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Alfred Hugo
Department of Mathematics and Statistics, The University of Dodoma, Dodoma, Tanzania

Eva M Lusekelo
Department of Mathematics and Statistics, The University of Dodoma, Dodoma, Tanzania

Corresponding Author:
Alfred Hugo
Department of Mathematics and Statistics, The University of Dodoma, Dodoma, Tanzania

Mathematical control of divorce stress among marriages

Alfred Hugo and Eva M Lusekelo

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Abstract
Marriage is the legally or formally acknowledged union of two individuals as partners (a man and a woman) in a personal connection; it refers to the scenario in which two people (a man and a woman) officially announce their connection permanently. Divorce, on the other hand, refers to the breakdown of a marriage between two people. This paper focuses on the dynamics of marriage and divorce scenarios. The stability analysis of the model was evaluated where counselling was used as the possible control measure to maintain peace and harmony among marriages. The numerical verification of the model was performed to justify the analytical results and it was observed that increasing counselling effort reduces the hardship life and complexity within marriage and or divorced individuals. Therefore, harmony, peace, and happier life in families could be maintained and stained through proper counselling and which in turn reduces divorce scenarios within marriages.

Keywords: divorce, stress, control, epidemiological model

1. Introduction
The legally or formally recognized union between two people as partners (a man and a woman) in a personal relationship is known as marriage, this involves the situation at which two people (a man and a woman) officially publicize their relationship permanently. On the other hand, the dissolution among the marriage partners is termed divorce; it's more permanent than separation and involves a legal process (Haviland *et al*, 2011) [8].

In Tanzania like any other country worldwide, parental separation is common rather than legal divorces, as the result of the big number of marital dissolution complaints (Rita 2007) [11]. Furthermore, the results show that widespread globalization; non-tolerant among couples, early marriage, and poverty are the main causes of divorces. The other courses of divorce pointed out by many researchers are infertility, financial problems, physical or emotional abuse, infidelity, alcoholism, cheating, and poor communications (Rita 2007, Emily and Shelley 2015) [11, 4]. Also, some divorce cases may depend on whether the mother or father acts as the cause of the conflict and break of relationship (Sun and Li 2002), on the other hand, Gambrah and Adzadu (2018) predicted parents' divorce could be probably caused by socio-demographic characteristics.

The rate of divorce has increased from 1.1% in 2008/2009 to 2.1% in 2014/2015 and whereby most of the divorce cases were not registered (Rita 2008) [12]. Although the widespread belief regarding family life is that marriage should be a lifetime commitment, still, the divorce rate in the world is growing up and becoming a social problem in many families (Sun and Li 2002, Amato and Anthony 2014) [14, 2]. Some literature shows that stepfamilies experience more stress than others and its output contributes more to marital termination (Amato 2000, Rita 2008) [1, 12]. In addition to that, the post-separation factors related to divorcing mothers have a direct connection with health output as the result of mental disorders that suppress the socioeconomic and cultural settings in the society ((Stephenson and DeLongis 2018, Øystein and Emily 2019) [13, 10].

Modelling social aspects such as divorce in marriage creates more awareness in many families and builds intrinsic harmony among the partners and family. Education campaigns and counselling services in the community help to maintain peace, love, and trust among the marriage. The particular study intends to apply the mathematical approach to investigate the dynamics of marriage and divorce using an epidemiological model.

2. Mathematical Model formulation and Analysis

2.1 Mathematical Model formulation

This section presents the procedure of model formulation; it describes the dynamics of divorce among marriage based on the context of Tanzania, however, the field data was not incorporated. It is assumed that the populations are homogeneously mixing (have the same interaction to the same degree). It is further, assumed that sex, race, and social status do not affect the probability of being divorced and the divorce epidemic occurs in a closed population (Emily and Shelley 2015) [4]. We divide the population into six compartments: marriage (M), healthy marriage (H), Unhealthy marriage (U), Counselling (C), Complexity life (K) and Divorced (D). Table 1 presents the parameters used in model formulation.

Table 1: Parameters descriptions

Parameters	Description
α	Rate of individuals receiving counselling and joining a healthy marriage.
β	Rate of individuals experiencing unhealthy marriage.
δ	Rate of stressed individuals after counselling.
ε	Rates of individuals attain counselling but remain under unhealthy marriage.
λ	Rate of divorced individuals due to complex marriage life.
μ	The human natural mortality rate.
θ	The rate of individuals joins unhealthy from a healthy marriage.
γ	Rate of individuals attaining counselling due to the unhappy life of marriage-related hardship.
π	Marriage growth rate.
τ	Rate of individuals seeking counselling after stress and hardship (complexity) life of marriage.
q	The factor of individuals experiencing unhealthy marriage.
a	The death rate of individuals is related to the complexity of life.

Basing on the assumptions provided the following model compartment is presented

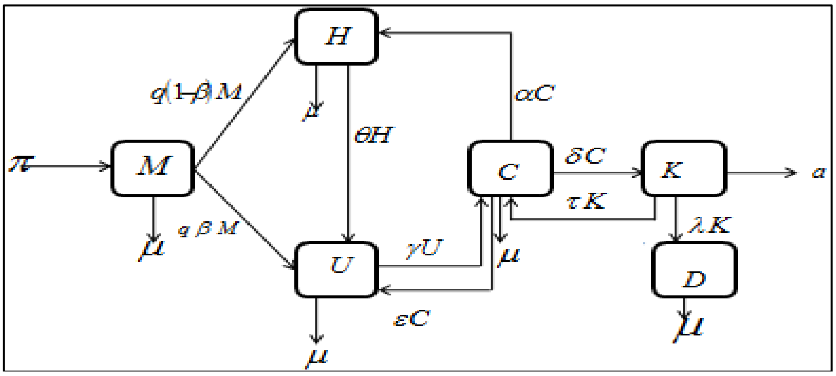

Fig 1: Flow diagram for the divorce model

From Figure 1, the following set of differential equations are generated

$$
\left.
\begin{aligned}
\frac{dM}{dt} &= \pi - (q+\mu)M, \\
\frac{dH}{dt} &= q(1-\beta)M + \alpha C - (\theta+\mu)H, \\
\frac{dU}{dt} &= q\beta M + \theta H + \varepsilon C - (\gamma+\mu)U, \\
\frac{dC}{dt} &= \gamma U_h + \tau K - (\varepsilon+\delta+\alpha+\mu)C, \\
\frac{dK}{dt} &= \delta C - (\tau+\lambda+a+\mu)K, \\
\frac{dD}{dt} &= \lambda K - \mu D,
\end{aligned}
\right\} \quad (1)
$$

with initial $M \geq 0, H \geq 0, U \geq 0, C \geq 0, K \geq 0, D \geq 0$.

2.2 Model Analysis

In this subsection, we analyze the basic properties of the model system (1) and determine its stability. The model properties are employed to establish criteria for boundedness and positivity of solutions of the system.

2.3 Invariant Region

The behavior of model (1) is analyzed to understand its biological validation. It is assumed that all the state variables and parameters of the model are non-negative for all $t \geq 0$. In this case, we take a total population as

$$N = M + H + U + C + K + D \tag{2}$$

Thus the time derivative and simplification of (2) gives the following inequality

$$\frac{dN}{dt} \leq \pi - \mu N. \tag{3}$$

Applying Birkhoff and Rota theorem (Turner *et al*,1995) on differential inequality (3) as well as applying the technique of separable variables, we have

$$\frac{dN}{\pi - \mu N} \leq dt. \tag{4}$$

Applying integration on both sides in (4) together with the initial condition $N(t) = N(0)$ $t = o$, we get

$$N \leq \frac{\pi}{\mu} - \left[\frac{\pi - \mu N_0}{\mu}\right] e^{-\mu t}. \tag{5}$$

As $t \to \infty$ the equation (5) tends to $N \leq \frac{\pi}{\mu}$ imply that $N \to \frac{\pi}{\mu}$ which means $0 \leq N \leq \frac{\pi}{\mu}$.

Therefore, the feasible solution of the system of equations (1) defined in the positive invariant region

$$\Omega = \left\{ \left(M, H, U, C, K, D\right) \in R_+^6 : N \leq \frac{\pi}{\mu} \right\}. \tag{6}$$

Thus, model (1) is epidemiologically and mathematically well-posed.

2.4 Positivity of solution

In this section, we present the positivity of the solution of the system of equations (1); where we show that all state variables are confined in the positive quadrant for all $t \geq 0$.

Lemma 2.1

Let T *be a non-negative region defined in* R_+^6 :

$$T = \left\{ \left(M, H, U, C, K, D\right) \in \mathrm{R}_+^6 ; M(0) > 0, H(0) > 0, U(0) > 0, C(0) > 0, K(0) > 0, D(0) > 0 \right\}.$$

Then, the solution $\{M(t), H(t), U(t), C(t), K(t), D(t)\}$ *from the system of equations (1) remains positive for all* $t \geq 0$.

Proof

Using the first equation of the system of equations (1), that is

$$\frac{dM}{dt} = \pi - \left(q + \mu\right) M . \tag{7}$$

we form inequality as

$$\frac{dM}{dt} \geq -\left(q + \mu\right) M, \tag{8}$$

Solving (8), and substituting the initial value at $t = 0$, we obtain

$$M(t) \geq M(0)e^{-(q+\mu)t} \tag{9}$$

as $t \to \infty$ we get

$M(t) \geq 0$. Hence proved.

For the second equation in the model (1), that is

$$\frac{dH}{dt} = q(1-\beta)M + \alpha C - (\theta + \mu)H \tag{10}$$

The inequality is formed as

$$\frac{dH}{dt} \geq -(\theta + \mu)H \tag{11}$$

The solution of (11) at $t = 0$ is given by

$$H(t) \geq H(0)e^{-(\theta+\mu)t} \tag{12}$$

as $t \to \infty$ $H(t) \geq 0$. Hence proved.

Therefore, the rest of the equations in the system of equations (1) can be solved in the same manner.

2.4.1 Steady states of the model

In this subsection, the system of equations (1) is qualitatively analyzed to determine its equilibrium points and its corresponding stability analysis. The equilibrium points are obtained by setting the right-hand side of the model (1) equal to zero and solve for state variables. That is

$$\left.\begin{aligned}
&\pi - (q + \mu)M = 0 \\
&q(1-\beta)M + \alpha C - (\theta + \mu)H = 0 \\
&q\beta M + \theta H + \varepsilon C - (\gamma + \mu)U = 0 \\
&\gamma U_h + \tau K - (\varepsilon + \delta + \alpha + \mu)C = 0 \\
&\delta C - (\tau + \lambda + a + \mu)K = 0 \\
&\lambda K - \mu D = 0
\end{aligned}\right\} \tag{13}$$

Solving system of the equations (13), we get the divorce free equilibrium point of the system as

$$E_O = (M^o, H^o, U^o, C^o, K^o, D^o) = \left(\frac{\pi}{q+\mu},\ \frac{q(1-\beta)\pi}{(\theta+\mu)(q+\mu)},\ 0,\ 0,\ 0,\ 0 \right). \tag{14}$$

2.4.2 Endemic Equilibrium (EE) Point

In the presence of infection, that is: $U \neq C \neq K \neq D \neq 0$ the model system (13-18) has a non-trivial equilibrium point, E_1 called the endemic equilibrium point which is given by

$E_1 = (M^*, H^*, U^*, C^*, K^*, D^*)$ for $M^*, H^*, U^*, C^*, K^*, D^* > 0$.

Where

$$\left.\begin{aligned}
M^* &= \frac{\pi}{\pi+\mu}, \\
H^* &= \frac{q(1-\beta)\pi}{q+\mu} + \frac{\alpha(\tau+\lambda+\alpha+\mu)\mu D}{\delta\lambda}, \\
U^* &= \frac{(\varepsilon+\delta+\alpha+\mu)(\tau+\lambda+\alpha+\mu)\mu D}{\delta\lambda\gamma} - \frac{\tau\mu D}{\lambda\gamma}, \\
C^* &= \frac{(\iota+\lambda+\alpha+\mu)\mu D}{\delta\lambda}, \\
K^* &= \frac{\mu D}{\lambda}, \\
D^* &= \frac{q\beta\pi(\theta-1)\text{-}q\pi\theta}{(q+\mu)N},
\end{aligned}\right\} \tag{15}$$

$$N = \frac{\mu}{\lambda}\left[\frac{(\tau+\lambda+\alpha+\mu)(\alpha\theta+\varepsilon)}{\delta} - \frac{\gamma+\mu}{\gamma}(\varepsilon+\delta+\alpha+\mu)(\tau+\lambda+\alpha+\mu)+\tau\right].$$

2.4.3 The Reproduction Number, R_0

The basic reproduction number denoted by R_0 is the average number of secondary infections caused by an infectious individual during his or her entire period of infectiousness (Hardesty *et al*, 2019). Furthermore, the stability of equilibrium can be analyzed using R_0; if $R_0 < 1$ it means that the divorce dies out from population and when $R_0 > 1$ the divorce persists in the population. In this paper, the reproductive number accounts for the average number of new divorce cases generated by people suffering from hardship life of marriage (either from unhealthy marriage, complexity or illness) which is to be calculated using the next-generation matrix method (Hardesty *et al*, 2019, Stephenson and DeLongis 2018) [3]. The value R_0 is computed as the largest positive eigenvalue of the next-generation matrix, which is the same as the spectral radius of the matrix.

Basing on the system of equations (1), we describe F and V as follows:

Let *F* be a matrix that defines the rate of new divorces and stress in different compartments, differentiates concerning *M*, *H*, and *C*, and then evaluates it at disease-free equilibrium.

Again let *V* be the matrix that defines the rate of transfer of divorces from one compartment to another.

$F_i = (q\beta M + \theta H + \varepsilon C)$ and

$$V_i = \begin{pmatrix} -\varepsilon C + (\gamma+\mu)U \\ -\gamma U - \tau K + (\varepsilon+\delta+\alpha+\mu)C \\ -\delta C + (\tau+\lambda+d_l+\mu)K \\ -\lambda K + \mu D \end{pmatrix} \tag{16}$$

Suppose matrix $F = \frac{\partial F_i}{\partial x_i}$ and $V = \frac{\partial V_i}{\partial x_i}$ where $x_i = U, C, K, D$. The Spectral radius of the next generation matrix FV^{-} gives the value of R_0.

$$F = \begin{pmatrix} 0 & \varepsilon & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \text{ and } V = \begin{pmatrix} \gamma+\mu & -\varepsilon & 0 & 0 \\ -\gamma & \varepsilon+\delta+\alpha+\mu & -\tau & 0 \\ 0 & -\delta & \tau+\lambda+a+\mu & 0 \\ 0 & 0 & -\lambda & \mu \end{pmatrix} \tag{17}$$

The spectral radius $\sigma(FV^{-})$ gives,

$$R_0 = \sigma(FV^{-}) = \frac{\gamma\varepsilon(a+\lambda+\mu+\tau)}{(a+\lambda+\mu+\tau)(-\gamma\varepsilon+(\alpha+\delta+\varepsilon+\mu)(\gamma+\mu))-\delta(\gamma\tau+\tau\mu)}. \quad (18)$$

2.4.4 Local Stability of Disease-Free Equilibrium (DFE)

Lemma 2.2

The divorce-free equilibrium point E_0 *is locally asymptotically stable if the trace of the Jacobin variation matrix* $J(E_0)$ *has a negative value* $(trace<0)$ *and the determinant is a positive value* $(\det>0)$.

Proof:

The partial differentiation of the system of equations (1) concerning M,H,U,C,K,D the divorce free equilibrium point gives:

$$J(E_0)=\begin{pmatrix} -(1+\mu) & 0 & 0 & 0 & 0 & 0 \\ 1-\beta & -(\theta+\mu) & 0 & \alpha & 0 & 0 \\ \beta & \theta & -(\gamma+\mu) & \varepsilon & 0 & 0 \\ 0 & 0 & \gamma & -(\varepsilon+\delta+\alpha+\mu) & \tau & 0 \\ 0 & 0 & 0 & \delta & -(\tau+\lambda+a+\mu) & 0 \\ 0 & 0 & 0 & 0 & \lambda & -\mu \end{pmatrix}. \quad (19)$$

Evaluating the trace of the matrix $J(E_0)$, we have

$$tr\left(J\left(E_0\right)\right) = -\left\{1+5\mu+\theta+\varepsilon+\delta+\alpha+\tau+\lambda+a\right\}. \quad (20)$$

Hence trace of $J\left(E_0\right)<0$.

Using the basic properties of matrix algebra, it is clear that one of the eigenvalues of (19) is $\lambda_1=-\mu$ which has negative real parts. The reduced matrix becomes

$$J(E_0)=\begin{pmatrix} -1-\mu & 0 & 0 & 0 & 0 \\ 1-\beta & -(\theta+\mu) & 0 & 0 & 0 \\ \beta & \theta & -(\gamma+\mu) & \varepsilon & 0 \\ 0 & 0 & \gamma & -(\varepsilon+\delta+\alpha+\mu & \tau \\ 0 & 0 & 0 & \delta & -(\tau+\lambda+a+\mu) \end{pmatrix}. \quad (21)$$

From reduced matrix (21), we determine the determinant as

$$\begin{aligned}\mathrm{Det}\left[J\left(E_0\right)\right] = {} & (\theta+\mu)(1+\mu)((\alpha\gamma+\gamma\delta+\alpha\mu+\gamma\mu+\delta\mu+\varepsilon\mu+\mu^2)(-a-\lambda-\mu-\tau)+ \\ & \delta(\gamma\tau+\mu\tau)).\end{aligned} \quad (22)$$

Simplifying the equation (22), we obtain

$$\begin{aligned}\mathrm{Det}\left[J\left(E_0\right)\right] = {} & \mu^3+\mu^2+\left(\delta+\alpha+\gamma+\varepsilon+a+\lambda\right)\mu^2+\left(\alpha+\gamma+\varepsilon\right)\mu\tau+\gamma\tau\alpha+ \\ & \left(a+\lambda+\gamma\right)\left(\delta+\alpha\right)\mu+\left(a+\lambda\right)\left(\gamma+\varepsilon\right)\mu+\left(a+\lambda\right)\left(\delta+\alpha\right)\gamma.\end{aligned} \quad (23)$$

It is seen that trace of (21) is $(trace<0)$ and the determinant is positive value $(\det>0)$ as presented in (23), then E_0 is locally asymptotically stable.

2.4.5 Global stability analysis

We perform a global stability analysis of the system (1) around the positive equilibrium point E_1 of the coexistence. We use the Lyapunov theorem by defining the function V as applied by(Kar and Chaudhuri 2002) and (Dubey and Upadhyay 2004).

Theorem 1

The co-existence equilibrium point E_1 is globally asymptotically stable if the following conditions are satisfied

i. $q+\mu>0.$
ii. $\gamma+\mu>0.$
iii. $\varepsilon+\delta+\alpha+\mu>0.$
iv. $\tau+\lambda+a+\mu>0.$
v. $\mu>0.$

Proof
Let

$$V=\left(M-M^{*}\right)-M^{*}\log\left(\frac{M}{M^{*}}\right)+\left(H-H^{*}\right)-H^{*}\log\left(\frac{H}{H^{*}}\right)+\left(U-U^{*}\right)-U^{*}\log\left(\frac{U}{U^{*}}\right)+\left(C-C^{*}\right)-C^{*}\log\left(\frac{C}{C^{*}}\right)+\left(D-D^{*}\right)-D^{*}\log\left(\frac{D}{D^{*}}\right)+\left(K-K^{*}\right)-K^{*}\log\left(\frac{K}{K^{*}}\right) \quad (24)$$

Thus,

$$\frac{\partial V}{\partial M}=\frac{M-M^{*}}{M},\ \frac{\partial V}{\partial H}=\frac{H-H^{*}}{H},\ \frac{\partial V}{\partial U}=\frac{U-U^{*}}{U}, \quad (25)$$

$$\frac{\partial V}{\partial C}=\frac{C-C^{*}}{C},\ \frac{\partial V}{\partial K}=\frac{K-K^{*}}{K},\ \frac{\partial V}{\partial D}=\frac{D-D^{*}}{D}$$

The time derivative of (24) gives

$$\frac{dV}{dt}=\frac{\partial V}{\partial M}.\frac{dM}{dt}+\frac{\partial V}{\partial H}.\frac{dH}{dt}+\frac{\partial V}{\partial U}.\frac{dU}{dt}+\frac{\partial V}{\partial C}.\frac{dC}{dt}+\frac{\partial V}{\partial K}.\frac{dK}{dt}+\frac{\partial V}{\partial D}.\frac{dD}{dt} \quad (26)$$

Substituting (1) and (25) into (26) we get

$$\frac{dV}{dt}=\left(\frac{M-M^{*}}{M}\right)\left[\pi-\left(q+\mu\right)M\right]+\left(\frac{H-H^{*}}{H}\right)\left[\mathrm{q}(1-\beta)M+\alpha C-\left(\theta+\mu\right)H\right]+\left(\frac{U-U^{*}}{U}\right)\left[\mathrm{q}\beta M+\theta H+\varepsilon C-(\gamma+\mu)U\right]+\left(\frac{C-C^{*}}{C}\right)\left[\gamma U+\tau K-\left(\varepsilon+\delta+\alpha+\mu\right)C\right]+\left(\frac{K-K^{*}}{K}\right)\left[\delta C-(\tau+\lambda+a+\mu)K\right]+\left(\frac{D-D^{*}}{D}\right)\left[\lambda K-\mu D\right] \quad (24)$$

At the endemic equilibrium point, we have

$$\frac{dV}{dt} = -(q+\mu)\frac{\left(M-M^*\right)^2}{M} - (\theta+\mu)\frac{\left(H-H^*\right)^2}{H} + \left(\frac{H-H^*}{H}\right)\left[q(1-\beta)\left(M-M^*\right)+\alpha\left(C-C^*\right)\right]$$
$$-(\gamma+\mu)\frac{\left(U-U^*\right)^2}{U} + \left(\frac{U-U^*}{U}\right)\left[q\beta\left(M-M^*\right)+\theta\left(H-H^*\right)+\varepsilon\left(C-C^*\right)\right]$$
$$-(\varepsilon+\delta+\alpha+\mu)\frac{\left(C-C^*\right)^2}{C} + \left(\frac{C-C^*}{C}\right)\left[\gamma\left(U-U^*\right)+\tau\left(K-K^*\right)\right]$$
$$-(\tau+\lambda+a+\mu)\frac{\left(K-K^*\right)^2}{K} + \left(\frac{K-K^*}{K}\right)\left[\delta\left(C-C^*\right)\right]$$
$$-\mu\left(D-D^*\right)\frac{\left(D-D^*\right)^2}{D} + \left(\frac{D-D^*}{D}\right)\left[\lambda\left(K-K^*\right)\right] \quad (27)$$

Hence (27) is in quadratic form, that can be expressed as

$$\frac{dV}{dt} = -X^T AX \quad (28)$$

Whereby $X^T = M-M^*, H-H^*, U-U^*, C-C^*, K-K^*, D-D^*$ and

A is a symmetric matrix given by

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} \\ a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & a_{66} \end{pmatrix} \quad (29)$$

where

$a_{11} = q+\mu,$

$a_{12} = a_{21} = q\left(1-\beta\right),$ $a_{22} = \theta+\mu,$

$a_{13} = a_{31} = q\beta,$ $a_{23} = a_{32} = \theta,$ $a_{33} = \gamma+\mu,$

$a_{14} = a_{41} = 0,$ $a_{24} = a_{42} = \alpha,$ $a_{34} = a_{43} = \varepsilon,$ $a_{44} = \varepsilon+\delta+\alpha+\mu,$

$a_{15} = a_{51} = 0,$ $a_{25} = a_{52} = 0,$ $a_{35} = a_{53} = 0,$ $a_{45} = a_{54} = \tau,$

$a_{16} = a_{61} = 0,$ $a_{26} = a_{62} = 0,$ $a_{36} = a_{63} = 0,$ $a_{46} = a_{64} = 0,$

$a_{55} = \tau+\lambda+a+\mu,$

$a_{56} = a_{65} = \lambda,$ and $a_{66} = \mu$.

The endemic equilibrium point is globally asymptotically stable whenever $\frac{dV}{dt} < 0$ which implies that a matrix A must be positive definite. For the matrix A to be positive definite the following conditions must hold $a_{i,j} > 0$ for $i,j = 1,2,3,...,6$ which implies $\beta < 1$, showing that the rate of people experiencing hardship in marriage should be fewer. Hence the E_1 is globally stable. This concludes the proof.

3. Numerical Simulation
This section presents, numerical simulations of the system (1) by analyzing the impact of divorce on population and its respective control measure to minimize the stress among the marriage. For numerical simulations, some of the parameter values obtained from different researchers as presented in Table 2.

Table 2: Ranges of parameters and variables used in the model with data source

Symbol	Default Value	Range	Reference source
α	0.15	0-15	[Emily and Shelley 2015] [4]
β	0.1	0.1-0.9	[Sun and Li 2002] [14]
δ	0.01	0-0.2	Assumed
ε	0.09	0-1	[Emily and Shelley 2015] [4]
λ	0.1	0.1-0.9	[Sun and Li 2002] [14]
μ	0.02	0.01-0.1	[Sun and Li 2002] [14]
θ	0.2	0-0.5	Assumed
γ	0.1	0.1-0.9	[Sun and Li 2002] [14]
π	100	100-150	Assumed
τ	0.09	0-0.1	[Emily and Shelley 2015] [4]
a	0.01	0.01-0.1	Assumed

In figure 2 it is shown that as the rate of individuals experiencing hardship life in marriage increases, the rate of an unhealthy marriage (β) and the complexity of life due to divorce are observed to increase simultaneously. As β decreases also divorce population decreases and the number of people living without stress increases for a healthy marriage.

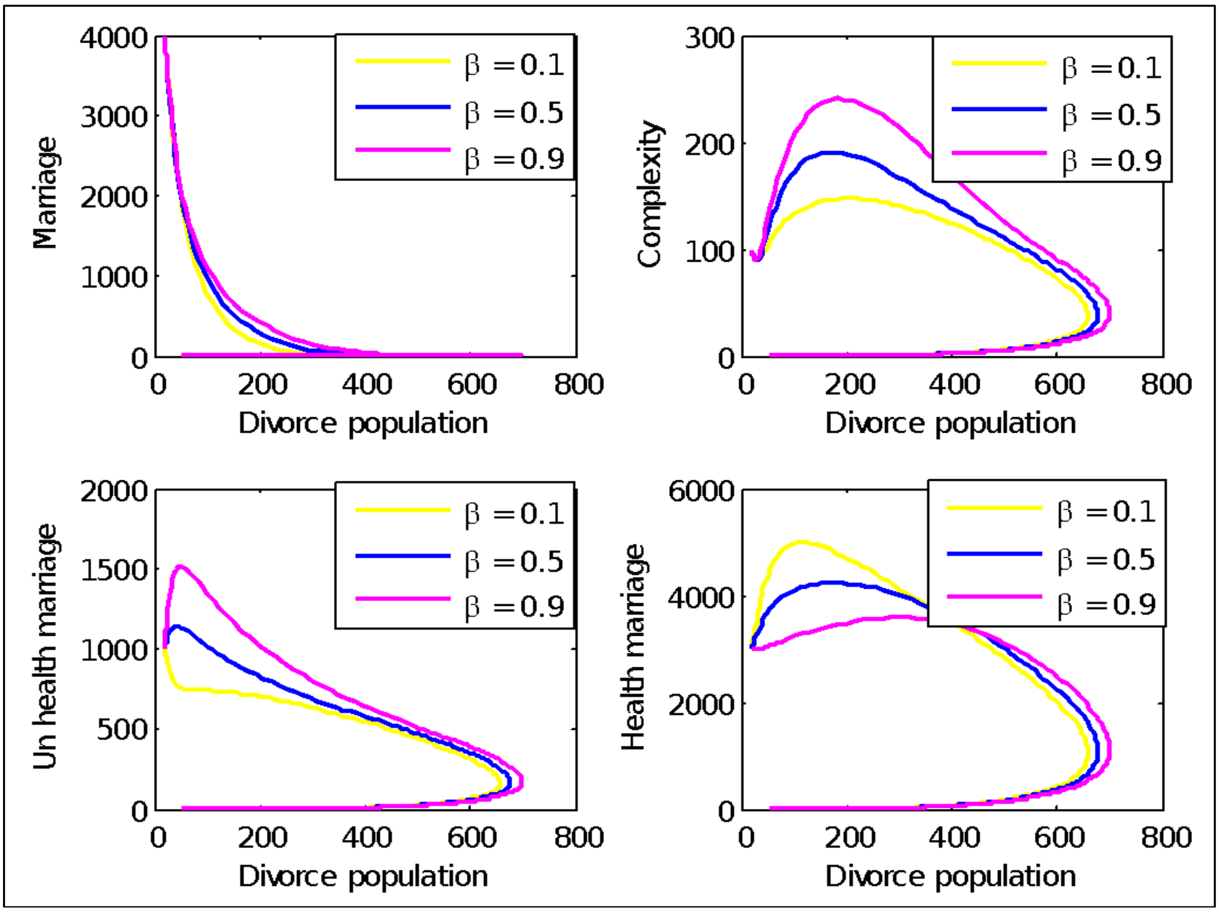

Fig 2: The impact of marriage hardship on the population

Figure 3 presents the rate of the status of individual behaviors after attaining counselling. It observed that when counselling is applied to the complexity and unhealthy marriage tends to reduce the number of divorce population and number of healthy marriage population increases. This proves that counselling is a powerful measure or strategy in minimizing the number of divorce individuals.

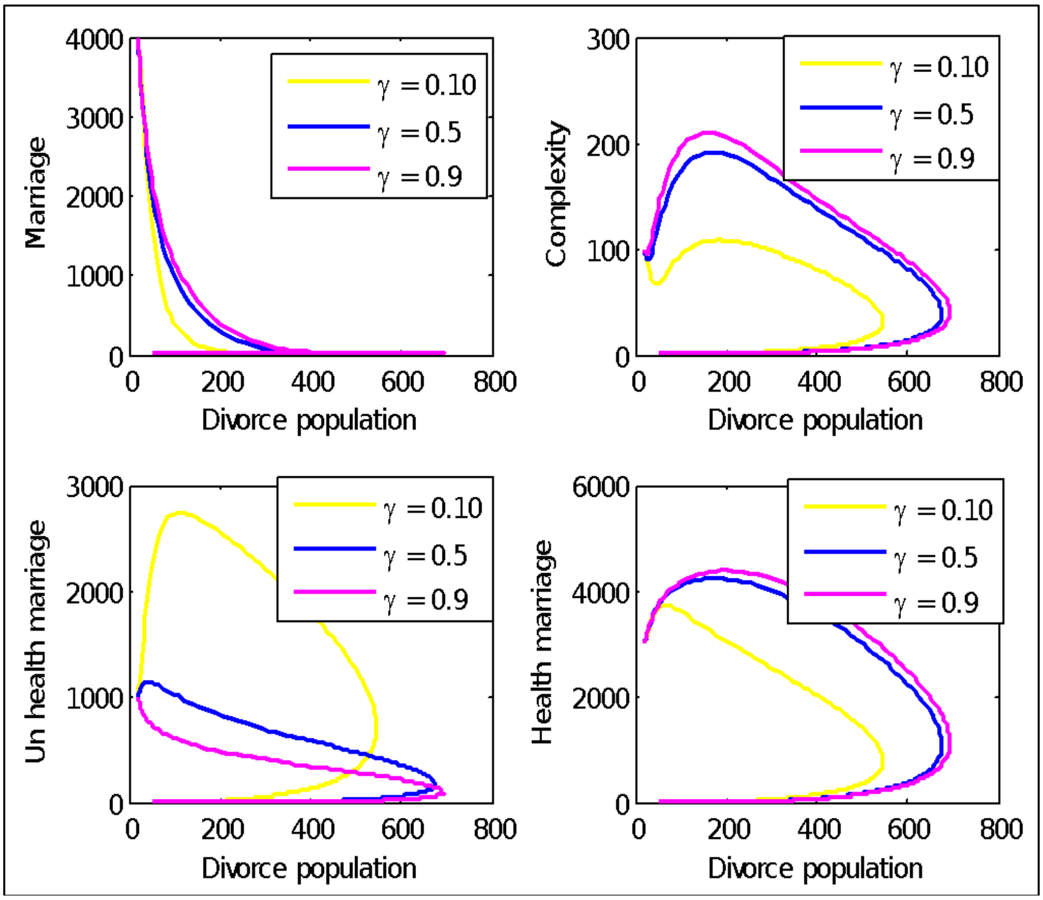

Fig 3: The impact of counselling among married individuals

Figure 4, depicts that as the divorce rate increases tend to increase unhealthy marriage, the complexity of life as well as the number of people who go into marriage decreases.

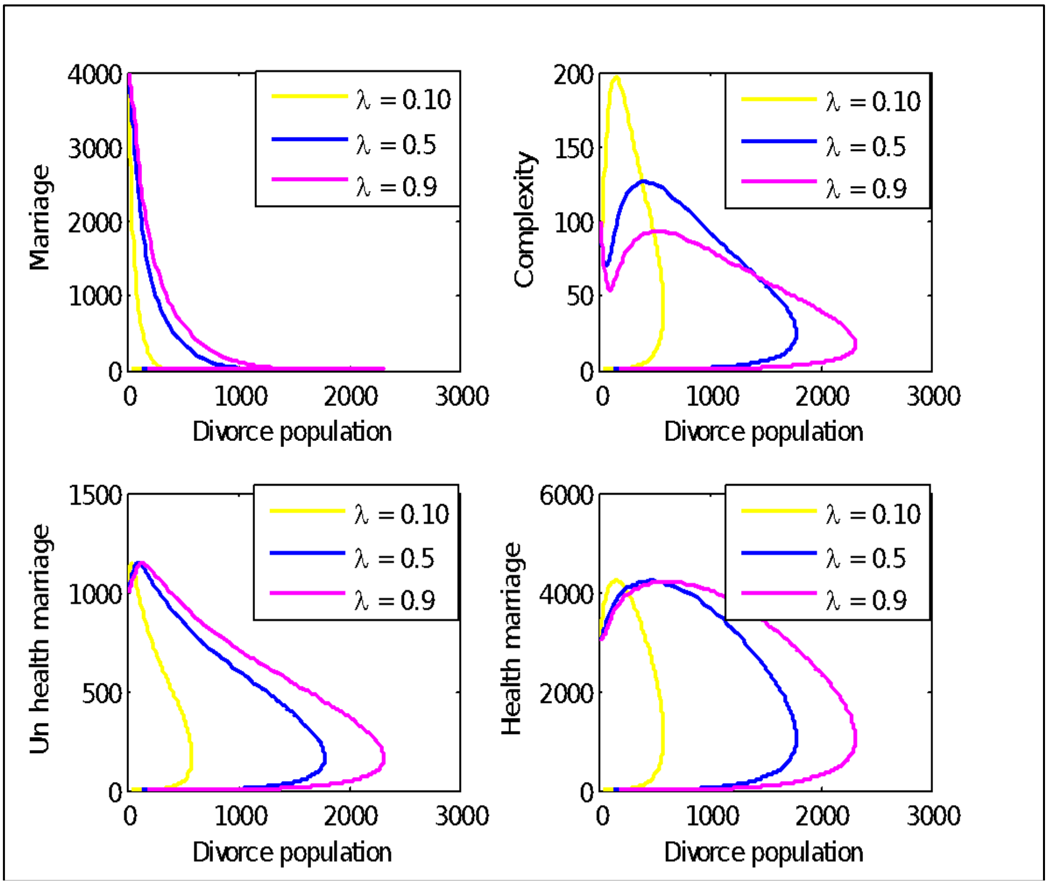

Fig 4: The impact of the varying divorce rate among the marriage classes

4. Conclusion

In this paper, we have developed a deterministic model that describes the dynamics of divorce among marriage based on the context of Tanzania. The systems were proven to be locally and globally asymptotically stable at both equilibrium points. The counselling control strategy has been introduced to the model to determine the strategy that reduces divorce in the population. The numerical results indicated that as the rate of counselling increases ultimately increase the number of healthy marriage and lower unhealthy marriage as well as complexity and divorce classes respectively. The study reveals that divorces may be minimized through counselling. Therefore, we advise the community members to have regular counselling whenever; there is a misunderstanding in the family which helps to motivate healthy marriage and happy life among the marriages.

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