International Journal of Statistics and Applied Mathematics 2021; 6(5): 59-61

International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
Maths 2021; 6(5): 59-61

www.mathsjournal.com
Received: 25-07-2021
Accepted: 27-08-2021

E Nafula
School of Mathematics,
University of Nairobi, Kenya

Stephen K Moindi
School of Mathematics,
University of Nairobi, Kenya

Peter W Njori
School of Pure and Applied Schiences, Kirinyaga University, Kenya

Corresponding Author:
E Nafula
School of Mathematics,
University of Nairobi, Kenya

A study of W_7 curvature tensor in para Kenmotsu manifolds

E Nafula, Stephen K Moindi and Peter W Njori

Abstract
The object of the present paper is to study certain curvature conditions in Para Kenmotsu manifolds admitting a W_7-curvature tensor.

Keywords: para Kenmotsu manifold, W_7-curvature tensor, symmetric, semi-symmetric, and W_7-flat, recurrent.

1. Introduction

A manifold M^n is called an almost para-contact metric manifolds if there exists in M^n a (1,1) tensor field ϕ, a vector field ξ, and a 1-form η such that

(1.1) $\eta(\xi) = 1, \phi^2(X) = X - \eta(X)\xi$

(1.2) $g(X,\xi) = \eta(X), g(\phi X, \phi Y) = g(X,Y) - \eta(X)\eta(Y)$

(1.3) $\phi\xi = 0, \eta(\phi X) = 0, rank\ \phi = n - 1$

Where g is the Riemmannian metric.

If in addition the manifold M^n satisfies

(1.4) $(\nabla_X \eta)(Y) - (\nabla_Y \eta)X = 0$

(1.5) $(\nabla_X \nabla_Y \eta)Z = [-g(X,Z) + \eta(X)\eta(Z)]\eta(Y) + [-g(X,Y) + \eta(X)\eta(Y)]\eta(Z)$

(1.6) $\nabla_X \xi = \phi^2 X = X - \eta(X)\xi$

Then it is called a Para-Kenmotsu manifold or briefly P-Kenmostu manifold.

It is well known that is a P-Kenmotsu manifold,

(1.7) $S(X,\xi) = -(n-1)\eta(X)$

(1.8) $(L_{\xi S})(X,Y) = 2S(X,Y) + 2(n-1)\eta(X)\eta(Y)$

Where S is the Ricci tensor of type (0,2) and L denotes the Lie derivative.

2. W_7 Curvature Tensor in Para Kenmotsu Manifold

Mishra and Pokhariyal [2] gave the definition of W_7 curvature tensor as

(2.1) $W_7 = R(X,Y)Z + \frac{1}{n-1}[g(Y,Z)QX - Ric(Y,Z)X]$

Or

(2.2) $W_7'(X,Y,Z,T) = R'(X,Y,Z,T) + \frac{1}{n-1}[g(Y,Z)Ric(X,T) - Ric(Y,Z)g(X,T)$

Definition 2.1 A Para Kenmotsu manifold is said to be flat if the Riemannian curvature tensor vanishes identically i.e. $R(X,Y)Z = 0$

Definition 2.2 A Para Kenmotsu manifold is said to be W_7 flat if W_7 curvature tensor vanishes identically i.e. $W_7(X,Y)Z = 0$

Theorem 2.1 A W_7 flat Para Kenmotsu manifold is a flat manifold

Proof
If a P-Kenmotsu space is W_7 flat then $W_7 = 0$. Thus from equation (2.2)

(2.3) $0 = R'(X,Y,Z,T) + \frac{1}{n-1}[g(Y,Z)Ric(X,T) - Ric(Y,Z)g(X,T)]$

Using equation (1.7) in (2.3) we have

(2.4) $R'(X,Y,Z,T) = \frac{1}{n-1}[Ric(Y,Z)g(X,T) - g(Y,Z)Ric(X,T) = \frac{1}{n-1}[-(n-1)g(Y,Z)g(X,T) + g(Y,Z)(n-1)g(X,T) = g(Y,Z)g(X,T) - g(Y,Z)g(X,T)$

But in P-Kenmotsu manifold we have
(2.5) $R'(X,Y,Z,T) = g(Y,Z)g(X,T) - g(X,Z)g(Y,T)$
Equations (2.4) and (2.5) can only be equal if and only if $R'(X,Y,Z,T) = 0$. Thus also $Ric(X,Y) = 0$.
Hence the theorem.

Corollary 2.1 A W_7-flat P-Kenmotsu manifold is neither Einstein nor η-Einstein.

3. A W_7-Semi-symmetric P-Kenmotsu manifold

U.C.De and N.Guha [3] gave the definition of semi-symmetry as $R(X,Y)R(Z,T)V = 0$

Definition 3.1 A P-Kenmotsu manifold is said to be W_7-semi-symmetric if $R(X,Y)W_7(Z,T)V = 0$

Theorem 3.1 A W_7-semi-symmetric P-Kemotsu manifold is a W_7-flat manifold.

Proof
If a P-Kenmotsu space is W_7-semi-symmetric then $R(X,Y)W_7(Z,T)V = 0$
Taking the inner product with ξ

$g(R(X,Y)W_7(Z,T)V,\xi) = R'(X,Y,W_7(Z,T)V,\xi)$

$= g(X,\xi)g(W_7(Z,T)V,Y) - g(Y,\xi)g(W_7(Z,T)V,X)$

$= \eta(X)W_7'(Y,Z,T,V) - \eta(Y)W_7'(X,Z,T,V) = 0$

But since $\eta(X) \neq 0$ and $\eta(Y) \neq 0$ we must have $W_7'(Y,Z,T,V) = 0$ and $W_7'(X,Y,T,V) = 0$.
Hence the theorem.

Corollary 3.1 A W_7-semi-symmetric P-Kenmostu manifold is neither Einstein nor η-Einstein.

4. A W_7-symmetric P-Kenmotsu manifold

A P-Kenmotsu manifold is said to be W_7-symmetric if $\nabla_U W_7(A,B)C = W_7'(U,A,B,C) = 0$

Theorem 4.1 A W_7-symmetric and W_7-flat P-Kenmotsu manifold is a flat manifold.

Proof
From theorem (3.1), we found out that a W_7-semi-symmetric manifold is a W_7-flat manifold and if a P-Kenmotsu space is W_7-symmetric this implies
(4.1) $R(X,Y)W_7(A,B)C - W_7(R(X,Y)A,B)C - W_7(A,R(X,Y)B)C - W_7(A,B)R(X,Y)C = 0$
For each of the terms in equation (4.1), we take its inner product with ξ and expand to obtain the expressions
(4.2) $R'(X,Y,W_7(A,B)C,\xi) = g(X,\xi)g(Y,W_7(A,B)C) - g(Y,\xi)g(X,W_7(A,B)C)$

$= \eta(X)W_7'(Y,A,B,C) - \eta(Y)W_7'(X,A,B,C)$

(4.3) $W_7'(R(X,Y)A,B,C,\xi) = R'(R(X,Y)A,B,C,\xi) + \frac{1}{n-1}[Ric(R(X,Y)A,\xi)g(B,C) - Ric(B,C)g(R(X,Y)A,\xi)]$
Using equation (1.7) in (4.3) we have

(4.4) $W_7'(R(X,Y)A,B,C,\xi) = R'(X,Y,A,\xi)g(B,C) - \eta(B)R'(X,Y,A,C)$

The third term of (4.1) becomes
(4.5) $W_7'(A,R(X,Y)B,C,\xi) = R'(A,R(X,Y)B,C,\xi) + \frac{1}{n-1}[g(C,R(X,Y)B)Ric(A,\xi) - Ric(R(X,Y)B,C)g(A,\xi)]$
$= \eta(A)R'(X,Y,B,C) - R'(X,Y,B,\xi)g(A,C)$ Similarly,

(4.6) $W_7'(A,B,R(X,Y)C,\xi) = R'(A,B,R(X,Y)C,\xi) + \frac{1}{n-1}[g(B,R(X,Y)C)Ric(A,\xi) - Ric(B,R(X,Y)C)g(A,\xi)]$

$= \eta(A)R'(X,Y,C,B) - \eta(B)R'(X,Y,C,A)$

Putting together equations (4.2), (4.4), (4.5), and (4.6) we have
$\eta(X)W_7'(Y,A,B,C) - \eta(Y)W_7'(X,A,B,C) - R'(X,Y,A,\xi)g(B,C) + \eta(B)R'(X,Y,A,C) - \eta(A)R'(X,Y,B,C) + R'(X,Y,B,\xi)g(A,C) - \eta(A)R'(X,Y,C,B) + \eta(B)R'(X,Y,C,A) = 0$

From the requirement that $W_7(A,B)C$ be flat, the terms $W_7'(Y,A,B,C)$ and $W_7'(X,A,B,C)$ vanish. Due to the skew-symmetric property of $R'(X,Y,B,C)$ in its last two variables, the coefficients of $\eta(A)$ cancel out. The same applies to the coefficients of $\eta(B)$. We then remain with the expression

$R'(X,Y,B,\xi)g(A,C) - R'(X,Y,A,\xi)g(B,C) = 0$

Since $g(A,C) \neq 0$ and $g(T,V) \neq 0$ for arbitrary vectors A,B,C, we must have $R'(X,Y,B,\xi)$ and $R'(X,Y,A,\xi) = 0$. Hence the theorem.

5. A W_7-Recurrent P-Kenmotsu manifold

Definition 5.1 A manifold M^n is said to be a recurrent manifold [1] if
$(\nabla_U R)(X,Y,Z) = B(U)R(X,Y,Z)$
Where B is the associated recurrence 1-form.

Definition 5.2 Similarly, it is Ricci recurrent if
$(\nabla_U Ric)(X,Y) = B(U)Ric(X,Y)$

Definition 5.3 We shall refer to a P-Kenmotsu manifold as W_7-recurrent if
(5.1) $(\nabla_U W_7'(X,Y,Z,T) = B(U)W_7'(X,Y,Z,T)$

Theorem 5.1 If a P-Kenmotsu manifold is W_7-recurrent and Ricci recurrent, then for the same recurrence 1-form, it is recurrent.
Proof
From (5.1), we have

$$(\nabla_U W_7')(X,Y,Z,T) = B(U)W_7'(X,Y,Z,T) = (\nabla_U R')(X,Y,Z,T) + \frac{1}{n-1}[g(Y,Z)(\nabla_U Ric)(X,T) - g(X,T)(\nabla_U Ric)(Y,Z)]$$

$$B(U)W_7'(X,Y,Z,T) = (\nabla_U R')(X,Y,Z,T) + \frac{B(U)}{n-1}[g(Y,Z)Ri\square(X,T) - g(X,T)Ric(Y,Z)]$$

$$(\nabla_U R')(X,Y,Z,T) = B(U)\left\{W_7'(X,Y,Z,T) - \frac{1}{n-1}[g(Y,Z)Ric(X,T) - g(X,T)Ric(Y,Z)]\right\}$$

$$(\nabla_U R')(X,Y,Z,T) = B(U)R'(X,Y,Z,T)$$

Hence the theorem.

6. Conclusion

The contracted part of W_7-curvature tensor does not vanish in an Einstein space. Thus we can not extend the Pirani's formalism of gravitational wave to the Einstein space with the help of W_{7jk}.

7. References

1. Mishra RS, Pokhariyal GP. Curvature tensors in Riemannian manifold. Indian Jour. of Pure and Appl. Maths 1971, 2(3).
2. Pokhariyal GP. Relativistic significance of curvature tensors. Internat. J Math. Sci 1982; 5(1): 133-139.
3. De UC, Guha N. On conharmonic recurrent sasakian manifold. Indian J Math 1992;34:209-215.