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Inference for the generalized inverse Lindley distribution under type-II censored data

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Abstract

In this paper, we consider the classical and the Bayesian inferences for the generalized inverse Lindley (GIL) distribution and their corresponding reliability characteristics (reliability function and hazard rate function) under the type-II censoring scheme. In the classical setup, first we obtain the maximum likelihood estimator for the unknown parameters of the distribution and their corresponding reliability characteristics. Further, we consider symmetric (squared error) and asymmetric (LINEX and general entropy) loss functions for the estimation of parameters and their corresponding reliability characteristics under the Bayesian paradigm. The performances of various derived estimators were recorded using Markov chain Monte Carlo (MCMC) simulation technique in Open BUGS for different sample sizes under type-II censoring schemes. Finally, a real data set is provided to illustrate the computation of various estimators.

Keywords: Generalized inverse Lindley distribution, Type-II censoring, maximum likelihood estimator (MLE), Bayesian estimation, Symmetric and asymmetric loss functions, MCMC

Introduction

In the random phenomenon of life, there are several life-time models are available for analysing uncertainty of various fields. Initially, exponential distribution played an extensive role to analyse the life-time data because of its simplicity and analytical flexibility. Although, the exponential distribution has limitations in the study of life-time models due to its constant hazard rate, which is not appropriate to analyse the life-time models. Therefore, many of the researchers have proposed new life-time distributions which overcome the limitation of constant hazard rate. Such few specific distributions are Weibull distribution, gamma distribution, Lindley distribution, lognormal distribution which are extension of exponential distribution. The Lindley distribution was proposed by Lindley (1958) ^[17], in the context of Bayes theorem as a counter example of fiducial statistics and also shows that non constant hazard rate distribution, Ghitany *et al.* (2008) ^[8] worked on statistical properties of Lindley distribution. Nadrajadeh *et al.* (2011) and Ghitany *et al.* (2013) ^[9] proposed power Lindley and the extension of Lindley distribution which is also called generalized Lindley distribution respectively. Sharma *et al.* (2016) ^[21] extended the inverse Lindley distribution by applying the power transformation of given method by Gupta and Kundu (2009) ^[11] and found the generalized inverse Lindley (GIL) distribution. In the recent past, Kumar *et al.* (2020) ^[12] discussed GIL distribution based on generalized order statistics.

The probability density function (PDF) $f(x)$ and Cumulative distribution function (CDF) $F(x)$ of the GIL distribution are given, respectively, by

$$f(x; \alpha, \theta) = \frac{\alpha\theta^2}{(1+\theta)} \left[\frac{(1+x^\alpha)}{x^{2\alpha+1}} \right] e^{-\frac{\theta}{x^\alpha}}, x > 0, \sim\alpha, \theta > 0 \quad (1.1)$$

and

$$F(x) = \left[1 + \frac{\theta}{(1+\theta)} \frac{1}{x^\alpha} \right] e^{-\frac{\theta}{x^\alpha}} \quad (1.2)$$

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The corresponding reliability function $R(t)$ and hazard function $H(t)$ of this distribution at same time $t > 0$ are given, respectively, by

$$R(t) = 1 - \left[1 + \frac{\theta}{(1+\theta)} \frac{1}{t^\alpha} \right] e^{-\frac{\theta}{t^\alpha}}, \tag{1.3}$$

And

$$H(t) = \frac{\alpha\theta^2(1+t^\alpha)e^{-\frac{\theta}{t^\alpha}}}{t^{\alpha+1} \left[(1+\theta)t^\alpha \left(e^{-\frac{\theta}{t^\alpha}} - 1 \right) - \theta \right]} \tag{1.4}$$

Where, α is a shape parameter and θ is a scale parameter of the GIL distribution.

In any life-testing experiments, it is very cumbersome to complete the experiment for a long time period due to time and cost constraints. There are various types of censoring schemes which have been introduced in the literature to reduce the time and cost involved into the experiments. Type-I (time) and Type-II (failure) are the most common censoring schemes among the various censoring schemes. The number of observed failures in the type-I censoring scheme are random in nature, whereas the termination time of the experiments is random in the type-II censoring scheme. In this censoring, we put n items on test and the test is terminated after r failure observations which is pre-defined. The type-I and type-II censoring schemes was discussed by Nelson (2003) [19] and Lawless (2011) [16] for the problem of estimation. Singh *et al.* (2005) [22] and Kundu and Howlader (2010) [15] discussed different lifetime models under type-II censoring for the parameter estimation in the Bayesian inference. Teena *et al.* (2019) discussed a new lifetime model in both classical and Bayesian technique under type-II censored data.

The rest of the paper is organized as follows, In Section 2, the Maximum likelihood estimations are obtained for the parameters and the reliability characteristics of GIL distribution are presented. The Bayes estimation under symmetric (squared error) loss function and asymmetric (LINEX, general entropy) loss functions are obtained in Section 3. In Section 4, interval estimation under type-II censoring scheme for classical (confidence interval) and the Bayesian (credible and HPD intervals) paradigms. A simulation study is presented to report the performances of the various estimators in section 5. In Section 6, a real data set is provided to illustrate the computation of various estimators. Lastly, the conclusions appear in section 7.

Maximum Likelihood Function

Suppose n items are put on test and a test is terminated after the first r ordered observations are recorded. Let $X_{(1)} \leq X_{(2)} \leq X_{(3)} \leq \dots \leq X_{(r)}, 0 < r < n$, be the failure time of first r ordered observations and the time started at $X_{(1)} \geq 0$. Obviously $(n - r)$ items survived until $X_{(r)}$. The likelihood function based on the type-II censored sample is given by-

$$l(\alpha, \theta|X) = \frac{n!}{(n-r)!} \prod_{i=1}^r f(x_{(i)}; \alpha, \theta) [1 - F(x_{(r)}; \alpha, \theta)]^{n-r} \tag{2.1}$$

Substituting Equations (1.1) and (1.2) into Equation (2.1), the likelihood function is-

$$l(\alpha, \theta|X) = \frac{n!}{(n-r)!} \prod_{i=1}^r \left[\frac{\alpha\theta^2}{(1+\theta)} \left(\frac{1+x_{(i)}^\alpha}{x_{(i)}^{2\alpha+1}} \right) e^{-\frac{\theta}{x_{(i)}^\alpha}} \right] \left[1 - \left(1 + \frac{\theta}{(1+\theta)} \frac{1}{x_{(r)}^\alpha} \right) e^{-\frac{\theta}{x_{(r)}^\alpha}} \right]^{n-r} \tag{2.2}$$

The log-likelihood function can be written as

$$L = \ln l(\alpha, \theta|\mathbf{x})$$

$$= A + r \ln \alpha \theta^2 - r \ln(1 + \theta) + \sum_{i=1}^r (1 + x_{(i)}^\alpha) - (2\alpha + 1) \sum_{i=1}^r \ln x_{(i)} - \theta \sum_{i=1}^r x_{(i)}^{-\alpha} + (n - r) \ln \left[1 - \left(1 + \frac{\theta}{(1+\theta)} \frac{1}{x_{(r)}^\alpha} \right) e^{-\frac{\theta}{x_{(r)}^\alpha}} \right] \tag{2.3}$$

where,

$$A = \ln \left(\frac{n!}{(n-r)!} \right)$$

Further, partial derivative of the Equation (2.3) under the parameter α and θ and equate as zero to find the ML estimates of the unknown parameters as

$$\frac{\partial L}{\partial \alpha} = \frac{r}{\alpha} + \sum_{i=1}^r \frac{\ln(x_{(i)}) x_{(i)}^\alpha}{(1 + x_{(i)}^\alpha)} - 2 \sum_{i=1}^r \ln x_{(i)} + \theta \sum_{i=1}^r \ln x_{(i)} x_{(i)}^{-\alpha}$$

$$-\frac{(n-r)e^{-\frac{\theta}{x_{(r)}^\alpha} \ln(x_{(r)})} (x_{(r)}^\alpha + 1) \theta^2 \ln x_{(r)}^{-2\alpha}}{(1+\theta)} \tag{2.4}$$

And

$$\frac{\partial L}{\partial \theta} = \frac{2r}{\theta} - \frac{r}{(1+\theta)} - \sum_{i=1}^r x_{(i)}^{-\alpha} + \frac{(n-r)e^{-\frac{\theta}{x_{(r)}^\alpha} \ln(x_{(r)})} (1+\theta+x_{(r)}^\alpha(2+\theta)) \ln x_{(r)}^{-2\alpha}}{(1+\theta)^2} \tag{2.5}$$

From the expressions (2.4) and (2.5), it is clear that the likelihood equations do not yield the MLEs of α and θ because of its implicit form. Therefore, the MLEs of the parameters cannot be obtained analytically. Thus, one may use any numerical approximation techniques, such as, Newton-Raphson (N-R) method, fixed point iterations, etc. At present, we have used N-R method to evaluate the MLE of the parameters.

Following are the expressions for the MLEs of reliability characteristics:

$$\widehat{R}(t) = 1 - \left[1 + \frac{\widehat{\theta}}{(1+\widehat{\theta})} \frac{1}{t^{\widehat{\alpha}}} \right] e^{-\frac{\widehat{\theta}}{t^{\widehat{\alpha}}}} \tag{2.6}$$

and

$$\widehat{H}(t) = \frac{\widehat{\alpha} \widehat{\theta}^2 (1+t^{\widehat{\alpha}}) e^{-\frac{\widehat{\theta}}{t^{\widehat{\alpha}}}}}{t^{\widehat{\alpha}+1} \left[(1+\widehat{\theta}) t^{\widehat{\alpha}} \left(e^{-\frac{\widehat{\theta}}{t^{\widehat{\alpha}}}} - 1 \right) - \widehat{\theta} \right]} \tag{2.7}$$

Bayesian Inference

In this section, we obtain Bayes estimation of unknown parameters of the distribution and their reliability characteristics under the symmetric (squared error) and asymmetric (LINEX and general entropy) loss functions. Some authors used symmetric/asymmetric or both loss functions for parameter estimation in Bayesian inference (see, Soliman *et al.* (2012) [24], Soliman *et al.* (2013) [25] and Goyal *et al.* (2019) [10]).

For GIL distribution, we consider both parameters shape as well as scale are unknown. To estimate these parameters, we take shape parameter α as informative prior distribution Gamma, i.e., $\alpha \sim \text{Gamma}(a, b)$ and non-informative prior distribution for scale parameter θ . Therefore, the prior distribution for α is

$$\pi(\alpha) = \frac{a^b \alpha^{b-1} e^{-a\alpha}}{\Gamma b}, \alpha > 0, a, b > 0 \tag{3.1}$$

and the prior distribution for θ is,

$$\pi(\theta) = \frac{1}{\theta}, \theta > 0. \tag{3.2}$$

The joint prior of α and θ is,

$$\pi(\alpha, \theta) = \frac{a^b \alpha^{b-1} e^{-a\alpha}}{\theta \Gamma b}, \alpha, \theta > 0, a, b > 0 \tag{3.3}$$

Combining the likelihood function (2.2) and prior density (3.3), the posterior density is

$$\Pi(\alpha, \theta | x) = \frac{\pi(\alpha, \theta) l(\alpha, \theta | x)}{\int_0^\infty \int_0^\infty \pi(\alpha, \theta) l(\alpha, \theta | x) d\alpha d\theta} \tag{3.4}$$

$$\Pi(\alpha, \theta | x) = \frac{K_1}{K_0}$$

where,

$$K_1 = \prod_{i=1}^r \left[\frac{\alpha \theta^2}{(1+\theta)} \left(\frac{1+x_{(i)}^\alpha}{x_{(i)}^{2\alpha+1}} \right) e^{-\frac{\theta}{x_{(i)}^\alpha}} \right] \left[1 - \left(1 + \frac{\theta}{(1+\theta)} \frac{1}{x_{(r)}^\alpha} \right) e^{-\frac{\theta}{x_{(r)}^\alpha}} \right]^{n-r} \frac{a^b \alpha^{b-1} e^{-a\alpha}}{\theta \Gamma b}$$

&

$$K_0 = \int_0^\infty \int_0^\infty K_1 d\alpha d\theta$$

Bayesian estimate under symmetric loss function

Squared Error Loss Function (SELF): In the SELF, the magnitude of underestimation and overestimation are equal. It is also known as Quadratic loss function. In SELF, the Bayes estimator is represented by the posterior mean.

The squared error loss function is defined as

$$L(\hat{\delta}, \delta) = (\hat{\delta} - \delta)^2, \hat{\delta} \in D, \delta \in \Theta,$$

where, $\hat{\delta}$ is the Bayes estimator of δ . D is a decision space and Θ is parameter space.

The Bayes estimators $\tilde{\alpha}_{BS}$ and $\tilde{\theta}_{BS}$ of parameters α and θ , respectively, are

$$\tilde{\alpha}_{BS} = \int_0^\infty \int_0^\infty \alpha \Pi(\alpha, \theta | x) d\alpha d\theta$$

$$\tilde{\alpha}_{BS} = \frac{1}{K_0} \int_0^\infty \alpha \left(\int_0^\infty K_1 d\theta \right) d\alpha \quad (3.1.1)$$

and

$$\tilde{\theta}_{BS} = \int_0^\infty \int_0^\infty \theta \Pi(\alpha, \theta | x) d\alpha d\theta$$

$$\tilde{\theta}_{BS} = \frac{1}{K_0} \int_0^\infty \theta \left(\int_0^\infty K_1 d\alpha \right) d\theta. \quad (3.1.2)$$

The Bayes estimator \tilde{R}_{BS} and \tilde{H}_{BS} of the reliability function $R(t)$ and hazard rate function $H(t)$, respectively, are

$$\tilde{R}_{BS} = \int_0^\infty \int_0^\infty \left(1 - \left[1 + \frac{\theta}{(1+\theta)t^\alpha} \right] e^{-\frac{\theta}{t^\alpha}} \right) \Pi(\alpha, \theta) d\alpha d\theta$$

$$\tilde{R}_{BS} = \frac{1}{K_0} \int_0^\infty \int_0^\infty \left(1 - \left[1 + \frac{\theta}{(1+\theta)t^\alpha} \right] e^{-\frac{\theta}{t^\alpha}} \right) K_1 d\alpha d\theta \quad (3.1.3)$$

and

$$\tilde{H}_{BS} = \int_0^\infty \int_0^\infty \left(\frac{\alpha \theta^2 (1+t^\alpha) e^{-\frac{\theta}{t^\alpha}}}{t^{\alpha+1} \left[(1+\theta)t^\alpha \left(e^{-\frac{\theta}{t^\alpha}} - 1 \right) - \theta \right]} \right) \Pi(\alpha, \theta | x) d\alpha d\theta$$

$$\tilde{H}_{BS} = \frac{1}{K_0} \int_0^\infty \int_0^\infty \left(\frac{\alpha \theta^2 (1+t^\alpha) e^{-\frac{\theta}{t^\alpha}}}{t^{\alpha+1} \left[(1+\theta)t^\alpha \left(e^{-\frac{\theta}{t^\alpha}} - 1 \right) - \theta \right]} \right) K_1 d\alpha d\theta \quad (3.1.4)$$

Bayesian estimate under asymmetric loss function

When the magnitude of overestimation and underestimation are not equal then we used the asymmetric loss function. In the asymmetric loss function, we consider LINEX loss function and General Entropy loss function.

LINEX Loss function: The LINEX loss function is defined as

$$L(\hat{\delta} - \delta) \propto e^{c(\hat{\delta} - \delta)} - c(\hat{\delta} - \delta) - 1, c \neq 0, \hat{\delta} \in D, \delta \in \Theta.$$

The Bayes estimator $\hat{\delta}_{BL}$ of δ under the LINEX loss function is

$$\hat{\delta}_{BL} = -\frac{1}{c} \ln [E_\delta(\exp(-c\delta))], \quad (3.2.1)$$

provided, $E_\delta(\exp(-c\delta))$ exists and finite.

The Bayes estimators $\tilde{\alpha}_{BL}$ and $\tilde{\theta}_{BL}$ of parameters α and θ under LINEX loss function Equation (3.2.1), respectively, are

$$\tilde{\alpha}_{BL} = -\frac{1}{c} \ln \left[\int_0^\infty \int_0^\infty \exp(-c\alpha) \Pi(\alpha, \theta | x) d\alpha d\theta \right]$$

$$\tilde{\alpha}_{BL} = -\frac{1}{c} \ln \left[\frac{1}{K_0} \int_0^\infty \exp(-c\alpha) \left(\int_0^\infty K_1 d\alpha \right) d\theta \right] \quad (3.2.2)$$

and

$$\tilde{\theta}_{BL} = -\frac{1}{c} \ln \left[\int_0^\infty \int_0^\infty \exp(-c\theta) \Pi(\alpha, \theta | x) d\alpha d\theta \right]$$

$$\tilde{\theta}_{BL} = -\frac{1}{c} \ln \left[\frac{1}{K_0} \int_0^\infty \exp(-c\theta) \left(\int_0^\infty K_1 d\theta \right) d\alpha \right] \quad (3.2.3)$$

The Bayes estimators \tilde{R}_{BL} and \tilde{H}_{BL} of the reliability function $R(t)$ and hazard function $H(t)$, respectively, are

$$\begin{aligned} \tilde{R}_{BL} &= -\frac{1}{c} \ln \left[\int_0^\infty \int_0^\infty \exp \left(-c \left(1 - \left[1 + \frac{\theta}{(1+\theta)} \frac{1}{t^\alpha} \right] e^{-\frac{\theta}{t^\alpha}} \right) \right) \Pi(\alpha, \theta | x) d\alpha d\theta \right] \\ \tilde{R}_{BL} &= -\frac{1}{c} \ln \left[\frac{1}{K_0} \int_0^\infty \int_0^\infty \exp \left(-c \left(1 - \left[1 + \frac{\theta}{(1+\theta)} \frac{1}{t^\alpha} \right] e^{-\frac{\theta}{t^\alpha}} \right) \right) K_1 \right] d\alpha d\theta \end{aligned} \quad (3.2.4)$$

and

$$\begin{aligned} \tilde{H}_{BL} &= -\frac{1}{c} \ln \left[\int_0^\infty \int_0^\infty \exp \left(-c \left(\frac{\alpha \theta^2 (1+t^\alpha) e^{-\frac{\theta}{t^\alpha}}}{t^{\alpha+1} \left[(1+\theta) t^\alpha \left(e^{-\frac{\theta}{t^\alpha}} - 1 \right) - \theta \right]} \right) \right) \right] \Pi(\alpha, \theta | x) d\alpha d\theta \\ \tilde{H}_{BL} &= -\frac{1}{c} \ln \left[\frac{1}{K_0} \int_0^\infty \int_0^\infty \exp \left(-c \left(\frac{\alpha \theta^2 (1+t^\alpha) e^{-\frac{\theta}{t^\alpha}}}{t^{\alpha+1} \left[(1+\theta) t^\alpha \left(e^{-\frac{\theta}{t^\alpha}} - 1 \right) - \theta \right]} \right) \right) K_1 \right] d\alpha d\theta. \end{aligned} \quad (3.2.5)$$

General Entropy Loss Function: The General Entropy loss function is defined as

$$L(\hat{\delta}, \delta) \propto \left(\frac{\hat{\delta}}{\delta} \right)^q - q \ln \left(\frac{\hat{\delta}}{\delta} \right) - 1, \quad q \neq 0, \hat{\delta} \in D, \delta \in \Theta.$$

The Bayes estimator $\hat{\delta}_{BG}$ of δ under GE loss function is

$$\hat{\delta}_{BG} = [E_\delta(\delta^{-q})]^{-\frac{1}{q}}, \quad (3.2.6)$$

provided, $E_\delta(\delta^{-q})$ exists and finite.

The Bayes estimators $\tilde{\alpha}_{BG}$ and $\tilde{\theta}_{BG}$ of parameters α and θ under General Entropy loss function Equation (3.2.2), respectively, are

$$\begin{aligned} \tilde{\alpha}_{BG} &= \left[\int_0^\infty \int_0^\infty (\alpha^{-q}) \Pi(\alpha, \theta | x) d\alpha d\theta \right]^{-\frac{1}{q}} \\ \tilde{\alpha}_{BG} &= \left[\frac{1}{K_0} \int_0^\infty \alpha^{-q} \left(\int_0^\infty K_1 d\theta \right) d\alpha \right]^{-\frac{1}{q}} \end{aligned} \quad (3.2.7)$$

and

$$\begin{aligned} \tilde{\theta}_{BG} &= \left[\int_0^\infty \int_0^\infty (\theta^{-q}) \Pi(\alpha, \theta | x) d\alpha d\theta \right]^{-\frac{1}{q}} \\ \tilde{\theta}_{BG} &= \left[\frac{1}{K_0} \int_0^\infty \theta^{-q} \left(\int_0^\infty K_1 d\alpha \right) d\theta \right]^{-\frac{1}{q}} \end{aligned} \quad (3.2.8)$$

The Bayes estimators \tilde{R}_{BG} and \tilde{H}_{BG} of the reliability function $R(t)$ and hazard function $H(t)$, respectively, are

$$\begin{aligned} \tilde{R}_{BG} &= \left[\int_0^\infty \int_0^\infty \left(1 - \left[1 + \frac{\theta}{(1+\theta)} \frac{1}{t^\alpha} \right] e^{-\frac{\theta}{t^\alpha}} \right)^{-q} \Pi(\alpha, \theta | x) d\alpha d\theta \right]^{-\frac{1}{q}} \\ \tilde{R}_{BG} &= \left[\frac{1}{K_0} \int_0^\infty \int_0^\infty \left(1 - \left[1 + \frac{\theta}{(1+\theta)} \frac{1}{t^\alpha} \right] e^{-\frac{\theta}{t^\alpha}} \right)^{-q} K_1 d\alpha d\theta \right]^{-\frac{1}{q}} \end{aligned} \quad (3.2.9)$$

and

$$\begin{aligned} \tilde{H}_{BG} &= \left[\int_0^\infty \int_0^\infty \left(\frac{\alpha \theta^2 (1+t^\alpha) e^{-\frac{\theta}{t^\alpha}}}{t^{\alpha+1} \left[(1+\theta) t^\alpha \left(e^{-\frac{\theta}{t^\alpha}} - 1 \right) - \theta \right]} \right)^{-q} \Pi(\alpha, \theta | x) d\alpha d\theta \right]^{-\frac{1}{q}} \\ \tilde{H}_{BG} &= \left[\frac{1}{K_0} \int_0^\infty \int_0^\infty \left(\frac{\alpha \theta^2 (1+t^\alpha) e^{-\frac{\theta}{t^\alpha}}}{t^{\alpha+1} \left[(1+\theta) t^\alpha \left(e^{-\frac{\theta}{t^\alpha}} - 1 \right) - \theta \right]} \right)^{-q} K_1 d\alpha d\theta \right]^{-\frac{1}{q}} \end{aligned} \tag{3.2.10}$$

Equations (3.1.1) to (3.1.4), Equations (3.2.2) to (3.2.5) and Equations (3.2.7) to (3.2.10) are not computed analytically. Therefore, for these kinds of equations, we generate the samples from equation (3.4) from one of the simulation technique Markov Chain Monte Carlo (MCMC) and compute these Bayes estimators under symmetric and asymmetric loss functions (see, El-Din *et al.* (2017) ^[6], Riad *et al.* (2020) ^[20] and Almongy *et al.* (2021) ^[2]).

Interval estimation under type-II censoring scheme

In this section, we deal with the confidence intervals (CIs) for the parameters under the classical setup. In the Bayesian paradigm, we obtained credible intervals and highest posterior density (HPD) for the parameters. The intervals under classical and Bayesian setup are as follows

Confidence interval

In the classical setup, the CIs can be obtained the diagonal elements of the inverse Fisher information matrix $I^{-1}(\hat{\alpha}, \hat{\theta})$ that gives the asymptotic variance for the parameters α and θ respectively. Thus, the two sided $100(1 - \eta)\%$ confidence interval for α and θ can be defined respectively as

$$\left[\hat{\alpha} - Z_{\eta/2} \sqrt{var(\hat{\alpha})}, \hat{\alpha} + Z_{\eta/2} \sqrt{var(\hat{\alpha})} \right],$$

and

$$\left[\hat{\theta} - Z_{\eta/2} \sqrt{var(\hat{\theta})}, \hat{\theta} + Z_{\eta/2} \sqrt{var(\hat{\theta})} \right].$$

Where, $Z_{\eta/2}$ is a standard normal variate.

The Fisher information matrix can be defined as

$$I(\hat{\alpha}, \hat{\theta}) = \begin{bmatrix} -\frac{\partial^2 L}{\partial \alpha^2} & -\frac{\partial^2 L}{\partial \alpha \partial \theta} \\ -\frac{\partial^2 L}{\partial \theta \partial \alpha} & -\frac{\partial^2 L}{\partial \theta^2} \end{bmatrix}_{(\hat{\alpha}, \hat{\theta})}$$

where,

$$\frac{\partial^2 L}{\partial \alpha^2} = -\frac{r}{\alpha^2} + \sum_{i=1}^r \frac{\ln^2(x_{(i)}) x_{(i)}^\alpha}{(1+x_{(i)}^\alpha)^2} - \theta \sum_{i=1}^r \ln^2 x_{(i)} x_{(i)}^{-\alpha} + \frac{(n-r) e^{-\frac{\theta}{x_{(r)}^\alpha}} \ln(x_{(r)})^2 (x_{(r)}^{2\alpha} - x_{(r)}^\alpha (\theta - 2) - \theta) \theta^2 \ln x_{(r)}^{-2\alpha}}{(1+\theta)} \tag{4.1.1}$$

$$\frac{\partial^2 L}{\partial \theta^2} = -\frac{2r}{\theta^2} + \frac{r}{(1+\theta)^2} + \frac{(n-r) e^{-\frac{\theta}{x_{(r)}^\alpha}} \theta (2x_{(r)}^{2\alpha} - \theta(1+\theta)^2 - x_{(r)}^\alpha (3\theta^2 + \theta^2 + \theta - 1)) \ln x_{(r)}^{-2\alpha}}{(1+\theta)^2} \tag{4.1.2}$$

and

$$\frac{\partial^2 L}{\partial \alpha \partial \theta} = \frac{\partial^2 L}{\partial \theta \partial \alpha} = \sum_{i=1}^r \ln x_{(i)} x_{(i)}^{-\alpha} + \frac{(n-r) e^{-\frac{\theta}{x_{(r)}^\alpha}} \ln(x_{(r)}) (x_{(r)}^\alpha + 1) \theta (\theta(1+\theta) - x_{(r)}^\alpha (2+\theta)) \ln x_{(r)}^{-2\alpha}}{(1+\theta)^2} \tag{4.1.3}$$

4.2 Credible interval and HPD interval

In the Bayesian paradigm, let parameter τ is a random variable and the probability for this parameter τ lies within the specified intervals. The credible and the highest posterior density (HPD) intervals discussed by Edwards *et al.* (1963) ^[6]. All the Bayesian credible intervals, the HPD interval is the shortest interval. The HPD interval for parameter τ based on the simulation method MCMC samples, i.e., $\tau_{(1)}, \tau_{(2)}, \dots, \tau_{(M)}$ discussed by Chen and Saho (1999) ^[5]. For the parameter τ , the credible interval $100(1 - \eta)\%$ is obtained as

$$((\tau_{-}\{(1)\}, \tau_{-}\{(1 - \eta)M + 1\}), \dots, (\tau_{-}\{M\eta\}, \tau_{-}\{M\})),$$

where, $[K]$ defines the largest integer value which is less than or equal to K . Therefore, for the parameter τ , the shortest length interval is the HPD interval. There are few researchers who discussed HPD interval in very detailed form (see, Edward *et al.* (1963), Box and Tiao (1973) ^[3] and Sinha (1987) ^[23]).

Simulation study

In this section, we perform Markov chain Monte Carlo (MCMC) simulation method for the GIL distribution under the type-II censored sample with the help of OpenBUGS software (see, Thomas *et al.* (2006) ^[28], Thomas (2010) ^[27] and Kumar *et al.* (2010) ^[14]). This simulation study is conducted to measure the performances of various estimators obtained in this article. In the classical setup, the estimators are measured by MLE with their mean square error (MSE) and the estimators along with their MSEs under different two loss functions which are symmetric (squared error) and asymmetric (LINEX, general entropy) loss functions are obtained in the Bayesian paradigm. In the interval estimation, the confidence interval in the classical setup and the credible and HPD intervals under the Bayesian paradigm are also obtained. For the parameters $(\alpha, \theta) \sim (1.5, 1.5)$, we generate different posterior samples of the different values on n and r with the Gibbs sampling technique of MCMC method under the OpenBUGS software. Some of the researchers used MCMC technique by OpenBUGS software (see, Kumar *et al.* (2012) ^[13], Adegoke *et al.* (2018) ^[1], Chaudhary *et al.* (2020) ^[4] and Srivastava *et al.* (2020) ^[26]). After that, from those samples, we estimate the parameters of GIL distribution and their reliability characteristics under classical and Bayesian inference. In classical, we estimate MLE along with their MSEs and also estimates the parameters under two different loss functions in the Bayesian set up. The reliability characteristics estimates and their MSEs are obtained in the both inference methods. In Bayesian, there are two choices for asymmetric loss functions (general entropy, $(q = -2, q = 2)$ and LINEX $(c = -2, c = 2)$). For the interval estimation, 5% level of significance we calculated, confidence interval in classical and credible and HPD interval under the Bayesian paradigm.

Table 1: MLE and Bayes estimates of α with their MSEs under type-II censoring scheme with $(\alpha, \theta) \sim (1.5, 1.5)$.

n	r	MLE (MSE)	BS (MSE)	BG (MSE)		BL (MSE)	
				q = -2	q = 2	c = -2	c = 2
25	10	1.7261 (0.2566)	1.4745 (0.0012)	1.4117 (0.0086)	1.4936 (0.0057)	1.4179 (0.0075)	1.5311 (0.0015)
	15	1.6391 (0.1287)	1.0188 (0.2321)	0.9637 (0.2882)	1.0363 (0.2156)	0.9839 (0.2669)	1.0561 (0.1978)
	25	1.5812 (0.0729)	1.6682 (0.0287)	1.6138 (0.0134)	1.6859 (0.0349)	1.6108 (0.0127)	1.7295 (0.0532)
50	10	1.7587 (0.2935)	1.8992 (0.1597)	1.8810 (0.1456)	1.9052 (0.1645)	1.8766 (0.1422)	1.9221 (0.1785)
	15	1.6701 (0.1582)	2.0497 (0.3025)	2.0351 (0.2866)	2.0545 (0.3078)	2.0299 (0.2812)	2.0695 (0.3247)
	25	1.5857 (0.0619)	0.8604 (0.4117)	0.8198 (0.4633)	0.8748 (0.3955)	0.8364 (0.4441)	0.8888 (0.3831)
	50	1.5422 (0.0307)	1.5794 (0.0065)	1.5516 (0.0028)	1.5886 (0.0080)	1.5509 (0.0027)	1.6092 (0.0121)
75	10	1.8032 (0.3512)	2.0993 (0.3594)	2.0863 (0.3440)	2.1036 (0.3645)	2.0814 (0.3381)	2.1175 (0.3815)
	15	1.6813 (0.1544)	1.9041 (0.1634)	1.8966 (0.1574)	1.9065 (0.1654)	1.8946 (0.1558)	1.9134 (0.1711)
	25	1.5967 (0.0699)	2.0616 (0.3157)	2.0549 (0.3082)	2.0638 (0.3182)	2.0525 (0.3055)	2.0707 (0.3260)
	50	1.5413 (0.0282)	0.3199 (1.3937)	0.3053 (1.4272)	0.3298 (1.3746)	0.3136 (1.4074)	0.3368 (1.3665)
	75	1.5271 (0.0202)	1.7035 (0.0415)	1.6867 (0.0350)	1.70911 (0.0438)	1.6847 (0.0342)	1.7225 (0.0496)

BS= Squared error; BG= General entropy; BL= LINEX; MSE=Mean square error.

In order to calculate MSEs under classical and with different loss functions under the Bayesian paradigm, we have replicated our results 30000 times. All the results of the MCMC method are reported in Tables (1)-(6). From Tables (1) to (6), we see the different choices of (n, r) as (25,10), (25,15), (25,25), (50,10), (50,15), (50,25), (50,50), (75,10), (75,15), (75,25), (75,50) and (75,75). The parameters value of (α, θ) as (1.5,1.5) show the estimates for both the parameters (α, θ) along with their MSEs under classical and Bayesian paradigm (see Tables (1) and (2) respectively). In Tables (4) and (5) show the estimators of reliability characteristics $(R(t), H(t))$ at $t = 1$ along with their MSEs in classical and Bayesian paradigm respectively. In classical set up, the estimators of parameters (α, θ) and reliability characteristics $(R(t), H(t))$ along with their MSEs are obtained under type-II censoring. With the different loss functions (symmetric (squared error) and asymmetric (LINEX, general entropy)), the estimators of parameters (α, θ) and reliability characteristics $(R(t), H(t))$ along with their MSEs are obtained under type-II censoring scheme in the Bayesian paradigm. There are two choices for every asymmetric loss functions, for general entropy loss function $(q = -2, q = 2)$ and for LINEX loss function $(c = -2, c = 2)$. In the interval estimations, confidence intervals (lower limits and upper limits) also obtained in the classical setup. The credible and HPD intervals (lower limits and upper limits) also obtained in the Bayesian paradigm. Tables (5) and (6) show the interval estimation for both the parameters α and θ respectively.

Table 2: MLE and Bayes estimates of θ with their MSEs under type-II censoring scheme with $(\alpha, \theta) \sim (1.5, 1.5)$.

n	r	MLE(MSE)	BS (MSE)	BG (MSE)		BL (MSE)	
				q = -2	q = 2	c = -2	c = 2
25	10	1.4727 (0.1008)	1.9606 (0.2122)	1.9595 (0.2112)	1.9610 (0.2125)	1.9592 (0.2109)	1.9619 (0.2134)
	15	1.5125 (0.0792)	1.9706 (0.2215)	1.9701 (0.2209)	1.9708 (0.2217)	1.9699 (0.2208)	1.9713 (0.2222)
	25	1.5318 (0.0737)	1.5890 (0.0081)	1.5457 (0.0023)	1.6025 (0.0107)	1.5454 (0.0023)	1.6305 (0.0172)
50	10	1.4046 (0.1017)	1.9509 (0.2033)	1.9492 (0.2018)	1.9514 (0.2038)	1.9487 (0.2014)	1.9528 (0.2051)
	15	1.4561 (0.0605)	1.9763 (0.2268)	1.9758 (0.2264)	1.9764 (0.2269)	1.9757 (0.2263)	1.9768 (0.2273)
	25	1.5018 (0.0387)	1.9937 (0.2437)	1.9937 (0.2437)	1.9938 (0.2438)	1.9937 (0.2437)	1.9938 (0.2438)
	50	1.5197 (0.0335)	1.3468 (0.0236)	1.3216 (0.0139)	1.3554 (0.0211)	1.3245 (0.0309)	1.3707 (0.0169)
75	10	1.3459 (0.1259)	1.9541 (0.2061)	1.9525 (0.2048)	1.9545 (0.2066)	1.9521 (0.2044)	1.9558 (0.2077)

15	1.4223 (0.0620)	1.9783 (0.2288)	1.9780 (0.2285)	1.9784 (0.2289)	1.9779 (0.2284)	1.9787 (0.2292)
25	1.4769 (0.0324)	1.9934 (0.2434)	1.9933 (0.2434)	1.9934 (0.2434)	1.9933 (0.2434)	1.9934 (0.2434)
50	1.5071 (0.0206)	1.9969 (0.2469)	1.9969 (0.2469)	1.9969 (0.2470)	1.9969 (0.2469)	1.9970 (0.2470)
75	1.5121 (0.0199)	1.1937 (0.0938)	1.1807 (0.1019)	1.1982 (0.0911)	1.1833 (0.1003)	1.2048 (0.0871)

From Tables (1)-(6), we conclude that

1. Tables (1) and (2) show the MSEs of both the parameters (α, θ) decreases when the different choices of (n, r) increases respectively for both classical and Bayesian inferences.
2. The Bayes estimator for LINEX loss function exhibits lower MSEs among other Bayes estimators for the parameters α and θ respectively [Tables (1) and (2)].
3. For the reliability characteristics, the MSEs for both reliability function $R(t)$ and hazard rate function $H(t)$ decreases in the increment of (n, r) with given different choices in Tables (3) and (4) respectively under both estimation methods (classical and Bayesian).
4. Tables (3) and (4) show the Bayes estimators for general entropy loss function exhibits lower MSEs for $R(t)$ and $H(t)$ among other Bayes estimators respectively.
5. Tables (5) and (6) show that the HPD interval length is smaller than other intervals length (confidence interval and credible interval) of both the parameters (α, θ) .

Table 3: MLEs and Bayes estimates of Reliability function $R(t)$ with their MSEs under type-II censoring scheme and $R(t = 1) = 0.6429$.

n	r	MLE (MSE)	BS (MSE)	BG (MSE)		BL (MSE)	
				q = -2	q = 2	c = -2	c = 2
25	10	0.6215 (0.0111)	0.7658 (0.0151)	0.7657 (0.1501)	0.7659 (0.0151)	0.7658 (0.0151)	0.7659 (0.0151)
	15	0.6372 (0.0075)	0.7681 (0.1565)	0.7680 (0.0156)	0.7681 (0.0156)	0.7680 (0.0156)	0.7681 (0.0156)
	25	0.6442 (0.0066)	0.6653 (0.0005)	0.6552 (0.0001)	0.6683 (0.0006)	0.6611 (0.0003)	0.6692 (0.0007)
50	10	0.5989 (0.0139)	0.7636 (0.0145)	0.7634 (0.0145)	0.7637 (0.0145)	0.7635 (0.0145)	0.7637 (0.0145)
	15	0.6210 (0.0072)	0.7693 (0.0159)	0.7693 (0.0159)	0.7694 (0.0159)	0.7693 (0.0159)	0.7694 (0.0159)
	25	0.6386 (0.0039)	0.7731 (0.0169)	0.7731 (0.0169)	0.7731 (0.0169)	0.7731 (0.2919)	0.7731 (0.0169)
	50	0.6451 (0.0031)	0.5875 (0.0030)	0.5798 (0.0040)	0.5901 (0.0028)	0.5846 (0.0034)	0.5905 (0.0027)
75	10	0.5768 (0.0191)	0.7642 (0.0146)	0.7642 (0.0146)	0.7644 (0.0147)	0.7643 (0.0147)	0.7644 (0.0147)
	15	0.6098 (0.0081)	0.7698 (0.0161)	0.7697 (0.0161)	0.7698 (0.0161)	0.7697 (0.0161)	0.7698 (0.0161)
	25	0.6313 (0.0036)	0.7730 (0.0169)	0.7730 (0.0169)	0.7730 (0.0169)	0.7730 (0.0169)	0.7730 (0.0169)
	50	0.6426 (0.0021)	0.7738 (0.0171)	0.7737 (0.0171)	0.7738 (0.0171)	0.7737 (0.0171)	0.7738 (0.0171)
	75	0.6443 (0.0019)	0.5304 (0.0126)	0.5257 (0.0137)	0.5321 (0.0123)	0.5287 (0.0131)	0.5322 (0.0122)

Table 4: MLEs and Bayes estimates of hazard rate function $H(t)$ with their MSEs under type-II censoring scheme and $H(t = 1) = 0.9369$.

n	r	MLE (MSE)	BS (MSE)	BG (MSE)		BL (MSE)	
				q = -2	q = 2	c = -2	c = 2
25	10	1.1316 (0.2409)	0.7041 (0.0543)	0.6728 (0.0699)	0.7137 (0.0499)	0.6905 (0.0608)	0.7177 (0.0481)
	15	1.0353 (0.1012)	0.4833 (0.2058)	0.4569 (0.2305)	0.4918 (0.1983)	0.4752 (0.2132)	0.4917 (0.1983)
	25	0.9832 (0.0604)	0.9990 (0.0041)	0.9382 (0.0002)	1.0202 (0.0072)	0.9594 (0.0007)	1.0456 (0.0121)
50	10	1.2120 (0.3545)	0.9125 (0.0007)	0.9011 (0.0014)	0.9165 (0.0005)	0.9055 (0.0011)	0.9199 (0.0004)
	15	1.0958 (0.1571)	0.9692 (0.0011)	0.9615 (0.0006)	0.9717 (0.0012)	0.9642 (0.0008)	0.9742 (0.0014)
	25	0.9999 (0.0503)	0.4024 (0.2862)	0.3834 (0.3064)	0.4092 (0.2795)	0.3969 (0.2919)	0.4286 (0.2803)
	50	0.9583 (0.0250)	1.0811 (0.0210)	1.0411 (0.0111)	1.0943 (0.0250)	1.0532 (0.0137)	1.1111 (0.0306)
75	10	1.2964 (0.5162)	1.0067 (0.0049)	0.9977 (0.0037)	1.0098 (0.0053)	1.0006 (0.0014)	1.0131 (0.0058)
	15	1.1278 (0.1771)	0.8991 (0.0014)	0.8951 (0.0017)	0.9005 (0.0013)	0.8967 (0.0016)	0.9016 (0.0012)
	25	1.0235 (0.0641)	0.9646 (0.0008)	0.9614 (0.0006)	0.9656 (0.0008)	0.9625 (0.0007)	0.9666 (0.0009)
	50	0.9637 (0.0220)	0.1494 (0.6205)	0.1425 (0.6311)	0.1541 (0.6141)	0.1476 (0.6231)	0.1525 (0.6162)
	75	0.9513 (0.0167)	1.2645 (0.1074)	1.2393 (0.0915)	1.2728 (0.1129)	1.2438 (0.0943)	1.2856 (0.1217)

Table 5: Classical and Bayesian Interval estimation for α under type-II censoring scheme.

n	r	Confidence interval		Credible interval		HPD interval	
		LL	UL	LL	UL	LL	UL
25	10	0.9423	2.5101	1.0130	1.9510	1.1080	1.9090
	15	1.0252	2.2531	0.6718	1.4220	0.6650	1.4100
	25	1.0956	2.0667	1.2120	2.1750	1.1930	2.1500
50	10	0.9563	2.5612	1.6020	2.2010	1.5900	2.1870
	15	1.0511	2.2891	1.7700	2.3270	1.7630	2.3170
	25	1.1287	2.0432	0.5938	1.1870	0.5816	1.1580
	50	1.2082	1.8762	1.2570	1.9320	1.2410	1.9150
75	10	0.9745	2.6320	1.8380	2.3670	1.8380	2.3660
	15	1.0535	2.3091	1.7090	2.0910	1.7130	2.0940
	25	1.1374	2.0560	1.8690	2.2550	1.8630	2.2460
	50	1.2234	1.8592	0.2343	0.4065	2.3190	0.3995
	75	1.2573	1.7971	1.4310	1.9730	1.4350	1.9760

LL= Lower limit; UL= Upper limit.

Table 6 Classical and Bayesian Interval estimation for θ under type-II censoring scheme.

n	r	Confidence interval		Credible interval		HPD interval	
		LL	UL	LL	UL	LL	UL
25	10	0.9215	2.0239	1.8610	1.9980	1.8840	2.0000
	15	1.0079	2.0171	1.8960	1.9990	1.9150	2.0000
	25	1.0436	2.0201	1.1710	1.9560	1.2190	1.9850
50	10	0.8628	1.9464	1.8310	1.9990	1.8590	2.0000
	15	1.0204	1.8917	1.9150	1.9990	1.9300	2.0000
	25	1.1360	1.8676	1.9780	2.0000	1.9810	2.0000
	50	1.1789	1.8605	1.0740	1.6700	1.0570	1.6480
75	10	0.7721	1.9197	1.8380	1.9990	1.8660	2.0000
	15	0.9775	1.8672	1.9250	1.9990	1.9380	2.0000
	25	1.1371	1.8168	1.9760	2.0000	1.9800	2.0000
	50	1.2239	1.7904	1.9890	2.0000	1.9910	2.0000
	75	1.2356	1.7886	1.0200	1.4180	1.0010	1.3820

Real data analysis

In this section, we have considered a real data set from Wingo (1983) ^[29] which represents a clinical trial describe a relief time (in hours) for 50 arithmetic patients. The data set is-

0.70, 0.84, 0.58, 0.50, 0.55, 0.82, 0.59, 0.71, 0.72, 0.61, 0.62, 0.49, 0.54, 0.36, 0.36, 0.71, 0.35, 0.64, 0.84, 0.55, 0.59, 0.29, 0.75, 0.46, 0.46, 0.60, 0.60, 0.36, 0.52, 0.68, 0.80, 0.55, 0.84, 0.34, 0.34, 0.70, 0.49, 0.56, 0.71, 0.61, 0.57, 0.73, 0.75, 0.44, 0.44, 0.81, 0.80, 0.87, 0.29, 0.50.

In the terms of suitable fitting of the distribution, the GIL distribution is compared with other related distributions such as Inverse Lindley (IL) distribution, Generalized Lindley (GL) distribution, gamma distribution and Weibull distribution (see, Table 7). The considered real data set has been measured on the basis of -Log L which is the negative of the logarithmic value of likelihood and Kolmogrove-Smirnov (K-S) test statistic for the distribution selection criterion. We also used Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC). The AIC and BIC are defined as,

$$AIC = 2 \times k - 2 \times \log \hat{L}$$

$$BIC = k \times \log(n) - 2 \times \log \hat{L}$$

Where

k = Number of parameters,

n = Sample size, and,

\hat{L} = value of the maximum likelihood for the considered distribution.

The K-S test statistic D is defined as

$$D = \text{Sup}_x |F_n(x) - F(x)|,$$

Where

$F_n(x)$ = Empirical distribution function.

Table 7: ML estimates of the parameters, -Log L, K-S distance, AIC and BIC for the fitted models.

Models	Estimates		-Log L	K-S	AIC	BIC
	α	θ				
IL	-	0.7041	43.0361	0.3234	90.0722	93.8963
GL	0.3277	0.2947	94.7228	0.5560	193.4456	197.2697
GILD	2.2881	0.3707	4.1835	0.1559	12.367	16.1911
Gamma	0.4942	4.5608	79.3829	0.5119	162.7657	166.5898
Weibull	0.4371	2.2538	67.6233	0.8247	139.2466	143.0707

From the Table 7, we observe that the value of AIC, BIC and -Log L of the GIL distribution is minimum rather than the other distributions values. The minimum value of AIC, BIC and -Log L shows that the GIL distribution better fits for the considered real data set rather than the other distributions. We also observe that the real data set of 50 arithmetic patients support GIL distribution with the K-S distance as 0.1559 for $\alpha = 2.2881$ and $\theta = 0.3707$. The empirical cdf, Q-Q plot and P-P plot shows that GIL distribution fits best for the considered real data set (see, figure 1).

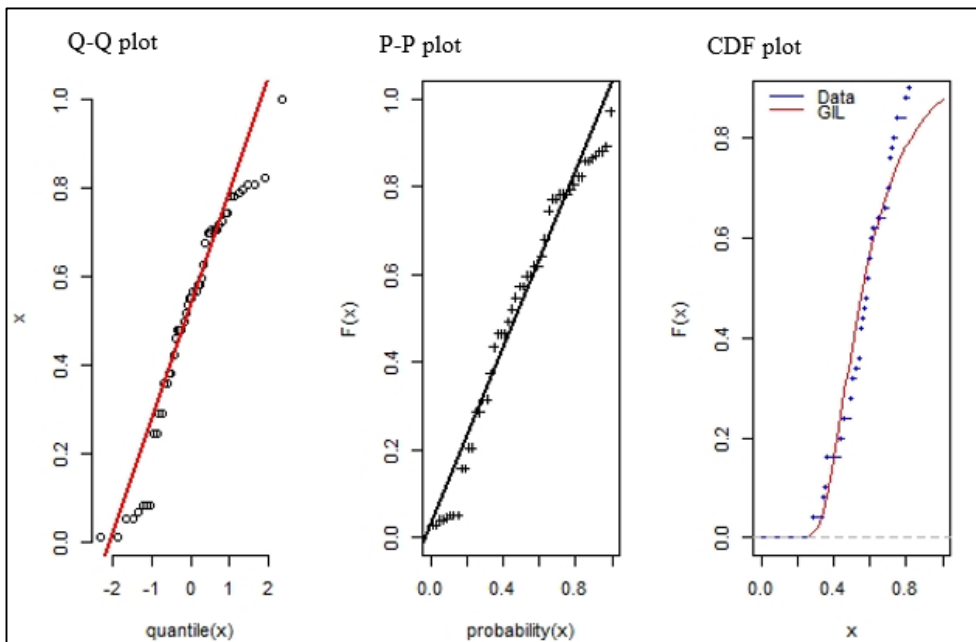


Fig 1: QQ-plot, PP-plot and empirical cdf plot of the GIL distribution for real data set.

Now, for the considered real data sets of 50 arithmetic patients, we generate the samples from type-II censoring scheme as done in the simulation study. For $n = 50$, we consider various combination of number of failures $r = 10, 25$ and 45 under type -II censoring scheme. For the real data set, we calculate the ML estimators and the Bayes estimators under symmetric and asymmetric loss functions. For asymmetric loss function, we consider $c = (-2, 2)$ in LINEX loss function and $q = (-2, 2)$ in general entropy loss functions. The calculated estimators of α and θ under type-II censoring for the real data set in Table 8 and Table 9 respectively. The reliability function $R(t)$ and hazard rate function $H(t)$ as $t = 1$ are given in Table 10 and Table 11 respectively for the real data set under type-II censoring.

Table 12 and Table 13 show the confidence intervals, credible intervals and HPD intervals for the parameter α and θ respectively at 5% level of significance.

Table 8: MLEs and Bayes estimates of α under type-II censoring scheme for the real data set.

n	r	MLE	BS	BG		BL	
				q = -2	q = 2	c = -2	c = 2
50	10	1.7587	1.8367	1.8185	1.8427	1.8148	1.8591
	25	1.5859	1.8603	1.8418	1.8664	1.8378	1.8828
	45	1.5449	1.7173	1.7091	1.7200	1.7080	1.7262

Table 9: MLEs and Bayes estimates of θ under type-II censoring scheme for the real data set.

n	r	MLE	BS	BG		BL	
				q = -2	q = 2	c = -2	c = 2
50	10	1.4046	1.4695	1.4468	1.4770	1.4477	1.4919
	25	1.5018	1.3231	1.3018	1.3303	1.3045	1.3426
	45	1.5190	1.0623	1.0583	1.0638	1.0593	1.0657

Table 10: MLEs and Bayes estimates of reliability function $R(t)$ under type-II censoring scheme for the real data set.

n	r	MLE	BS	BG		BL	
				q = -2	q = 2	c = -2	c = 2
50	10	0.5989	0.6301	0.6243	0.6320	0.6277	0.6325
	25	0.6386	0.5793	0.5727	0.5815	0.5768	0.5819
	45	0.6449	0.4758	0.4741	0.4764	0.4753	0.4764

Table 11: MLEs and Bayes estimates of hazard function $H(t)$ under type-II censoring scheme for the real data set.

n	r	MLE	BS	BG		BL	
				q = -2	q = 2	c = -2	c = 2
50	10	1.2120	1.1779	1.1315	1.1932	1.1429	1.2156
	25	0.9999	1.2930	1.2464	1.3080	1.2549	1.3326
	45	0.9605	1.3658	1.3526	1.3698	1.3543	1.3762

Table 12: Classical and Bayesian Interval estimation for α under type-II censoring scheme for the real data set.

n	r	Confidence interval		Credible interval		HPD interval	
		LL	UL	LL	UL	LL	UL
50	10	0.9563	2.5612	1.5530	2.1360	1.5680	2.1460
	25	1.1287	2.0432	1.5710	2.1530	1.5790	2.1570
	45	1.1983	1.8914	1.4850	1.8860	1.5080	1.9000

Table 13: Classical and Bayesian Interval estimation for θ under type-II censoring scheme for the real data set.

n	r	Confidence interval		Credible interval		HPD interval	
		LL	UL	LL	UL	LL	UL
50	10	0.8628	1.9464	1.1870	1.7800	1.1700	1.7510
	25	1.1360	1.8676	1.0670	1.6090	1.0570	1.5930
	45	1.1773	1.8608	1.0020	1.2300	1.0000	1.1870

Conclusion

In this paper, we have considered the statistical inference of the unknown parameters, reliability characteristics (reliability and hazard rate function) of the GIL distribution when the data are type-II censored. The maximum likelihood estimators (MLEs) and the Bayes estimators of the parameters and reliability characteristics are obtained. The MLEs are not in closed forms. therefore, numerical approximation techniques have been implemented to evaluate them. We have used symmetric (squared error) and symmetric (LINEX, general entropy) loss functions to compute the Bayes estimates and their MSEs. Simulation technique MCMC has been done for the different choices of (n, r) combinations to report the performances of the various estimators. Lastly, a real data set has been considered to illustrate the computation of various estimators.

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