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Glimpses of contributions of some top Indian mathematicians: A review article

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Abstract

Indian mathematics has its deep roots in the Vedas, different from what is known as Vedic Mathematics. Vedic age gave rise to a new era of progress in the field of Science, Technology and Development. The Hindu Scripture Vedas is synonymous with all kinds of original source of knowledge and intellectual wisdom in the universe leading to modern knowledge in modern mathematics. Indian mathematicians made tremendous contributions to the entire world of Mathematics and Science. Decimal number system as well as the invention of zero (0) are among the greatest contributions of Indian mathematicians. The theory of trigonometry, Mathematical Modelling, Algebra, algorithm, modern arithmetic, sine and cosine functions leading to modern trigonometry, Diophantine equation, square root, cube root, negative numbers are also developed by Indian mathematicians. In this review article, the work of some of the renowned Indian mathematicians from Indus Valley civilization and the Vedas to modern times are covered in short with the hope that it may reveal hidden fundamental mathematical ideas as basic ideal tools which may usefully motivate for further research work in every domain of mathematical sciences, natural and applied sciences, engineering and social sciences. Moreover, there are many more remarkable Indian mathematicians who contributed to the origin of mathematical sciences. They have made several general contributions to mathematics that have significantly influenced scientists and mathematicians in the modern times.

Keywords: glimpses, contributions, Indian mathematicians

1. Introduction

Aryabhata, also called Aryabhata I, was one of the greatest and the earliest Indian mathematicians and astronomers from the classical era of Indian mathematics and astronomy [15, 20, 21]. He developed in Kusumapura that is near Pataliputra, Patna, Bodh Gaya, Bihar, India. Kusumapura was then the capital of the Gupta dynasty known as an important center of learning. He discovered zero(0). For the detailed about discovery of zero(0), one can refer to F. Staal [5].

Aryabhata gave a systematic treatment of the position of the planets in space. He also believes with his knowledge that the orbits of the planets are elliptical. Aryabhata had the belief that the moon and planets had the brightness through the reflected light from the surface of the sun. He gave the circumference of the earth in 4 967 yojanas and its diameter in 1 581 $\frac{1}{24}$ yojanas. Since 1 yojana = 5 miles, this gives the circumference in 24 835 miles, which is an excellent approximation to the current value stated in 24 902 miles. Aryabhata was the first to state that the earth is spherical and it revolves around the sun and also that the earth rotates on its axis [13]. He calculated the length of the year in 365 days, 6 hours, 12 minutes and 30 seconds which is notably near to the exact measurement in 365 days and 6 hours approximated. He accurately describes why and what cause the sun and the moon eclipse. Furthermore, he worked on the place value system using letters to signify numbers stating qualitative measures. He also elaborated to obtaining the cube and cube root of an integer and gave rules with smooth computations of squares and square roots too. Aryabhata also gives for mulae for the areas of a right triangle, right pyramid and that of a circle and trapezium.

Jha ^[23] says:

“Aryabhata discovered this value independently and also realised that π is an irrational number. He had the Indian background, no doubt, but excelled all his predecessors in evaluating π . Thus the credit of discovering this exact value of π may be ascribed to the celebrated mathematician, Aryabhata I”.

Aryabhata is also said to have constructed an observatory at the Sun temple in Taregana, Bihar, India.

His work, a Sanskrit Astronomical Treatise, called Aryabhatiya ^[42], on Mathematics and Astronomy, was widely referred to in the Mathematical literatures and has preserved up to modern times. Aryabhatiya in Devanagari scripts is classified into four sections as follows:

1. Gitikapada, that is, Basics of Astronomy and Sine table;
2. Ganitapada, that is, Mathematics;
3. Kalakriyapada, that is, The Reckoning of Time;
4. Golapada, that is, The Sphere.

The mathematical part of his treatise Aryabhatiya ^[4] covers arithmetic, algebra, plane trigonometry, and spherical trigonometry. It also contains continued fractions, quadratic equations, sums of power series, and a table of sines, definitions of various units of time, eccentric and epicyclic models of planetary motion. The methods for finding integer solutions of Diophantine equations can be seen in Indian literature from the time of Aryabhata. The first clear investigation of the general integral solution of the linear Diophantine equation $ay - bx = 0$ is in his Aryabhatiya. This algorithm of Aryabhata is considered the most significant contribution in Pure Mathematics with important applications in astronomy. The integer solution of the linear Diophantine equation was studied by Bachet in Europe for the first time in 16AD, 1122 later than Aryabhata. Bachet's solution same, as Kuttaka, was rediscovered in 1735 by Euler. In Europe, this was further developed by Fermat, Brouncker, and Wallis in their Number Theory research work.

Some work on Algebra in the Aryabhatiya is the first of its kind giving integer solutions to equations of the formats looking like $by = ax + c$ and $by = ax - c$, with a , b , and c integers. Aryabhata also gives summing of the first n integers, the sum of squares of these integers and also sum of their cubes, and Sankalita of various kinds.

$$\sum_1^n n = \frac{n(n+1)}{2}, \sum_1^n n^2 = \frac{n(n+1)(2n+1)}{6}, \sum_1^n n^3 = \left\{ \frac{n(n+1)}{2} \right\}^2$$

According to Jha ^[22], Aryabhata was an author of at least three astronomical texts and wrote some free stanzas too.

His another Book called "Aryabhatasiddhanta" ^[42] is one of the earliest astronomical works to assign the beginning of each day to midnight. Via the Sasanian dynasty of Iran, it had a profound impact on the development of Islamic Astronomy too. Its contents are preserved to some extent in the works of Bhaskara I, Varahamihira, Brahmagupta, and some others.

In "Ganita", Aryabhata provides the names the first 10 decimal places and gives algorithms for finding square and cubic roots, applying the decimal number system ^[42]. Then he geometrically covers $62,832/20,000 = 3.1416$ for $\pi = \text{circumference/diameter}$. Aryabhata does not calculate his exact value for π but prefers to apply $\sqrt[4]{10} = 3.1622$ to make it applicable. Mathematical series, quadratic equations, compound interest with a quadratic equation, proportions and ratios, and the solution of various linear equations are among the arithmetic and algebraic topics developed. Aryabhata's

general solution for linear indeterminate equations, which Bhaskara I called Kuttakara i.e., Pulverizer, is performed by breaking the problem down into new problems with successively smaller coefficients that was basically the Euclidean algorithm related to the techniques of continued fractions. The "Kuttaka" algorithm of Aryabhata may be useful too for the solution of a system of linear equations by matrix operation as well. The continued fractions is a useful topic in number theory. He develops characteristics of similar right angled triangles and studies two intersecting circles. By the Pythagorean Theorem, he found one of the two techniques for making his table of sines. He also vividly understood that the second order sine differences is proportional to sine.

Aryabhatiya culminates his work with spherical astronomy in Gola. In this work, he applied plane trigonometry to spherical geometry by projecting points and lines on the surface of a sphere onto appropriate planes.

The Government of India, in his honour, named its first satellite Aryabhata, and the lunar crater Aryabhata. Aryabhata Knowledge University (AKU), Patna has been established by Government of Bihar for the development and management of educational infrastructure related to technical, medical, management and allied professional education in his honour. Aryabhata's work was of great influence in the Indian astronomical tradition and influenced several neighbouring cultures through translations.

In the 13th century, through the translated version of the Aryabhatiya into Latin, European mathematician learned methods for calculating the volumes of spheres, areas of triangles as well as squares and cube roots. But the European astronomers by then had acquired these knowledge base with the observations of Copernicus and Galileo. Aryabhata discovered these concepts 1500 years ago, and 1000 years before Copernicus and Galileo, making Aryabhata a pioneer in this field. The reliable Aryabhata Siddhanta practically formulated the purpose of the Panchanga, i.e., Hindu Calendar. In the 11th century, Al-Biruni informed that Aryabhatiya ^[42] was brought to the Central Asia by the Abbasid Khalifs, and he then made it translated into the Arabic language by Abul Hasan Ahwazi. They formed the basis of the Jalali Calendar introduced by astronomers including Omar Khayyam himself to the Islamic worlds. The versions of Jalali Calendars modified in 1925 are the calendars used in Iran and in Afghanistan. The Arabic translated work during the Islamic Golden Age had had its own impact. Al-Khwarizmi cited some of his results. Aryabhata set the agenda for mathematical research in India for many centuries to come. The fraternity of a "mathematical family" in a library contained the mathematical writings on the commentaries of the previous mathematician's work such as "Aryabhatiya" and so on.

2. Brahmagupta

The 7th Century Indian mathematician and astronomer Brahmagupta ^[43] authored some important works on both mathematics and astronomy. He was from the modern Bhinmal, Rajasthan of India. He became the head of the astronomical observatory at Ujjain in the central India. His remarkable contributions is number system, negative numbers, and zero (0). He also had a profound and direct impact on Islamic and Byzantine astronomy. Brahmagupta's working methodology rests mostly on his astronomical treatise "Brahma Sphuta Siddhanta". It was translated into the Arabic language in Baghdad and had a major impact on

Islamic mathematics and astronomy. Brahmagupta also authored "Khandakhadyaka" on mathematics and astronomy. The "Brahma Sphuta Siddhanta" [43] studies longitudes of the planets, mean longitudes of the planets, the three problems of diurnal rotation, lunar eclipses, solar eclipses, sunrises and sunsets, the moon's crescent, the moon's shadow, how the planets connect with each other, and conjunctions of the planets with the static stars as well. He defined zero as a consequence of subtracting a number from itself. He extended the arithmetic to negative numbers and zero(0) as well. He applied the place value system to its complete advantage in almost the same way as it is used nowadays. He characterized zero(0) as follows:

"When zero is added to a number or subtracted from a number, the number remains unchanged; and a number multiplied by zero becomes zero".

Brahmagupta authored several chapters in "Brahma Sphuta Siddhanta" on mathematics. In particular, his chapters 12 and 18 laid the base of the two large fields of Indian mathematics, "Pati Ganita", that is mathematics of procedures, or algorithms and "Bija Ganita" that is mathematics of seeds, or equations. These less or more correspond to the arithmetic with mensuration and algebra, respectively. The Chapter 12 is simply named Mathematics, perhaps because of the fundamental operations, such as arithmetic operations and proportions, and the mathematics, such as mixture and series. These took the large parts of the mathematics of Brahmagupta's milestone. The Chapter 18, Pulverizer, is named after the first topic of the chapter.

Brahmagupta in the "Khandakhadyaka" devised use in a special case study of the Newton Stirling interpolation formula of the second order to interpolate new values of the sine function from other values already tabulated [26]. He also gave to the world the sine table as well as the Pythagorean Triples. His most significant contribution was the introduction of zero (0). He considered zero (0) as a number in its own right, not simply a placeholder digit as was interpreted by the Babylonians. The Greeks and Romans basically considered zero (0) in a symbolic form for a lack of quantitative measures, and also that standing concepts for nothing.

Brahmagupta established the basic mathematical rules for dealing with zero, i.e., $1 + 0 = 1$; $1 - 0 = 1$; and $1 \times 0 = 0$. However, he thought that 1 divided by 0 = 0, that is considered correct for centuries. In the 12th century, almost 500 years later, another Indian mathematician, called Bhaskara II was able to confirm that 1 divided 0 as infinity, not zero on the logical basis that 1 is divisible into an infinitely many pieces of size zero. However, this logic does not make point why 6 divided by 0, 5 divided by 0, 99 divided by 0 and so on, should also be infinity.

The modern viewpoint is that a number divided by zero is undefined denoted by ∞ . The infinity is not a number, + infinity means, no matter however large a number is, there always exists a larger number, and -infinity means, no matter however small a number is, there always exists a smaller number. A negative times a negative is a positive, and a negative times a positive is a negative. The number zero (0) is an additive identity with annihilating multiplicative property, not like absorbing element from left or right, however, it is multiplicatively removes the existence of every number to replace by itself.

Other than his work on solutions to general linear equations and quadratic equations, Brahmagupta went further ahead by considering systems of simultaneous equations. He solved quadratic equations with two unknowns. This was not even

considered in the West until a thousand years later, when Fermat considered similar problems in 1657. He gave a formula, now known as Brahmagupta's Formula, for the area of a cyclic quadrilateral. He also authored a theorem on the diagonals of a cyclic quadrilateral known as Brahmagupta's Theorem.

In his work on arithmetic, Brahmagupta explained how to find the cube and cube-root of an integer and gave rules facilitating the computation of squares and square roots. This is based on an algorithm that is found equivalent to the Newton-Raphson iterative formula [27]. For the original Sanskrit versions from Brahmagupta's Brahmasphuta Siddhanta, and their translated version in English language with modern interpretation, one can refer to [24]. He also gave five types of combinations of fractions. For Origin and Growth of Astronomy in India, one can see to view to listen to observe the lecture [41].

3. Madhava of the town Sangamagrama

As the founder of the Kerala School of Astronomy and Mathematics, India, Madhava of the town of Sangamagrama [16, 28] is the most prominent figure in Mathematics and Astronomy. He is creditable for introducing many concepts in mathematics like that of mathematical induction and some early calculus related concepts. Madhava made pioneer contribution to infinite series expansions, calculus, trigonometry, geometry and algebra. Even though, no systematic rules for calculus were developed by the Kerala School, but its proponents first conceived of many of the results that would later be rediscovered in Europe including Taylor series expansions, differentiation, and in finitessimals. He is credited with developing infinite series of trigonometric functions, for instance, $\arctan(\theta)$, $\sin(\theta)$, and $\cos(\theta)$ which appeared in Europe 300 years after ward. This went forward ahead from the finite techniques of classical mathematics to deal with the finite limit passage to infinity and infinite series expansions. For his work, one can also refer to Rajagopal and Rangachari [3].

Madhava [25] found an approximation for π correct to 11 decimal places as $\pi = 3.14159265359$ much before Leibnitz in Europe. Madhava discovered the series equivalent to the Maclaurin expansions of $\sin x$, $\cos x$, and $\arctan x$ around 1400, which is over two hundred years before they were rediscovered in Europe. Numerous detailed works appear written by his followers such as Mahajanayana Prakara which means "Method of Computational Technique" the great sines. In fact, some historians [see Sarma [18], for instance] had claimed this work to be Madhava's himself. But this seems highly unlikely. It is now generally accepted to be the 16th century work by a follower of Madhava.

Other than developing the series expansion, another noteworthy feature of Madhava's work is that he develops other series which converge faster. He also gives techniques to obtain good error estimates. Thus, it seems that he have had clearly grasped the ideas about the future trends in mathematical analysis.

Madhava's work, via the writings of Kerala School, went to Europe [2] through traders and Jesuit missionaries. This may have had an impact on later European developments in calculus and analysis [6]. He also solved some transcendental equations by iteration process, and with continued fractions, approximated transcendental numbers as well. He discovered an algorithm to find the location of the moon per every 36 minutes. He further went on to estimate the motions of the planetary heavenly bodies.

4. Srinivasa Ramanujan

Srinivasa Ramanujan was born in Erode, Tamil Nadu, India, [17, 19]. He began developing his theories in mathematics. He published his first paper in 1911 in the "Journal of the Indian Mathematical Society" [1]. Every year, Srinivasa Ramanujan's birth anniversary on December 22 is commemorated as "National Mathematics Day". He is a self-taught Indian mathematician who coined the idea of "taxicab numbers (Hardy Ramanujan Number)", that is, the n th Hardy-Ramanujan number is the smallest integer that can be expressed in the form of a sum of two positive integer cubes in n distinct ways like, for instance, the natural number 1729 following 1728 and preceding 1730. Interestingly, $1729 = 1^3 + 12^3 = 9^3 + 10^3$. The six taxi numbers are known so far. The same expression defines 1729 as the first in the sequence of "Fermat's near misses", that is, numbers of the form $1 + z^3$ that are also expressible as the sum of two other cubes defined in respect of Fermat's Last Theorem.

Ramanujan's life, had been very humble and difficult beginnings, but he continued his work anyhow. He wrote to G. H. Hardy, a British Mathematician, to ask him for help. Hardy responded positively. He guided, recognized and facilitated him at Cambridge and encouraged Ramanujan to publish his findings. Ramanujan is the most popular for his contribution in analytical theory of numbers, elliptic functions, continued fractions, infinite series, and the properties of the partition of function some of which he delivered with proof, and stated many more without proof. His other contributions include hypergeometric series, the Riemann series, the elliptic integrals, the theory of divergent series, and the functional equations of the zeta function. Mathematicians have been able to calculate the value of π correct to millions of places of decimal by using one of his identities. The field of number theory in mathematics has been enriched with his intuitive research work and his vast contributions. As a result of his work, the modern arithmetic theory of modular forms occupies a central place in number theory. Ramanujan also had remarkable mastery knowledge of continued fractions.

5. Prasanta Chandra Mahalanobis

P. C. Mahalanobis [31] was an Indian scientist and statistician from the erstwhile Calcutta, Bengal, now modern West Bengal. He is considered the father of modern statistics in India. He is the best known for being one of the members of the first Planning Commission of free India. He also came up with pioneering studies in anthropometry in India. Mahalanobis founded the Indian Statistical Institute (ISI). He started human genetical studies in ISI. With his interests, many scientists worked in genetics, epidemiology, health and nutrition, and human ecology and adaptation.

His most significant contribution in the field of statistics has been a statistical measure called the Mahalanobis Distance. He also contributed to the design of large scale sample surveys in India [29]. He [30] launched Sankhya: The Indian Journal of Statistics.

Mahalanobis' tutor, W. H. Macaulay in England, drew his attention to the journal Biometrika. He purchased some papers and brought them to India. He developed an interest in biometry. He researched statistical problems in agriculture, biology, meteorology, and anthropometry, that is, the scientific study of the measurements and proportions of human body. Mahalanobis gave "natural" generalized distance D^2 for correlated variates, as well as its Studentized

form using sample values of parameters. Both these measures play a basic crucial role in statistics and data analysis [14].

6. Calyampudi Radhakrishna Rao

Calyampudi Radhakrishna Rao, popularly known as C. R. Rao, is a well-known mathematician and statistician. He is famous for his "theory of estimation". He was born in Karnataka, India. His contributions to statistical theory and applications are well known, and many of his results, which bear his name, are included in the curriculum of courses in statistics at bachelor's and master's level all over the world. Rao authored the books like "Advanced statistical methods in biometric research", and a more mathematical nature "Linear statistical inference and its applications" for mathematical statistical courses in universities.

He also authored "Computers and the Future of Human Society", and jointly with Sujit Kumar Mitra, he wrote "Generalized inverse of matrices and its applications". His other books are "Characterization Problems of Mathematical Statistics", Estimation of variance components and applications (with J. Kleffe), Statistics and truth, Putting Chance to work, Choquet-Deny type functional equations with applications to stochastic models (with D. N. Shanbhag), Linear models (with H. Toutenburg), Least squares and alternatives, and Matrix algebra and its applications to statistics and econometrics (with M. B. Rao).

The Rao-Blackwell and the Cramer-Rao theorem that provides a bound for the variance of an unbiased estimate of a parameter. His Ph.D. studies at King's college, Cambridge were guided by R. A. Fisher. Rao bagged 38 honorary doctoral degrees from universities worldwide in 19 countries and several awards and medals for his contributions to statistics and science. The Times of India listed Rao as one of the top 10 Indian scientists of all time [32]. For this living legends in Indian Science, one can refer to B. L. S. Prakasa Rao: Living Legends in Indian Science [33]. He received so many honours and awards including Andhra Pradesh Academy of Sciences.

7. Dattaraya Ramchandra Kaprekar

Dattaraya Ramchandra Kaprekar, born in Dahanu, Mumbai, Maharashtra, India, worked in Number Theory. He was a recreational mathematician who described several classes of natural numbers including the "Kaprekar" [7], Harshad number (Niven number) and "Self-numbers (Swayambhu)" [12]. He also discovered the "Kaprekar constant" [8]. Harshad number in a given base number is an integer that is divisible by the sum of its digits when written in that base. Harshad numbers in base n is called n -Harshad number. Harshad is a Sanskrit word meaning joy-giver.

The number 18 is a Harshad number in base 10 since the sum of the digits 1 and 8 is 9, and 18 is divisible by 9. The Hardy-Ramanujan number 1729 is a Harshad number in base 10 as $1+7+2+9=19$, and $1729=19 \times 91$. The base number and its powers will always be a Harshad number as it is 10 and $1+0=1$. The number 99 is not divisible by $18=9+9$, hence it is not a Harshad number. A number which is a Harshad number in every number base is called an all-Harshad number, or an all-Niven number. For a prime number to be a Harshad number, it must be less than or equal to the base number. Otherwise, the digits of the prime will sum up to a number that is more than 1, but less than prime, and will not be divisible.

For an interesting property of number 6174, one can refer to [9]. He also introduced "Demlo numbers" and this name

generates from his mind at the station where he was changing trains on the Bombay to Thane line having had the idea of working with numbers of that kind ^[10, 11]. Without any formal mathematical education, he published extensively and was very well known in recreational mathematical circle.

8. Satyendra Nath Bose

Satyendranath Bose was born in Kolkata (Calcutta), India. He established modern theoretical physics in India. Satyendranath Bose ^[34] is best known for his work in quantum mechanics. He was specialized in mathematical physics. He is also popular

for his collaboration with Albert Einstein. He also worked on the Bose-Einstein statistics and the theory of Bose-Einstein condensate theory. He developed the statistical mechanics for bosons of a photon gas. He is honored as the namesake of the boson the sub-atomic particles ^[35].

Bose's numerous scientific contributions are statistical mechanics, the electromagnetic properties of the ionosphere, the theories of X-ray crystallography and thermoluminescence, and unified field theory. Bose's Planck's Law and the Hypothesis of Light Quanta led Einstein to seek him out for collaboration ^[36]. He has worked in the fields of nuclear physics, biotechnology, zoology, chemistry, geology, anthropology, engineering, and literature. Rabindranath Tagore dedicated his only book (Visva Parichay) in the field of science to Bose.

9. Bhaskara I

Bhaskara I ^[39] was an Indian mathematician and astronomer. He was born in modern Gujarat, India. He was the first to author numbers in the Hindu decimal system (Hindu Arabic) with a circle for zero (0). The satellite built by the Indian Space Research Organization (ISRO) was named Bhaskara-I in his honour. He was an author of two books, and also extended work of Aryabhata I: "Mahabhaskariya", and "the Laghubhaskariya". The Mahabhaskariya is an eight (8) chapter based work on Indian Mathematical Astronomy: a nice piece of authorship at this time. He also wrote the Aryabhaticabhasa on the Aryabhaticiya by Aryabhata I. Without the explanation of Bhaskara I, it would have been difficult to interpret Aryabhata's verses. Bhaskara I constructed a problem to find the least natural number N which leaves the remainder 1 when divided by 2, 3, 4, 5, or 6 but is exactly divisible by 7. This question can also be seen in Ibn Al Hatim and Leonardo Fibonacci. The next one from the same author was to find the least number N which when divided by 8 leaves remainder 5 (as remainder), divided by 9 leaves 4 and divided by 7 leaves 1.

He introduced the topic of quadrilaterals with four sides unequal. He developed Planetary longitudes, heliacal rising and setting of the planets, conjunctions among the planets and stars, solar and lunar eclipses, and the phases of the Moon are among the topics Bhaskara discussed in his astronomical treatises.

Bhaskara I explains in detail Aryabhata's method of solving linear equations and provides a number of illustrative astronomical examples. Bhaskara I termed this method "Kuttaka (Pulverizer)". Laghubhaskariya of Bhaskara I studies a problem related to the revolutions of Saturn. Shukla ^[15] gives detailed account of his mathematical contributions: quadratic equations, cubic equations, and equations with more than one unknown, symbolic algebra, numbers and symbolism, weights and measures, the Euclidean algorithm for solving linear indeterminate equations, explaining

Aryabhata I's rules. Bhaskara particularly stressed the importance of proving mathematical rules rather than just relying on tradition or expediency. In his writings, there are clues to possible locations for his life, such as Valabhi, and the location of a school of followers of Aryabhata, the capital of the Maitraka dynasty, and Ashmaka, a town in Andhra Pradesh, India.

10. Narendra Krishna Karmarkar

Narendra Krishna Karmarkar ^[40], a mathematician, was born in Gwalior, Madhya Pradesh, India. He introduced a polynomial-time algorithm known as Karmarkar's algorithm for solving linear programming problems that is the optimization subject of study of Operation Research in Mathematics. The algorithm is a milestone in the field of Linear Programming. This algorithm solved numerous Linear Programming Problem (LPP) in the polynomial time. The complex optimization problems are also solved much faster applying the Karmarkar's algorithm. With the efficiency of this algorithm, all the solution time involved in the communication optimization network reduced from weeks to days. Karmarkar's innovative and novel method solves through cutting the above solid in its transversal. He devised a new architecture for supercomputing based on ideas from finite geometry, projective geometry over finite fields.

LPP maximizes or minimizes (Optimizes) a linear objective function subject to certain constraints on the variables (linear inequalities) with programs models to optimize problems in business, science, and industry. The "Simplex" method of Dantzig took exponential time on certain instances. Khachian explained that "Ellipsoid" algorithm could solve LPP in polynomially bounded time but it was much slower than Simplex method. Karmarkar introduced a third approach, the "Interior Point" with improvement over that for Khachian's algorithm. This cleared the earlier barrier methodologies by researchers in nonlinear programming problem. Karmarkar's work were greeted with skepticism but he inspired a renaissance and paradigm shift in the theory and practice of LPP to improve and extend the Simplex Method.

He also served as the scientific advisor to the chairman of the TATA group. During this time period, he was granted funds by the Ratan Tata to make the supercomputer scaled up by making prototype at TIFR. He was the founding director of Computational Research labs in Pune, India. At this lab, he scaled up the performance. He keeps on working on his new architecture design for supercomputing. He is also listed among ISI highly cited researchers. He is synthesizing these ideas with some new ideas known as sculpturing free space extending his work to the physical design of machines.

11. Harish Chandra

Harish Chandra ^[37] was born in Kanpur, near Prayagraj (Allahabad) in Uttar Pradesh, India. He was an Indian American mathematician and physicist who did fundamental work in representation theory. He especially developed harmonic analysis on semisimple Lie groups. He was greatly influenced by the mathematicians H. Weyl and C. Chevalley. He did work in pure mathematics at the Institute for Advanced Study in Princeton ^[38].

He published papers on theoretical physics. He extended some of Dirac's results, and published a number of joint papers with Bhabha. He studied for his Ph.D. under Dirac's supervision. He worked on Infinite Lorentz group in which he gave a complete classification of the irreducible unitary representations of $SL(2, C)$.

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