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An average range modulated method for solving transportation problems

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Abstract
This paper proposes an alternative approach of the average penalty method for solving transportation problems. The new method makes use of average range method approach to obtain efficient optimal transportation problems. Numerical examples show simplicity of the algorithm developed, its efficiency and, it compares favorably with the state of art algorithms in literature.

Keywords: Sources, optimal solution, modulated average, penalty, destinations

1. Introduction

Transportation problems are everyday problems arising from the process of moving items/goods from one point to another. In other words, goods/products/items are shipped/transported from the place of origin (supply points, sources, and warehouses) to distinct places of needs or destinations (demand points, sinks, delivery centres, etc) on daily bases. The basic concern in transportation problem is how to minimize the cost of shipping/moving these goods from one point (source) to another (demand) [1]. Defines Transportation problem as an extraordinary kinds of linear programming problems in which the objective is to transport various quantities (goods) initially stored at different origins/plants/factories to various destinations/ distribution centres/warehouses in such a way that the total transportation cost is minimized [2]. Noted that the basic idea in transportation problems is to develop an integral schedule that meets all demands requirements from current inventory at minimum total shipping cost [3]. Puts it succinctly that transportation problem tries to provide optimal costs in transportation system.

Thus, there are many conventional transportation mathematical models in literature that used to obtain the initial Basic Feasible Solution (IBFS). The slack is that the (IBFS) may not reach the optimal solution; hence another method called Modified Distribution (MODI) is applied thereafter to obtain the optimality of the solution [4]. Some of these conventional methods are Northwest Corner Method (NWCM) [5]; Least Corner Method (LCM) and Vogel's Approximation Method (VAM) [6]. Some new efficient methods are recently evolved that can reach optimality or near optimality as thus: A new method for solving transportation problems considering average penalty by [3], An efficient methodology for solving transportation problem, by [7]; and An improved vogel's approximation method for the transportation problem (IVAM), by [8]. This method was conspicuously more efficient in finding initial solutions for large scale transportation problems.

In this paper a modulated average penalty approach is presented. The method, instead of taking arithmetic average penalty, adopts Range average penalty approach. This method is simple and reaches optimal or near optimal solution in few iterations and compares favorably with the state of art transportation algorithms.

The paper is divided into sections; presentation of the transportation mathematical models is in section 2, section 3 presents the modulated method, section 4 discusses some other different methods and its algorithms, section 5 provides numerical examples and solutions, section 6 presents table of comparison and discussion, and section 7 concludes and recommends.

2. Mathematical Model

Transportation problem involves m sources, each has available a_i units of homogeneous products, and n destinations, each requiring b_j units of these products. The numbers a_i and b_j are positive integers. The cost, C_{ij} of transporting a unit of products from the ith source to the j^{th} destination is given for each i and j; see [3] and [9].

The basic idea of transportation is to minimize the cost of shipping or moving goods from one location to another without violating the law tying the supply and demand.

Designate a_i ,=1,2, . . ., m and b_j , j=1,2, . . ., n as supply and demand points respectively. It is normally assumed that the total supply and total demand are equal; hence it is referred to as balanced transportation problem, otherwise unbalanced. This implies that $\sum_{i=1}^{m} a_i = \sum_{j-1}^{n} b_j$. This equation is guaranteed by creating either a fictitious (dummy) destination with demand equal to the supply if the total demand is less than total supply or a fictitious (dummy) source with supply equal to the shortage if the total demand exceeds total supply [2]. The transportation cost of the dummy demand or supply is zero.

Again, this transportation diagram could be transformed into table as below.

Table 2.1: Transportation table

Destination							
	D_1	D_2	.	.	.	Dn	supply a_i
S_1	C_{11} X_{11}	C_{12} X_{12}	.	.	.	C_{1n} X_{1n}	$a1$
S_2	C_{21} X_{21}	C_{22} X_{22}	.	.	.	C_{2n} $X2n$	$a2$
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
S_m	C_{m1} X_{mi}	$Cm2$ $Xm2$	.	.	.	C_{mn} X_{mn}	am
demand b_j	b_1	b_2	.	.	.	b_n	

The S_i, is the i^{th} source point and m, the number of source points; Dj is the j^{th} destination point and n is the number of destination (demand) points; X_{ij} is the number of good moved from source i to destination, j ; C_{ij} is the cost of moving a unit of item from source i, to destination j; a_i is the total number of items in source i, whereas b_j is the total number of items needed in destination j.

The mathematical model is given as

Minimize Z= $\sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij} X_{ij}$

Subject to $\sum_{j=n}^{n} X_{ij} = a_i; i = 1,2,..m$

$$\sum_{i=1}^{m} X_{ij} = b_j; j = 1,2,..n$$

Where $X_{ij} \geq 0$

3. The proposed algorithm/method (modulated penalty approach)

The new method was a modulated approach of the arithmetic average penalty of transportation matrix. The steps are:

I. Balance the demand and supply values of the transportation matrix table, if not balance, otherwise go to step 2.
II. Find the smallest element in each row of the cost matrix and subtract it from each element of that row, write it as a superscript in each element of that row.
III. In the same manner, find the smallest element in each column and then subtract it from each element of that column and write the result as subscript in each element of the column.
IV. Take average of the difference of the maximum value and least value of the superscript of each row and record it at the right hand side of the table (row penalty or row average range).
V. Again, take average difference of the maximum value and the least value in each column and record it at the bottom of the table (column penalty or column average range).
VI. Thereafter, pick the maximum value among the row and column average difference and allocate as many units as possible to the minimum cost unit in the table without violating the constraints. If there is a tie go for the one with the most minimum unit cost.
VII. Eliminate any row or column that is satisfied.
VIII. Repeat step 4 step 7 until all the supply and demand requirements are met.

Note that the range of in this method without taking the average also takes us to optimal result.

4. Some existing efficient methods and its algorithms

Other efficient algorithms for solving transportation problems in the literature are presented hereunder and compared with the proposed method.

4.1 Northwest corner method (NWCM)

This is among the first method in use for solving transportation problems. The method is summarized in the steps below.

(i) Balance the transportation problem if the total supply is greater than demand or total demand is greater than total supply.
(ii) Begin with cell (1, 1) in the transportation table, allocate to X_{11} as many units as possible without violating the constraints.
(iii) Continue by moving one cell to the right, if supply remains in that row or if not, one cell down.
(iv) At each step, allocate as much as possible to the cell under consideration without violating the constraints.
(v) Continue until all the demand and supply requirements are met.

4.2 Vogel's Approximation Method (VAM)

This method is another method of solving transportation problems. It is usually close to optimal, if not optimal in attempt to obtain the initial feasible solution.
The steps are presented below.

(i) Balance the transportation problem, if not balanced.
(ii) Obtain the difference between the smallest and the next smallest units in the transportation cost table along the rows and columns.
(iii) From the row or column that has the largest difference, allocate as much as possible to the least cost in that row or column without violating the constraint.
(iv) Then, adjust the demand and supply, and cross out the satisfied row or column.
(v) Repeat the step two to four until all the supplies are exhausted and all the demands are met.

4.3 Least Cost Method (LCM)

The algorithmic steps are given below

(i) Balance the transportation table.
(ii) Identified the cell with the least unit cost in the entire transportation table.
(iii) Allocate as much units as possible to the cell identified in 2 above without violating the constraints. Thereafter, the row or column whose demand is satisfied is eliminated for further considerations.
(iv) Adjust the demand and supply for all uncrossed rows and columns after step 3 above and select another cell with the next smallest unit cost in the table.
(v) Again, allocate as much units as possible to this next smallest unit cost without violating the constraints.
(vi) Repeat the process until the available supply and demand at various sources and demands are satisfied.

4.4 A new method for solving transportation problems (ANMSTP)

(i) Subtract the smallest entry from each of the element of every row of the transportation table and place them on the right top of the corresponding element.
(ii) Apply the same process on each of the columns and place them on the right bottom of the corresponding elements.
(iii) Place the average row penalty (ARP) and the average column penalty (ACP) just after and below the supply and demand amount respectively within first brackets, which are averages of the transportation table.
(iv) Find the highest element among the ARP and ACP, if there are two or more highest element; choose the highest element along which the smallest cost elements is present. If there are two or more smallest elements, choose any one of the arbitrarily.
(v) Allocate X_{ij} = min ($a_i b_j$) on the left top of the smallest entry in the (i, j)th of the transportation table.
(vi) If ($a_i < b_j$), leave the ith row and adjust b_j as $b_j^{'} = b_j - a_i$ or if ($a_i > b_j$), leave the jth column and readjust (a_i) as $a_i^{'} = a_i - b_j$.

5. Numerical examples and solutions

Al the examples are taken from [3]

Illustration 1: The per unit transportation cost (in thousand dollar) and the supply and demand (in number) of motor bike of different factories and showrooms are given in the following transportation table.

Table 5.1: Showrooms

Factories	1	2	3	4	Supply
A	9	8	5	7	12
B	4	6	8	7	14
C	5	8	9	5	16
Demand	8	18	13	3	42

Subtract the smallest cost from every element in row and column placed them as the superscript and the subscription on each cost element of the rows and columns respectively.

Table 5.2: Showrooms

Factories	1	2	3	4	Supply
A	9^4_5	8^3_2	5^0_0 (12)	7^2_2	12
B	4^0_0 (8)	6^2_0 (6)	8^4_3	7^3_2	14
C	5^0_1	8^3_2 (12)	9^4_4 (1)	5^0_0 (3)	16
Demand	8	18	13	3	42

The average rang differences of the rows and the columns are placed as the average range penalty of the rows and columns respectively.

Table 5.3: Showroom

Factories	1	2	3	4	Supply	Average row penalty				
A	9^4_5	8^3_2	5^0_0 (12)	7^2_2	12	2.0	1.5	-	-	
B	4^0_0 (8)	6^2_0 (6)	8^4_3	7^3_2	14	2.0	1.0	1.0	1.0	-
C	5^0_1	8^3_2 (12)	9^4_4 (1)	5^0_0 (3)	16	2.0	2.0	0.5	0.5	0.5
Demand	8	18	13	3	42					
Average column penalty	2.5	1.0	2.0	1.0						
	-	1.0	2.0	1.0						
	-	1.0	0.5	1.0						
	-	1.0	0.5	-						

Minimize Z= $\sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij} X_{ij}$

= 5x12+4x8+6x6+8x12+9x1+5x3
= 248

Illustration 2

A company manufactures Toy Robots for children and it has three factories whose weekly production capacities are 3, 7, and 5 hundred pieces respectively. The company supplies Toy Robots to its four showrooms located at four different locations whose weekly demands are 4, 3, 4, and 4 hundred pieces respectively. The transportation cost per a hundred pieces of Toy Robots is given below in the transportation table.

Table 5.4: Showrooms

Factories	1	2	3	4	Supply
A	2	2	2	1	3
B	10	8	5	4	7
C	7	6	6	8	5
Demand	4	3	4	4	15

Subtract the smallest cost from every element in row and column placed them as the superscript and the subscription on each cost element of the rows and columns respectively.

Table 5.5: Show rooms

Factories	1	2	3	4	Supply
A	2^1_0	2^1_0	2^1_0	1^0_0	3
B	10^6_8	8^4_6	5^1_3	4^0_3	7
C	7^1_5	6^0_4	6^0_4	8^2_7	5
Demand	8	18	13	3	15

The average rang differences of the rows and the columns are placed as the average range penalty of the rows and columns respectively.

Table 5.6: Showroom

Factories	1	2	3	4	Supply	Average row penalty				
A	2^1_0 (3)	2^1_0	2^1_0	1^0_0	3	0.5	-	-	-	
B	10^6_8	8^4_6	5^1_3	4^0_3	7	3.0	3.0	2.5	-	-
C	7^1_5 (1)	6^0_4 (3)	6^0_4 (1)	8^2_7	5	1.0	1.0	0.5	0.5	
Demand	4	3	4	4	15					
Average column penalty	4.0	3.0	2.0	3.5						
	1.5	1.0	0.5	2.0						
	1.5	1.0	0.5	-						

Minimize Z= $\sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij} X_{ij}$

= 2x3+5x3+4x4+7x1+6x3+6x1 =68

Illustration 3

A company manufactures toilet tissues and it has three factories whose weekly production capacities are 9, 8, and 10 thousand pieces of toilet tissues respectively. The company supplies tissues to its three showrooms located at three different places whose weekly demands are 7, 12, and 8 thousand pieces respectively. The transportation costs per thousand pieces are given in the transportation table below.

Table 5.7: Showrooms

Factories	1	2	3	Supply
A	4	3	5	9
B	6	5	4	8
C	8	10	7	10
Demand	7	12	8	27

Subtract the smallest cost from every element in row and column placed them as the superscript and the subscription on each cost element of the rows and columns respectively.

Table 5.8: Showrooms

Factories	1	2	3	Supply
A	4^{1}_{0}	3^{0}_{0}	5^{2}_{1}	9
B	6^{2}_{2}	5^{1}_{2}	4^{0}_{0}	8
C	8^{1}_{4}	10^{3}_{7}	7^{0}_{3}	10
Demand	7	12	8	27

The average rang differences of the rows and the columns are placed as the average range penalty of the rows and columns respectively.

Table 5.9: Showroom

Factories	1	2	3	supply	Average row penalty				
A	4^{1}_{0}	3^{0}_{0}(9)	5^{2}_{1}	9	1.0	-	-	-	
B	6^{2}_{2} (8)	5^{1}_{2}(3) (6)	4^{0}_{0}(5)	8	1.0	1.0	1.0	-	
C	8^{1}_{4}(7)	10^{3}_{7}	7^{0}_{3}(3)	10	1.5	1.5	0.5	0.5	-
Demand	7	12	8	27					
Average column penalty	2.0	3.5	1.5						
	1.0	2.5	1.5						
	1.0	-	1.5						

Minimize Z= $\sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij} X_{ij}$

= 3x9+5x3+4x5+8x7+7x3
= 139

6. Comparison and Discussion

Table 6.1: An evaluation table

Illustrations	Modulated	VAM	NWCM	ANMSTPP	LCM
1	248	248	320	248	248
2	68	68	93	68	79
3	139	150	150	139	145

It is observed in the table above that only the modulated (the new method) and the A new method for solving transportation problems considering average penalty got the optimal solution to the three illustrations given. However, it is observed that the modulated method simple and fastest with less serious calculations.

Other existing methods, the Vogel's Approximation Method (VAM), Northwest Corner Method (NCWM) and the Least Corner Method (LCM) that are compared with the new method could only get one or two optimal solutions with more laborious calculations.

7. Conclusion

The modulated penalty method has shown to have capacity to reach the optimal feasible solution of solving transportation problems very quickly. The simplicity of the method gives the method an edge over others.

8. Recommendation

- We recommend this method to all the practitioners because of its steady fast and simplicity.
- Researchers should as well try this method in large scale transportation problems to check for its veracity and efficiency.

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