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One-parameter linear-exponential distribution

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Abstract
In this paper, we introduce a new continuous probability distribution which is based on the product of linear and exponential functions having a single parameter and hence, we named it 'One-parameter Linear-exponential distribution'. Different structural properties such as probability density function, probability distribution function and moment generating function of this distribution have been derived. Moments about origin and hence, the first four central moments of the proposed distribution have been obtained. The hazard rate function and the mean residual life function of the proposed distribution have been discussed, and show its flexibility over Lindley distribution. Estimation of parameters has been discussed by the method of moments as well as the method of maximum likelihood. Goodness of fit has been applied to different data-sets which were used by others. It has been observed that the proposed distribution gives better fit to the most of data-sets of same nature, having variance greater than the mean, than Lindley distribution.

Keywords: Lindley distribution, parameters, moments, goodness of fit, estimation

1. Introduction

The nice feature of introducing One-Parameter Linear-Exponential distribution (OPLED) is that it stands with a single parameter which is to be proposed as to give a better alternative of Lindley (1958) [2] distribution and to create another plate-form for researcher working on probability distribution theory. In the last two decades, many researchers have published different types of Quasi-Lindley distributions having two or more than two parameters which are generalised form of Lindley distribution (1958) [2]. Lindley introduced a single parameter continuous probability distribution, known as Lindley distribution (LD), given by its probability density function

$$f_1(x) = \frac{\phi^2(1+x)e^{-\phi x}}{(1+\phi)};\ x>0,\ \phi>0 \qquad \dots (1)$$

Mishra and Sah (2015) [3] obtained a generalised Lindley-exponential distribution which gives better alternative to the Lindley distribution but it has two parameters. So, we think that it is not fare to compare with the distribution having a single parameter. Sah (2015) [5] introduced a one parameter continuous probability distribution, known as Mishra distribution (MD), given by its probability density function

$$f_2(x;\phi) = \frac{\phi^3}{(\phi^2+\phi+2)}(1+x+x^2)e^{-\phi x};\, x>, \phi>0 \qquad \dots (2)$$

Mishra distribution of Sah (2015) [5] which gives better fit to the same nature of data-sets, having variance greater than the mean, than Lindley distribution (1958) [2]. Sah has described importance of Poisson-Mishra distribution (2017) [6] and Generalised Poisson-Mishra distribution (2018) [7] in accident proneness.

The proposed distribution is a modified form of Lindley distribution replacing '1' by 'ϕ^2'in the expression (1+x) of the equation (1).

The proposed distribution is based on the product of linear function ($(\phi^2 + x)$ and exponential function $e^{-\phi x}$ with a single parameter 'ϕ' and hence, it is named as One-parameter Linear-exponential distribution (OPLED). Several structural properties such as probability density function, probability distribution function, moment generating function, moments about origin as well as moments about mean of the proposed distribution have been obtained. The hazard rate function and the mean residual life function of the proposed distribution have been discussed. The estimation of parameters has been discussed by the method of moments as well as the maximum likelihood methods. The proposed distribution has been fitted to some well-known data-sets which were earlier used by others and it is expected to give better alternative to the same nature of data-sets than Lindley (1958) [2] distribution (LD).

2. Material and Methods

The proposed distribution is based on theoretical concept of probability distribution. Probability density function of One-Parameter Linear-Exponential distribution (OPLED) has been constructed so that it follows all the basic properties of probability distribution. Probability distribution function of OPLED has constructed to calculate probability of the variable under study for each interval. Estimate of the parameter has been discussed by the methods of moments as well as the maximum likelihood methods. To test validity of the theoretical work, goodness of fit has been applied to some data-sets which were earlier used by others.

3. Results

3.1 One-parameter linear-exponential distribution (OPLED)

The proposed distribution, OPLED, has been obtained on the basis of the product of linear and exponential functions. This distribution with parameter ϕ is defined by its probability density function.

$$f(x;\phi)=\frac{\phi^2}{(1+\phi^3)}(\phi^2+x)e^{-\phi x}\text{; Where } x>0,\ \phi>0 \qquad \ldots (3)$$

The expression (3) is the probability density function of OPLED. Probability distribution function of this distribution has been obtained as

$$F(x)=P(X\le x)=\int_0^x f(x)dx \quad =\frac{\phi^2}{(1+\phi^3)}\int_0^x(\phi^2+x)e^{-\phi x}dx \quad =1-\frac{(1+\phi x+\phi^3)}{(1+\phi^3)}e^{-\phi x} \qquad \ldots (4)$$

Moment generating function (M.G.F) of the OPLED (3) has been obtained by

$$M_X(t)=E\left[e^{tx}\right]=\int_0^\infty e^{tx}f(x)dx$$

$$=\frac{\phi^2}{(1+\phi^3)}\int_0^\infty(\phi^2+x)e^{-(\phi-t)x}dx=\frac{\phi^2}{(1+\phi^3)}\frac{\left[\phi^2(\phi-t)+1\right]}{(\phi-t)^2} \qquad \ldots (5)$$

The expression (5) is the M.G.F. of OPLED (3).

Graphical representation of probability density function and distribution function for varying values of parameter are given below.

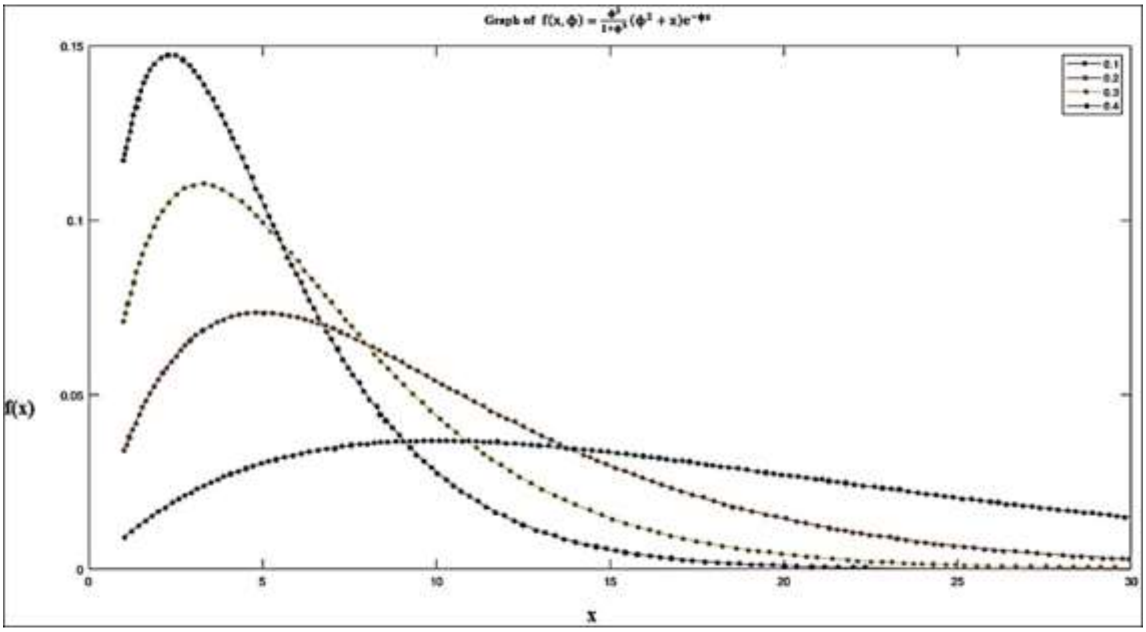

Fig 1: Graph of pdf of OPLED at $\phi = 0.1, 0.2, 0.3, 0.4$

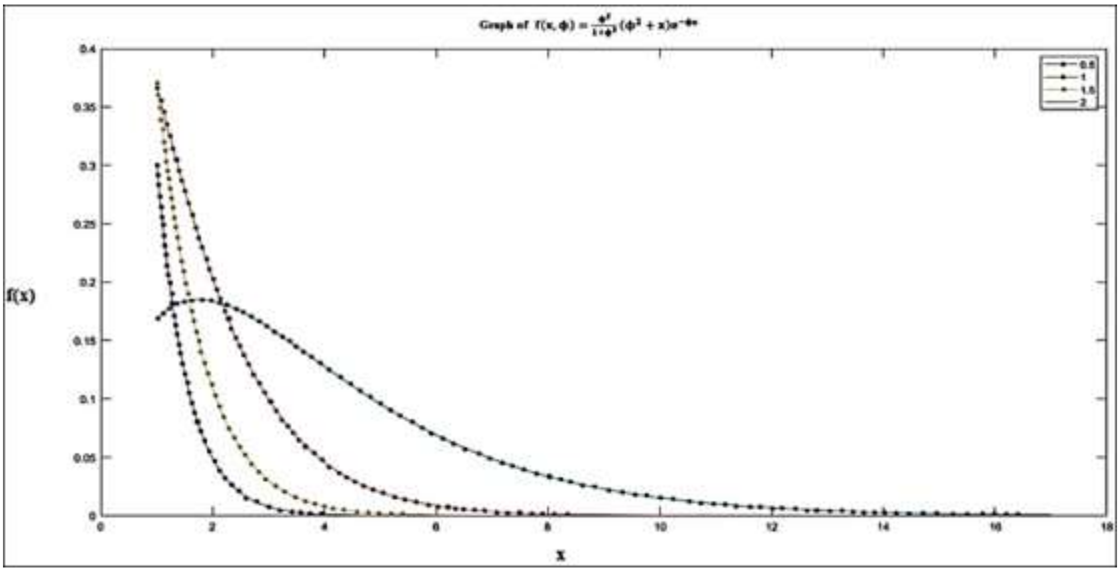

Fig 2: Graph of pdf of OPLED at $\phi = 0.5, 1.0, 1.5, 2.0$

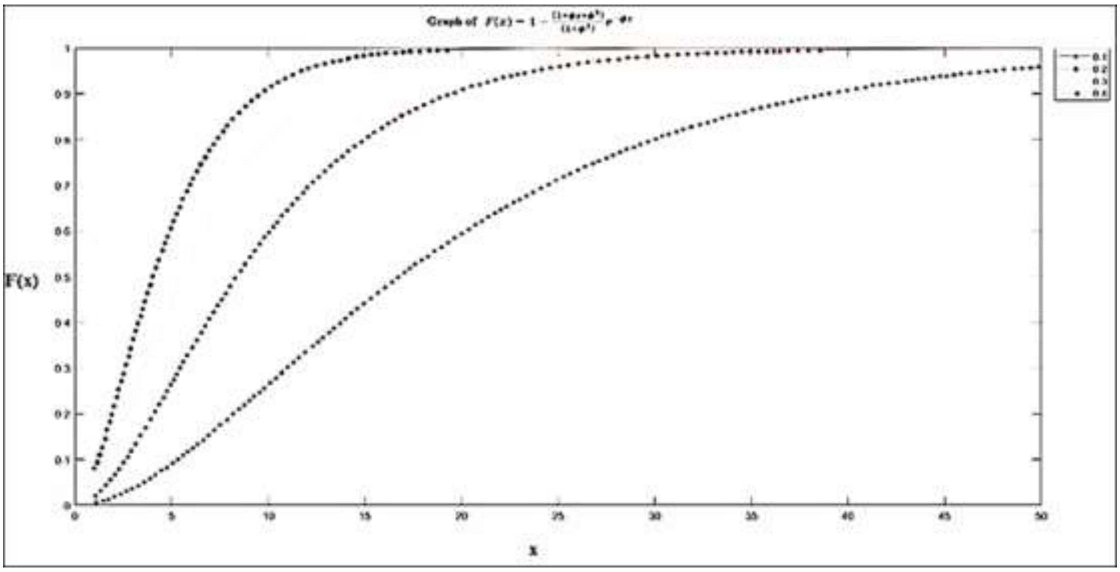

Fig 3: Graph of df of OPLED at $\phi = 0.1, 0.2, 0.3, 0.4$

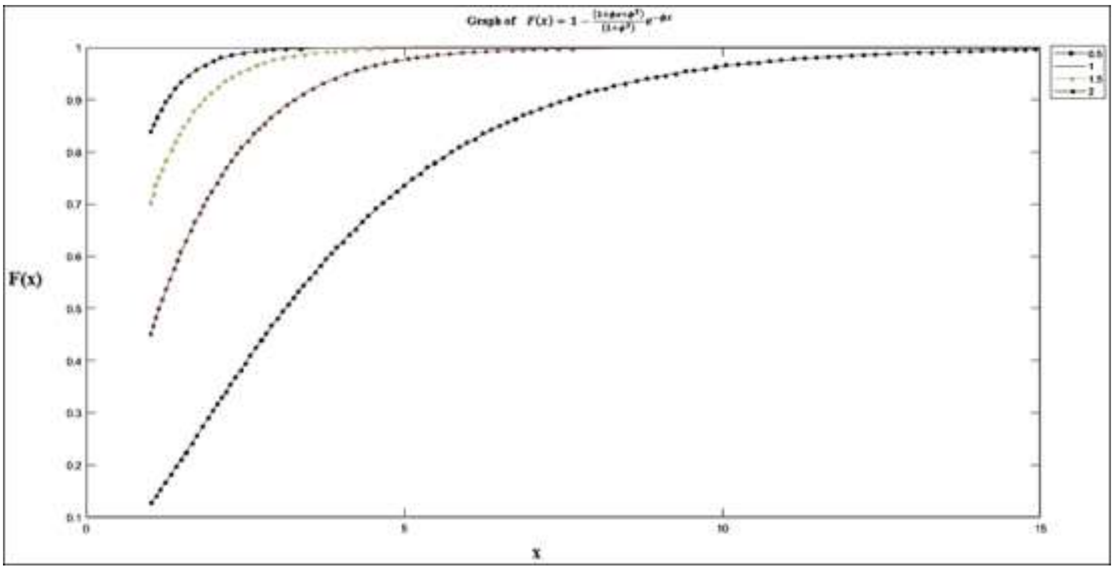

Fig 4: Graph of df of OPLED at $\phi = 0.5, 1.0, 1.5, 2.0$

3.2 Moments and related measures of OPLED

The r[th] moment about origin of the OPLED (3) can be obtained as

$$\mu_r' = E(X^r) = \int_0^\infty x^r f(x)dx = \frac{\phi^2}{(1+\phi^3)} \int_0^\infty x^r (\phi^2 + x) e^{-\phi x} dx$$

$$= \frac{\phi^2}{(1+\phi^3)} \frac{\Gamma(r+1)}{\phi^r} \frac{(\phi^3 + r + \alpha)}{\phi^2} = \frac{1}{(1+\phi^3)} \frac{r!}{\phi^r} (\phi^3 + r + 1) \qquad \ldots (6)$$

The expression (6) is the general form of the r[th] moment about origin of the OPLED (3). It is necessary to obtain statistical moments of the proposed distribution to estimate parameter of OPLED (3). Putting $r = 1, 2, 3$ and 4 in the expression (6), we get the first four moments about origin of OPLED (3) as

$$\mu_1' = .\frac{1!}{\phi} \frac{(2+\phi^3)}{(1+\phi^3)} \qquad \ldots (7)$$

$$\mu_2' = .\frac{2!}{\phi^2} \frac{(3+\phi^3)}{(1+\phi^3)} \qquad \ldots (8)$$

$$\mu_3' = .\frac{3!}{\phi^3} \frac{(4+\phi^3)}{(1+\phi^3)} \qquad \ldots (9)$$

$$\mu_4' = .\frac{4!}{\phi^4} \frac{(5+\phi^3)}{(1+\phi^3)} \qquad \ldots (10)$$

The first four moments about mean of the OPLED (3) has been obtained as

$$\mu_1 = 0$$

$$\mu_2 = \mu_2' - \mu'^2$$

$$= \frac{2!}{\phi^2} \frac{(3+\phi^3)}{(1+\phi^3)} - \left[\frac{1!}{\phi} \frac{(2+\phi^3)}{(1+\phi^3)}\right]^2 = \frac{(\phi^6 + 4\phi^3 + 2)}{\left\{\phi(1+\phi^3\right\}^2} \qquad \ldots (11)$$

The expression (11) is the variance of the OPLED (3).

$$\mu_3 = \mu_3' - 3\mu_2' \mu_1' + 2(\mu_1')^3$$

$$= \frac{3!}{\phi^3} \frac{(4+\phi^3)}{(1+\phi^3)} - 3\left[\frac{2!}{\phi^2} \frac{(3+\phi^3)}{(1+\phi^3)}\right]\left[\frac{1!}{\phi} \frac{(2+\phi^3)}{(1+\phi^3)}\right] + 2\left[\frac{1!}{\phi} \frac{(2+\phi^3)}{(1+\phi^3)}\right]^3$$

$$= \left[\frac{2\left\{3(4+\phi^3)(1+\phi^3)^2 - 3(3+\phi^3)(2+\phi^3)(1+\phi^3) + (2+\phi^3)^3\right\}}{\left\{\phi(\phi^3+1)\right\}^3}\right] \qquad \ldots (12)$$

The expression (12) is the third moment about the mean of OPLED (3). The fourth moment about the mean of OPLED (3) can be obtained as

$$\mu_4 = \mu_4' - 4\mu_3' \mu_1' + 6\mu_2' (\mu_1')^2 - 3(\mu_1')^4$$

$$= \frac{4!}{\phi^4} \frac{(5+\phi^3)}{(1+\phi^3)} - 4\left[\frac{3!}{\phi^3} \frac{(4+\phi^3)}{(1+\phi^3)}\right]\left[\frac{1!}{\phi} \frac{(2+\phi^3)}{(1+\phi^3)}\right] + 6\left[\frac{2!}{\phi^2} \frac{(3+\phi^3)}{(1+\phi^3)}\right]\left[\frac{1!}{\phi} \frac{(2+\phi^3)}{(1+\phi^3)}\right]^2 - 3\left[\frac{1!}{\phi} \frac{(2+\phi^3)}{(1+\phi^3)}\right]^4$$

$$= \left[\frac{3\begin{Bmatrix} 8(5+\phi^3)(1+\phi^3)^3 - 8(4+\phi^3)(2+\phi^3)(1+\phi^3)^2 \\ +4(3+\phi^3)(2+\phi^3)^2(1+\phi^3) - (2+\phi^3)^4 \end{Bmatrix}}{\left\{\phi(\phi^3+1)\right\}^4} \right] \quad \dots (13)$$

We can observe that variance (11) is greater than the mean (7). Hence, OPLED (3) is always over-dispersed. Shape and size of the proposed probability distribution can be studied by obtaining co-efficient of skewness and kurtosis

$$\gamma_1 = \frac{2\left\{3(4+\phi^3)(1+\phi^3)^2 - 3(3+\phi^3)(2+\phi^3)(1+\phi^3) + (2+\phi^3)^3\right\}}{\left[\phi^6 + 4\phi^3 + 2\right]^{3/2}} \quad \dots (14)$$

Here, $\gamma_1 > 0$. Hence, OPLED (3) is positively skewed.
And co-efficient of kurtosis can be obtained by using

$$\beta_2 = \frac{3\begin{Bmatrix} 8(5+\phi^3)(1+\phi^3)^3 - 8(4+\phi^3)(2+\phi^3)(1+\phi^3)^2 \\ +4(3+\phi^3)(2+\phi^3)^2(1+\phi^3) - (2+\phi^3)^4 \end{Bmatrix}}{(\phi^6 + 4\phi^3 + 2)^2} \quad \dots (15)$$

Here, $\beta_2 > 3$.Hence, OPLED (3) is leptokurtic.

3.3 The reliability function, hazard rate function and mean residual life function of OPLED

The reliability function

Let X follows OPLED (3) with parameter ϕ and its probability density function is *f(x).* Distribution function of X can be defined as

$$F(x) = 1 - e^{-\phi x} - \frac{\phi x}{(1+\phi^3)} e^{-\phi x}$$

$$1 - F(x) = \frac{(1+\phi x + \phi^3)}{(1+\phi^3)} e^{-\phi x}$$

Let the random variable X be the lifetime or the time to failure of a component. The probability that the component will survive until sometime 't' is called reliability R(t) of the component defined by

$$R(t) = P(X > t) = \int_t^\infty f(x)dx = 1 - F(t) = \frac{(1+\phi t + \phi^3)}{(1+\phi^3)} e^{-\phi t} \quad \dots (16)$$

The expression (16) is the reliability function of OPLED (3). The component is said to be working properly at time t=0 and no component work forever without failure i.e.

$$R(t=0) = 1 \qquad and \quad \lim_{t \to 0} R(t) = 0$$

R(t) is a monotone non-increasing function of *t*. For $t < 0$, the reliability has no meaning.

The hazard rate function

Hazard measures the conditional probability of a failure given that the system is working. The failure density (pdf) measures the overall speed of failures. Hazard rate or Instantaneous failure rate measures the dynamic speed of failures. For a continuous distribution with p.d.f. *f(x)* and c.d.f. *F(x),* the hazard rate function (also known as failure rate function) is defined as

$$h(x) = \lim_{\Delta x \to 0} \frac{P(X < x + \Delta x / X > x)}{\Delta x} = \frac{f(x)}{1 - F(x)} \quad \dots (17)$$

The main reason for defining the *h(x)* is that it is more convenient to work with than *f(x).* Putting the value of f(x) and 1-F(x) in the expression h(x), the Hazard rate function of OPLED (3) has been obtained as

$$h(x)=\frac{\phi^2(\phi^2+x)}{(1+\phi x+\phi^3)} \quad \ldots (18)$$

The failure rate function of OPLED (3) until time 't' is thus obtained as

$$h(x=t)=\frac{\phi^2(\phi^2+t)}{(1+\phi t+\phi^3)} \quad \ldots (19)$$

At $t=0$, $h(x=t=0)=\frac{\phi^4}{(1+\phi^3)}$ … (20)

It is also obvious that h(x) is an increasing function of x and ϕ .

Graphical representation of hazard rate function for varying values of parameters have been given below.

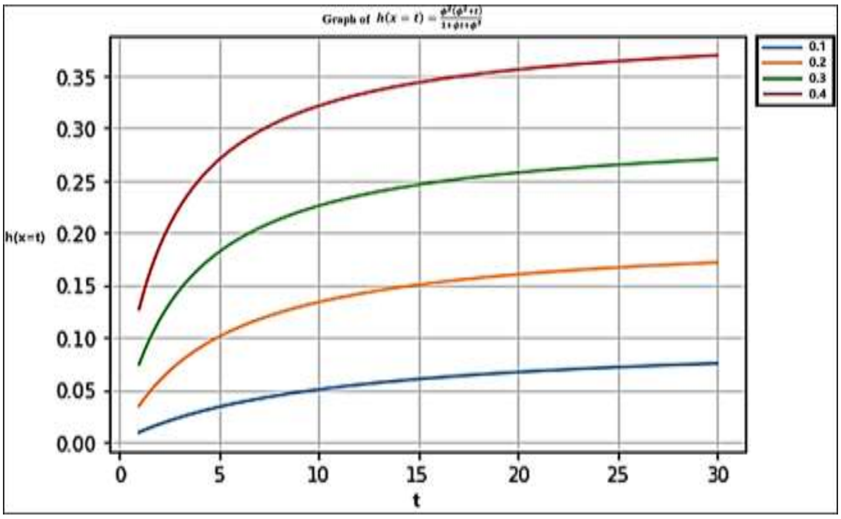

Fig 5: Graph of h(x) at $\phi = 0.1, 0.2, 0.3, 0.4$

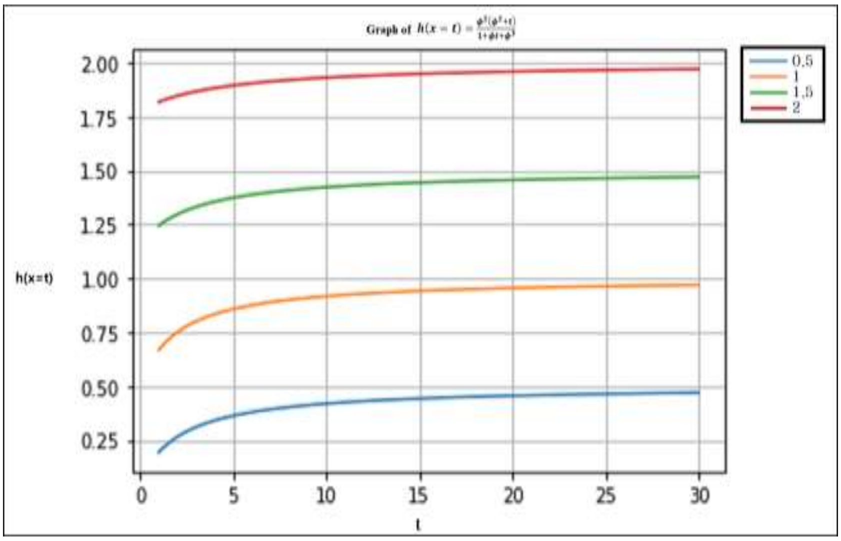

Fig 6: Graph of h(x) at $\phi = 0.5, 1.0, 1.5, 2.0, 2.5$

Residual life function

In reliability studies, the expected additional life time given that a component has survived until time 't' is called mean residual life time. Let a random variable X denotes the life of a component, the mean residual life function is given by

$$m(x)=E\left[X-x/X>x\right]=\frac{\int_x^{\infty}\left[1-F(t)\right]dt}{1-F(x)} \quad \ldots (21)$$

To obtain *m(x)* of the OPLED (3), we have to calculate the following measures

$$f(x;\phi)=\frac{\phi^2}{(1+\phi^3)}(\phi^2+x)e^{-\phi x}$$

Where x > 0, $\phi>0$ and $(1+\phi^3)>0$

$$F(x)=1-e^{-\phi x}-\frac{\phi x}{(1+\phi^3)}e^{-\phi x}$$

$$1-F(x)=\frac{(1+\phi x+\phi^3)}{(1+\phi^3)}e^{-\phi x}$$

$$1-F(t)=\frac{(1+\phi t+\phi^3)}{(1+\phi^3)}e^{-\phi t}$$

Where *f*(*x*) and *F*(*x*) are the probability density function and probability distribution function of OPLED (3).

$$\int_x^\infty \{1-F(t)\}dt=\int_x^\infty \frac{(1+\phi t+\phi^3)}{(1+\phi^3)}e^{-\phi t}dt=\frac{(2+\phi x+\phi^3)e^{-\phi x}}{\phi(1+\phi^3)} \quad \ldots (22)$$

Putting the value of $\int_x^\infty [1-F(t)]dt$ and $1-F(x)$ in equation (21), the mean residual life function has been obtained as

$$m(x)=\frac{(2+\phi x+\phi^3)}{\phi(1+\phi x+\phi^3)} \quad \ldots (23)$$

$$\text{At } x=0,\ m(x=0)=\frac{(2+\phi^2)}{\phi(1+\phi^3)}=\mu_1' \quad \ldots (24)$$

It can also be seen that at $x=0$, the mean residual life function (23) is the mean of OPLED (3). It can also be seen that *m*(*x*) is a decreasing function of x, The hazard rate function and the mean residual life function of the OPLED show its flexibility over Lindley distribution.

3.4 Stochastic orderings

Stochastic ordering of non-negative continuous random variables is an important tool for judging the comparative behavior. Let us consider two random variables X and Y. The random variable X is said to be smaller than Y in the

a) Stochastic order $(X\le_{st}Y)$ if $F_X(x)\ge F_Y(x)$ for all x
b) Hazard rate order $(X\le_{hr}Y)$ if $h_X(x)\ge h_Y(x)$ for all x
c) Mean residual life order $(X\le_{mri}Y)$ if $m_X(x)\ge m_Y(x)$ for all x
d) Likelihood ratio order $(X\le_{lr}Y)$ if $\frac{f_X(x)}{f_Y(y)}$ decreases in x

The following results due to Shaked and Shanthi Kumar (1994) [8] establishing stochastic ordering of the distributions

$$\begin{array}{l}(X\le_{lr}Y)\Rightarrow(X\le_{hr}Y)\Rightarrow(X\le_{mri}Y)\\ \quad\Downarrow\\ (X\le_{st}Y)\end{array}$$

The OPLED (3) is ordered with respect to the strongest ‘likelihood ratio’ ordering as shown in the following theorem

Theorem: Let $X \sim OPlED(\phi_1)$ and $Y \sim OPLED(\phi_2)$. If $(\phi_1\ge\phi_2)$, then $(X\le_{lr}Y)$ and hence $(X\le_{hr}Y)$, $(X\le_{mrl}Y)$ and $(X\le_{st}Y)$

Proof: We have

$$\frac{f_X(x)}{f_Y(x)} = \frac{\phi_1^2(1+\phi_2^3)}{\phi_2^2(1+\phi_1^3)}\frac{(\phi_1^2+x)}{(\phi_2^2+x)}e^{-(\phi_1-\phi_2)x} \quad ; x>0 \qquad \dots (25)$$

$$\log\left\{\frac{f_X(x)}{f_Y(x)}\right\} = \log\left\{\frac{\phi_1^2(1+\phi_2^3)}{\phi_2^2(1+\phi_1^3)}\right\} + \log\left\{(\phi_1^2+x)\right\} - \log\left\{(\phi_2^2+x)\right\} - (\phi_1-\phi_2)x \qquad \dots (26)$$

d.w.r.t. x of equation (5.2), we get

$$\frac{d}{dx}\log\left\{\frac{f_X(x)}{f_Y(x)}\right\} = \frac{1}{(\phi_1^2+x)} - \frac{1}{(\phi_2^2+x)} - (\phi_1-\phi_2) = \frac{(\phi_2^2-\phi_1^2)}{(\phi_1^2+x)(\phi_2^2+x)} - (\phi_1-\phi_2) \qquad \dots (27)$$

If $\phi_1 \geq \phi_2$, then $\frac{d}{dx}\log\left\{\frac{f_X(x)}{f_Y(x)}\right\} \leq 0$. It indicates that $(X \leq_{lr} Y)$ and hence $(X \leq_{hr} Y)$, $(X \leq_{mlr} Y)$ and $(X \leq_{st} Y)$

This theorem shows flexibility of OPLED (3) over Lindley (1) and exponential distributions.

3.5 Estimation of parameter of OPLED

OPLED (3) has a single parameter (ϕ). The estimate of parameter has been obtained by using (a) Method of moments and (b) Method of maximum likelihood.

a) Method of moments: To estimate value of the parameter (ϕ), the first moment about origin is required. So, the population first moment is replaced by respective sample moment and using the expression (7), we can obtain estimate of ϕ as follows.

$$\mu_1' = .\frac{1!}{\phi}\frac{(2+\phi^3)}{(1+\phi^3)}$$

Or, $\mu_1'(\phi+\phi^4) = (2+\phi^3)$

Or, $\mu_1'(\phi+\phi^4) - (2+\phi^3) = 0$

The expression (28) is the Polynomial equation of fourth degree. This equation may be solved by Regula-Falsi method to obtain estimated value of the parameter (ϕ).

b) The method of maximum likelihood: Let ($x_1, x_2, \dots, x_n$) be a random sample of size n from OPLED (3) and let f_x be the observed frequency in the sample corresponding to $X = x$ $(x = 1, 2, ..., k)$ such that $\sum_{x=1}^{k} f_x = n$, where k is the largest observed value having non-zero frequency. The likelihood function, L, of the OPLED (3) is obtained as

$$f(x;\phi) = \frac{\phi^2}{(1+\phi^3)}(\phi^2+x)e^{-\phi x}$$

$$L = \left(\frac{\phi^2}{1+\phi^3}\right)^n \left[\prod_{x=1}^{k}(\phi^2+x)^{f_x}\right] e^{-n\phi\bar{x}} \qquad \dots (29)$$

and so, the log likelihood function is obtained as

$$\ln L = n\ln\phi^2 - n\ln(1+\phi^3) + \sum_{x=1}^{k} f_x \ln(\phi^2+x) - n\phi\bar{x}$$

The log likelihood equation is thus obtained as

$$\frac{\partial \ln L}{\partial \phi} = \frac{2n}{\phi^2} - \frac{3n\phi^2}{(1+\phi^3)} + \sum_{x=1}^{k}\frac{2\phi f_x}{(\phi^2+x)} - n\bar{x} = 0$$

We can observe that the equation (33) does not seem to be solved directly. However, Fisher's scoring method can be applied to solve this equation.

4. Goodness of fit and discussion

The OPLED has been fitted to a number of data-sets to which earlier the Lindley distribution have been fitted by others and to almost all these data-sets this distribution provides closer fits than the Lindley distribution. The fittings of the OPLED (3) to the two such data-sets have been presented in the following tables. The first data-set is regarding the survival times (in days) of guinea pigs infected with virulent tubercle bacilli, observed and reported by Bjerkedal (1960) [1] and the second data-set is regarding mortality grouped data for blackbirds species, reported by Paranjpe and Rajarshi (1986) [4]. The expected frequencies according to the Lindley distribution have also been given for ready comparison with those obtained by the LED.

Table 1: Survival times (in days) of 72 guinea pigs infected with virulent tubercle bacilli

Survival Time (in days)	Observed frequency	Expected Frequency	
		Lindley	OPLED
0-80	8	16.1	15.9
80-160	30	21.9	22.0
160-240	18	15.4	15.5
240-320	8	9.0	9.0
320-400	4	5.5	4.8
400-480	3	1.8	2.5
480-560	1	2.3	2.3
Total	72	72.0	72.0
	μ_1' =181.11111 μ_2' =43911.11111	$\hat{\phi}$ =0.011 χ^2 (df)=7.77(3)	$\hat{\phi}$ =0.0110429 χ^2 (df)=7.61(3)

Table 2: Mortality grouped data for blackbirds species reported by Paranjpe and Rajarsi (1986)

Survival time (In days)	Observed frequency	Expected frequency	
		Lindley	OPLED
0-1	192	173.5	151.1
1-2	60	98.6	99.2
2-3	50	46.5	53.5
3-4	20	20.1	26.9
4-5	12	8.1	11..36
5-6	7	3.2	5.5
6-7	6	1.4	2.4
7-8	3	0.4	1.1
>8	2	0.3	0.8
Total	352	352.0	352.0
	μ_1' =1.568181 μ_2' =5.005682	$\hat{\phi}$ =0.984 χ^2 (df)=49.85(4)	$\hat{\phi}$ =0.970723 χ^2 (df)=35.67(5)

In the above tables, the expected frequencies according to the Lindley distribution have also been given for ready comparison with those obtained by the OPLED. From table (I), we can observe that the value of Chi-square of OPLED is less than the Lindley distribution with same degrees of freedom. From table (II), we can observe that the value of Chi-square of OPLED is also less than Lindley distribution with greater degrees of freedom. Hence, it may conclude that in most of the cases OPLED gives better fit to the same nature of data-sets, having variance greater than the mean, than Lindley distribution.

5. Conclusion

In this paper, we propose a single parameter continuous distribution which is named as One-Parameter Linear-Exponential distribution (OPLED). Several structural properties such as moment generating function, distribution function, moments about origin as well as mean have been derived. The reliability function, hazard rate function and mean residual life function have been obtained and discussed. The methods of estimation of parameter have been discussed. Finally, the proposed distribution has been fitted to a number of data-sets, having variance greater than mean, to test its goodness of fit and it has been observed that the OPLED (3) gives better fit to the most of the similar nature of the data-sets, having variance greater than the mean, than the Lindley distribution (1).

Conflict of interest

The authors declared that there is no conflict of interest.

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