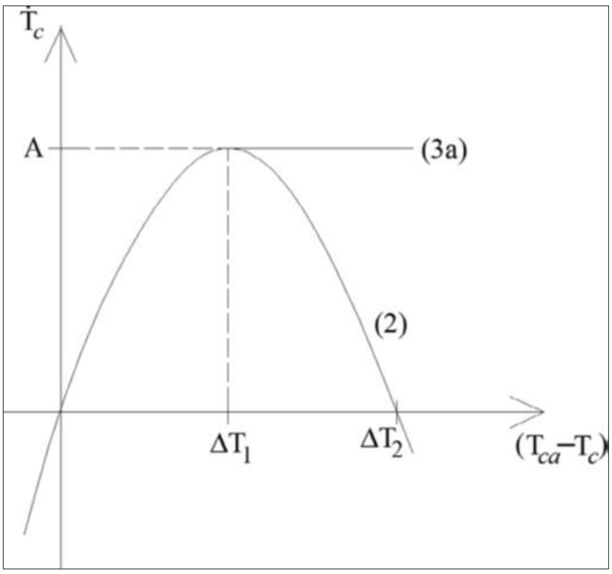

Fig 2: $\dot{T}_c$ as a function of $T_{ca} - T_c$ ($v_c = 0$, $I_c = 0$) according to Eqs. (2) and (3a)

Based on curve (2), T_c increases degressively from the temperature difference ΔT_1. It is evidently a physical unreality, since a temperature value of T_{ca} higher than the collector temperature to a greater extent must warm up the collector faster than in case of a smaller temperature difference. It is even more unreal that temperature differences higher than ΔT_2 not only fail to increase but even reduce T_c. It should be mentioned here that generally, the value of ΔT_1 is already pretty high and beyond the normal operation range of solar collectors, so Eq. (2) can be practically applied. Nevertheless, Eq. (2) is modified here so that it is more realistic for values higher than ΔT_1 according to curve (3a) in Fig. 2. The original function is unchanged below ΔT_1 (curves (2) and (3a) coincide below ΔT_1), and is set constant from ΔT_1.
Finally, the used nonlinear collector model, including also the irradiation and volumetric flow effects, is resulted in the following way:

$$\dot{T}_c = \begin{cases} \frac{A_c\eta_0}{\rho_c C_c V_c} I_c + \frac{k_1 A_c}{\rho_c C_c V_c}(T_{ca} - T_c) - \frac{k_2 A_c}{\rho_c C_c V_c}(T_{ca} - T_c)^2 + \frac{v_c}{V_c} + (T_{hh} - T_c) \; if \; (T_{ca} - T_c) \in]-\infty \Delta T_1] \\ \frac{A_c\eta_0}{\rho_c C_c V_c} I_c + \frac{k_1 A_c}{\rho_c C_c V_c}(\Delta T_1) - \frac{k_2 A_c}{\rho_c C_c V_c}(\Delta T_1)^2 + \frac{v_c}{V_c} + (T_{hh} - T_c) \quad if \; (T_{ca} - T_c) \in]\Delta T_1 + \infty[\end{cases} \tag{3a}$$

The final basic nonlinear model of the solar heating system is established if Eq. (3a) is completed by the models of the solar storage and the heat exchangers, based on [3] with the modification that the flow rate dependence for the heat transfer coefficient of the heat exchanger is taken into account:

$$\dot{T}_{hh} = \frac{\rho_c C_c v_c}{\frac{C_h m_h}{2} + \rho_c C_c \frac{V_h}{2}}(T_c - T_{hh}) + \left(k_2 \left(\frac{v_c + v_s}{2} \right)^{\frac{4}{5}} + k_4 \right) \frac{A_h}{\frac{C_h m_h}{2} + \rho_c C_c \frac{V_h}{2}}(T_{hc} - T_{hh}) + \frac{\frac{A_a k_a}{2}}{\frac{C_h m_h}{2} + \rho_c C_c \frac{V_h}{2}}(T_{ha} - T_{hh}), \tag{3b}$$

$$\dot{T}_{hc} = \frac{\rho_s C_s v_s}{\frac{C_h m_h}{2} + \rho_s C_s \frac{V_h}{2}}(T_s - T_{hc}) + \left(k_3 \left(\frac{v_c + v_s}{2} \right)^{\frac{4}{5}} + k_4 \right) \frac{A_h}{\frac{C_h m_h}{2} + \rho_s C_s \frac{V_h}{2}}(T_{hh} - T_{hc}) + \frac{\frac{A_a k_a}{2}}{\frac{C_h m_h}{2} + \rho_s C_s \frac{V_h}{2}}(T_{ha} - T_{hc}), \tag{3c}$$

$$\dot{T}_s = \frac{v_l}{V_s}(T_d - T_s) + \frac{v_s}{V_s}(T_{hs} - T_s) + \frac{A_s k_s}{\rho_s C_s V_s}(T_{sa} - T_s). \tag{3d}$$

The $k_2\left(\frac{v_c + V_s}{2}\right)^{\frac{4}{5}} + k_4$ heat transfer coefficient of the heat exchanger, with two constants k_3 and k_4, is according to the Dittus–Boelter correlation [8, 9].

Extended ODE models

In the improved, extended linear and nonlinear ODE models, the equations for the collector and for the storage are very similar as in ((1a) and (1d)) and ((3a) and (3d)), respectively.
The equations for the heat exchanger ((1b), (1c)) and ((3b), (3c)) are replaced with the equations of the well-known effectiveness-NTU method [10] according to ((4a) and (4b)) below. This concept is well in accordance with compact heat exchangers, when A_h is great and V_h is small [11, 12].

$$T_{hh} = T_{pc1} - \Phi(T_{pc1} - T_{pst}) \frac{\rho_s C_s v_s}{\rho_c C_c v_c} \tag{4a}$$

$$T_{hc} = \Phi(T_{pc1} - T_{ps1}) + T_{ps1} \tag{4b}$$

Where

Φ: Heat exchanger effectiveness (–).

The models are completed with four pipe, respectively, which describe the four pipes as separated working components with their own volumes and heat loss coefficients (similarly as in the case of the storage).

Results and Discussion

The simulation data of the extended model, with the above identified value U_{Le} = 5.2 W/(m^2 K), is compared with the measured data of two independent days 19th August 2019 and 10th September 2019. (The same comparison is made in case of the basic model with U_{Lb} = 7.3 W/(m^2 K).) Figs. 3 and 4 and Table 1 compare the modelled and measured storage temperatures in case of the basic and extended models for the two days. The operating state of the pumps is also shown in the figures.

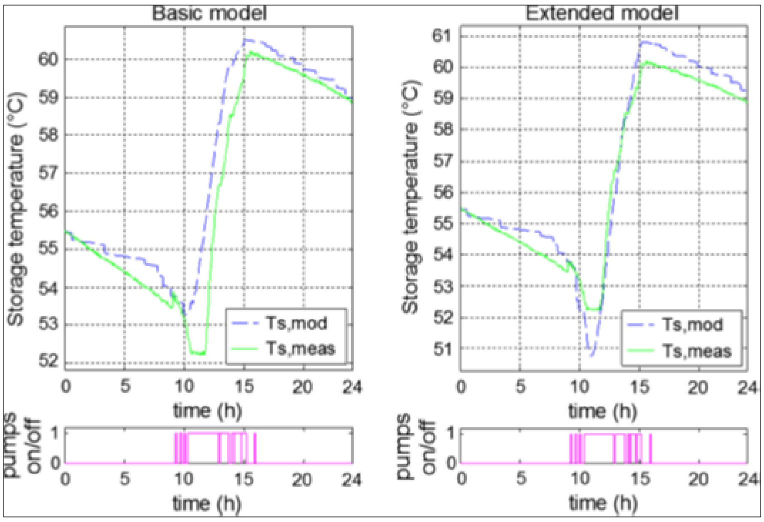

Fig 3: Modelled $T_{s,mod}$ and measured $T_{s,meas}$ storage temperatures on 19th August 2019 in case of the basic and extended models

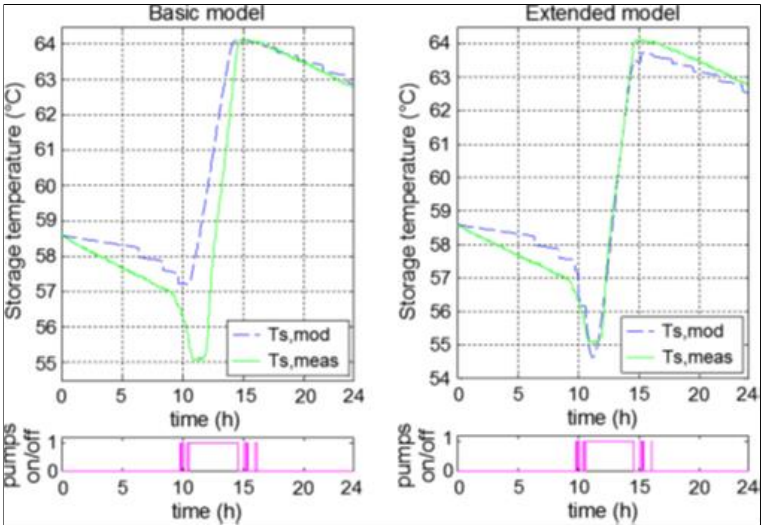

Fig 4: Modelled $T_{s,mod}$ and measured $T_{s,meas}$ storage temperatures on 10th September 2012 in case of the basic and extended models

Table 1: Averages of the error and the absolute error with the validated models

		Basic model	Extended model
Storage			
2nd July (identification)	Average of absolute error	0.76 °C; 7.9%	0.42 °C; 4.3%
19th August (validation)	Average of error	0.60 °C	0.23 °C
	Average of absolute error	0.61 °C; 7.6%	0.49 °C; 6.1%
10th September (validation)	Average of error	0.72 °C	0.07 °C
	Average of absolute error	0.73 °C; 8.1%	0.38 °C; 4.2%
Collector			
19th August	Average of error	0.81 °C	1.81 °C
	Average of absolute error	4.88 °C; 7.6%	5.12 °C; 8.0%
10th September	Average of error	−2.13 °C	−0.93 °C
	Average of absolute error	7.45 °C; 11.3%	7.85 °C; 11.9%

Conclusion
Pipe effects can easily become significant in systems with long pipes, for other case, when the collector field (eventually with a exchanger) is far from the system component to be heated.

References

1. Buzás J, Farkas I, Biró A, Németh R. Modelling and simulation of a solar thermal system. Math Comput Simulat 1998;48:33-46.
2. Zeghib I, Chaker A. Simulation of a solar domestic water heating system. Energy Procedia 2011;6:292-301.
3. Buzás J, Farkas I. Solar domestic hot water system simulation using blockoriented software. In: The 3rd ISES-Europe solar world congress (Eurosun 2000), Copenhagen, Denmark, CD-ROM Proceedings 2000, 9.
4. Kicsiny R, Varga Z. Real-time state observer design for solar thermal heating systems. Appl Math Comput 2012;218:11558-69.
5. Kicsiny R, Varga Z. Real-time nonlinear global state observer design for solar heating systems. Nonlinear Anal Real World Appl 2013;14:1247-64.
6. BS EN 12975-2:2006. Thermal solar systems and components. Solar collectors. Test methods. British-Adopted European Standard; [ISBN: 058048131X] 2006.
7. Zambolin E, Del Col D. Experimental analysis of thermal performance of flat plate and evacuated tube solar collectors in stationary standard and daily conditions. Sol Energy 2010;84:1382-96.
8. Ryant CJ. Nomograph of Dittus-Boelter equation. Ind Eng Chem 2018;35(11):1187-8.
9. APV Heat Transfer Handbook 2014.
10. Kays WM, London AL. Compact heat exchangers. New York: McGraw-Hill 1984.
11. Kicsiny R, Farkas I. Improved differential control for solar heating systems. Sol Energy 2012;86:3489-98.
12. Kicsiny R. Energetically-based control of a solar heating system. Thesis of PhD work. Szent István University, Gödöllö 2018, 121.