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An assessment of inverse functions in context of doing and undoing process

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Abstract
This study examines why students have difficulty with inverse functions (inverse functions is the process of doing and undoing operations and what we can do to support their learning. This was a one group pre-test post-test design. In a math classroom in an urban Areas of Anantnag District. After two weeks of instruction one group of students was taught the traditional way of inverse functions and another group was taught conceptually. About (N=80) mathematics students in the sampling were assessed before and after the study. Students were given a test to measure their learning of inverse functions and a questionnaire to measure their perspectives on the unit of study of inverse functions. Qualitative and quantitative methods were used to analyse the data. The results of the study reveal that there is significant variation in pre-test and post-test achievement of the students.

Keywords: inverse functions, doing and undoing process

1. Introduction
Mathematics is the study of quantity, structure, space and change; it has historically developed, through the use of abstraction and logical reasoning, from counting, calculation, measurement and the study of the shapes and motions of physical objects. At a psychological level, exposure to mathematics helps in developing an analytic mind and assists in better organization of ideas and accurate expression of thoughts. At a more general level, far away from dealing with the higher mathematical concepts, the importance of mathematics for a common man underpinned. A common man is being increasingly dependent upon the application of science and technology in the day-to-day activities of life, the role of mathematics has undoubtedly been redefined. Mathematics is around us. It is present in different forms; Right from getting up in early hours of the day to the ringing of an alarm, reading time on a watch, rounding a date on a calendar, picking up the phone, preparing a recipe in the kitchen, to wait for the counts of whistles of the cooker, manage the money, travel to some place, to exchange currency at a ticket outlet while availing a public conveyance or checking up the mileage of your car, halting at the filling station, attending to a roll call at school, getting scores in the class exams, even meet new friends the list is just endless if one goes on to note down the situations when our computational skill, or more specifically, simple mathematics comes to play a role, almost every next moment we do the simple calculations at the back of our mind. Of course these are all done pretty unconsciously without a thought being spared for the use of mathematics on all such occasions. A popular approach to finding the inverse of a function is to switch the xx and yy variables and solve for the yy variable. The strategy of swapping variables is not grounded in mathematical operations and, we will argue, is nonsensical. Nevertheless, the procedure is so ingrained in textbooks and other curricula that many teachers accept it as mathematical truth without questioning is conceptual validity. As a result, students try to memorize the strategy but struggle to "accurately carry out mathematical procedures, understand why those procedures work, and know how they might be used and their results interpreted" (Harvey, R., Kerslake, D., Shuard, H. & Torbe, M. (2014)) [11]. As we will illustrate, this common process for finding the inverse of a function makes it *harder* for students to understand fundamental inverse function concepts. In context to same, the researcher selected the below mentioned research problem:

1.1 Statement of the research problem: In the present study an attempt has been made by the investigator to study the research problem which reads as:

1.2 Objectives/Research questions of the study: In this study the investigator made an attempt to answer the following research questions:
1) What skills and conceptual understanding of functions among students?
2) Can serve as possible predictors of misconceptions of inverse functions?
3) Which learning skills help foster students' growth in mathematics?

1.3 Delimitations of the study: Keeping budget, time and other constraints under consideration, the research delimited the present study to following domains:
1) The study was delimited to 80 respondents only.
2) The study was delimited to
3) Anantnag districts of South Kashmir
4) The study was delimited to 80 students with due representation of the gender.
5) The study was delimited class 11th and 12th students.

1.5 Design of the study: The present study has been operated through "*one group pre-test post-test design".*
- **Sample:** The sample for the present study consists of 80 respondents. These respondents were selected from higher secondary schools of Anantnag District.
- **Sapling technique:** The required sample was selected with the help of random sampling technique.

1.4 Analysis of the data: The data has been analysed with the help of suitable statistical treatment. Descriptive and comparative analysis was used for processing the data. The detailed description of the statistical treatment is given as under:
After two weeks of intensive instruction, the same assessment was given again to both groups at the end of the unit of lessons. Both groups showed overall growth. However, the arrow diagram group showed more growth compared to the control group. Consistent with the pre-test the post-assessment was worth 13 points. The mean score for the Pre-Post assessment for both groups of students was computed. The pre-post mean difference was computed for each individual to measure growth (see Figures 1 and 2). Figure 1 shows the mean of the pre-post assessment scores. The mean for the pre-assessment was $\bar{x}_{pre} = 2.8$ and the post-assessment $\bar{x}_{post} = 8.1$.

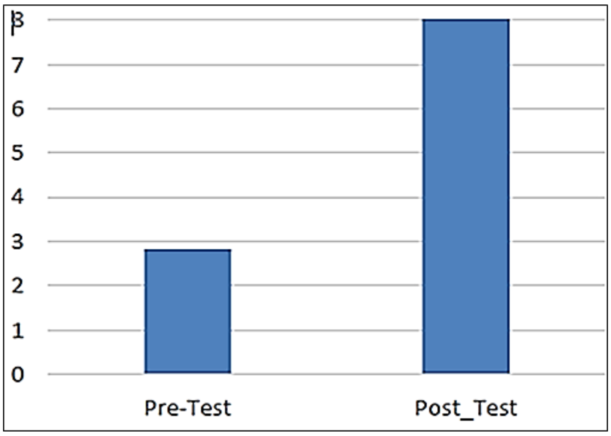

Fig 1: Means of the Treatment Group

A closer examination of growth by item indicated that effective scaffolding in the treatment group and revisiting the concept of functions using arrow diagrams, students were more comfortable with the concept of inverse functions, since students had experience with composition of functions during the first quarter. Question 2 received the most correct responses from the rest of the items, which asked students to evaluate the composition of functions. On the pre-assessment, several students did not give responses on question 4. Other students attempted to answer the question but were unsuccessful. On the post-assessment, several students used the strategy of arrow diagrams to tackle the question and several were successful in answering the question. In Table 8 I randomly selected three students from the pile of 40 assessments which used arrow diagrams to work backwards. On Question five, 21 more students answered the question correctly compared to the pre-assessment from the arrow diagram group.
For the control group of students, students also showed overall growth.
Similar to the arrow diagram group, question 2 was the item that most students answered correctly. Considering that I was the instructor for both groups since the beginning of the academic year that began in August both groups had prior knowledge on the composition of functions. In question 1, 13 more students responded correctly. In question 2, 9 more students responded correctly in which students were able to answer the question which asked them to create their own set of ordered pairs and to find its inverse. Questions 3, 4, and 5 showed the least growth of the 5 items: On question 3, only 1 more student answered correctly since question 3 was tied to question 2, if students were unsuccessful in answering question 2, they would have had difficulty in answering question 3. Figure 2 shows the mean of the pre-post assessment scores, the mean for the pre-assessment was $\bar{x}pre$ = 4.7 and the post-assessment $\bar{x}post$ = 6.7.

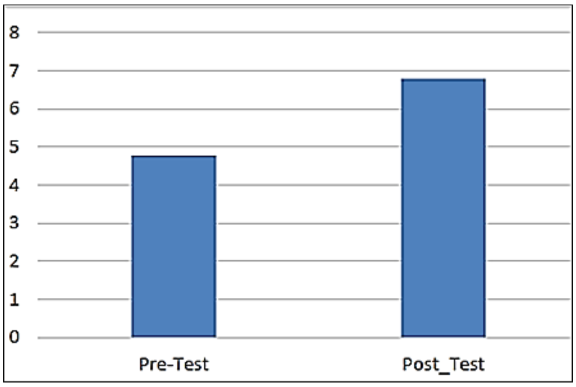

Fig 2: Means of the Control Group

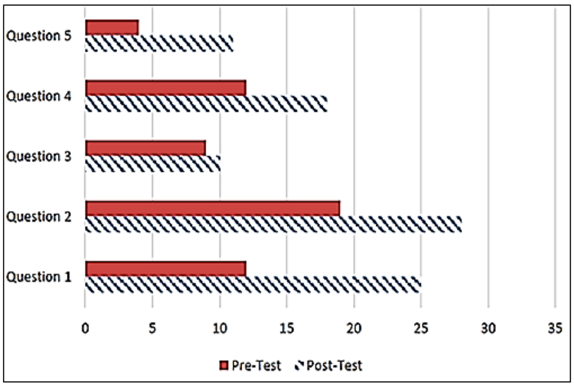

Fig 3: Control Group Assessment by Item.

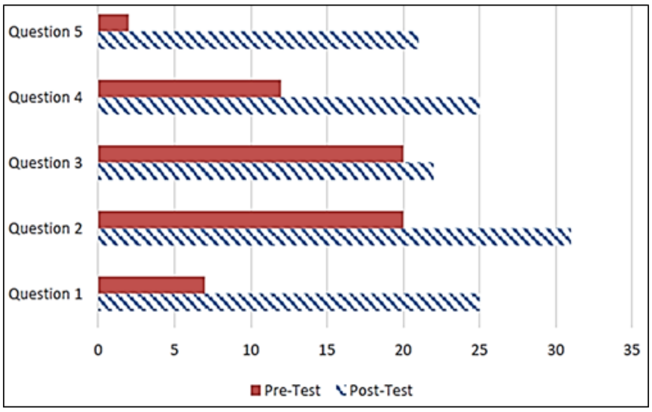

Fig 4: Treatment Group Assessment by Item

On question 1, 25 students from each group answered correctly. On question 2, three more students from the arrow diagram group answered correctly. On question 3, twelve more students from the arrow diagram group answered correctly. In question 4, 29 students used arrow diagrams to answer the word problem, the rest of the students used number sense to answer the problem. In the traditional group tried using number sense or some sort of algebraic manipulation in order to answer the question. In question 5, ten more students were successful in answering all four items in working with function machines and function notation.

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Inferential Analysis: By preforming an item analysis of each question, by finding the mean score for each item. I was able to conclude which items were difficult and which ones were easy. In the control group for the pre-assessment item 5 was the most difficult question and in the post-assessment item 5 was still the most difficult question. In the treatment group, for the pre-assessment item 3 and 5 were the most difficult question and in the post-assessment item 3 was the most difficult question. The range of scores were from 0-13 points, by using Microsoft Excel I was able to find the mean scores from the pre-and post-assessments for comparisons. Since there were 13 points possible, I was able to compare each individual student's pre- assessment and compared it to their post-assessment. I was able to perform an analysis of covariance (ANCOVA) to test the differences between the means of the treatment and control groups. The pre-test scores were used as the covariate to adjust for any initial differences in the two groups. The null hypothesis: $H0$: $\mu adj.treatment - \mu adj.control = 0$, if teaching inverse functions by using the arrow diagram has the same result as teaching inverse functions as the traditional way. Then the end result would be the same and the null hypothesis would be 0. The alternative hypothesis was Ha: $\mu adj.treatment - \mu adj.control \neq 0$ with $\alpha = 0.05$. To find out if there is a statistical significance which means the p-value is less than 5%. The ANCOVA results showed that the p-value was less than .01 (see Figure 10) which suggested that there was a statistically significant difference between the adjusted means of the two groups. As shown below in Figure 10, I rejected the null hypothesis in favour of the alternative hypothesis. By teaching inverse functions with the use of arrow diagrams, students were able to have a better conceptual understanding of inverse functions compared to the control group.

Source	SS	df	MS	F	P
Adjusted means	90.64	1	90.64	9.42	0.002963
Adjusted error	740.64	77	9.62		
Adjusted total	831.28	78			

Table 1: Showing Student Responses of Questionnaire

Questions	Arrow Diagram group	Traditional group
What did you learn from this lesson?	71%	52%
Can you explain the process on how to find the Inverse of a function?	65%	42%
How can you determine whether the inverse of a function is a function?	55%	38%
What did you enjoy most about this lesson? Why?	71%	52%

Student questionnaire: The questionnaire consisted of 4 items as shown in Table 2, in which I entered the percentage of positive responses from the questionnaires. On item 1, about 71% of students in the arrow diagram group were able to successfully describe what they learned. I was able to determine this by seeing the results in their post test to see if they used arrow diagrams to solve the word problem. On Item 2 about 65% of the arrow diagram group was able to explain the process. For example, one response from a student was "working backwards for example undoing and doing the opposite of an operation to get your answer" (Participant May 2019). On item 4, a higher percentage of students that were taught conceptually gave positive feedback in comparison to the control group. One student's remark who answered correctly said, "I enjoyed learning the concept of working backwards to find the solution! Also, you have multiple steps to get your answer. I actually knew what was going on and I understood it" (Participant 2, December 2017). As for the control group, about half of the students enjoyed the lesson since the concept of exchanging x for y was not really new to them.

2. Conclusions of the study

Finding in this study support the use of arrow diagrams as a means to promote students' conceptual understanding of inverse functions. These findings are consistent with those researchers such as Mosteller, F., Light, R.J. & Sachs, J.A. (2016) [26] and Moses, R.P. (2014) [25]. Who all support the idea of arrow diagrams as a means to promote students' conceptual understanding of inverse functions. In conclusion, this research study explored whether teaching inverse functions with the use of arrow diagrams would improve students' conceptual understanding of inverse functions. All findings support the conclusion that teaching inverse functions by using arrow diagrams does in fact improve students' understanding of inverse functions. Furthermore, the students' responses on the questionnaires confirmed that they enjoyed learning inverse functions by using arrow diagrams, and that it helped them understand inverse functions, as well as functions. The researcher suggest that further exploration on this topic, including student interviews and groups of teachers willing to teach my lesson on inverse functions via arrow diagrams, would only strengthen my study.

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