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Analysis of S&P 500 stocks: Long-term vs short-term investment

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Abstract

This paper is concerned with analyzing the stocks in the Standard & Poors 500 Index that potentially yield high returns. To achieve this goal, historical stock prices and their corresponding simple moving averages are used together to capture the stock trend directions. When the time series of the stock crosses over its corresponding simple moving average line from below (or above), it indicates an up (or down) trend signal and a buy (or sell) is initiated. A trading strategy consisting of these buying and selling indicators is implemented. Simple moving averages of various lengths are constructed and employed in order to maximize the return for each stock. The results suggest that long-term trading can potentially yield high returns and short-term trading generally perform poorly.

Keywords: Time series, simple moving average, long-term trading, short-term trading, stocks

1. Introduction

A time series of daily stock prices can be classified as a geometric Brownian motion (GBM), a mean-reverting process (MR), or a trend-following process (TR). Trading strategies have been developed for each type of processes.

A GBM is technically a Markov process. This means the stock prices follow a random walk. Merhi and Zervos (2007) ^[10] studied an investment capacity expansion/reduction problem following a dynamic programming approach under a GBM market model. In addition, Guo and Zhang (2005) ^[3] studied the optimal selling rule under a model with switching GBM. Using a smooth-fit technique, they obtained the optimal threshold levels by solving a set of algebraic equations.

A MR refers to a time series that displays a tendency to revert to its historical mean value. Such a time series is referred to as an Ornstein-Uhlenbeck process. The mean-reverting property of a time series can be exploited in order to produce profitable trading strategies. A common strategy in mean-reversion trading is to buy low and sell high. For example, Luu *et al.* (2018a) ^[8] focused on a threshold type policy and developed a method that was easy to implement in practice. They formulated the optimal trading problem in terms of a sequence of optimal stopping times. The optimal threshold levels could be obtained by solving quasi-algebraic equations. Recently, Luu *et al.* (2018b) ^[9] developed pairs trading strategies under both mean-reverting and geometric Brownian motion models. They showed that pairs trading ideas were more general and not necessary to be cast under a mean-reverting framework.

A TR is assumed to continue to move in the same direction as the current trend. Dai *et al.* (2010) ^[1] developed a trend-following rule based on a conditional probability indicator. They showed that the optimal trading rule can be determined by two threshold curves which can be obtained by solving the associated Hamilton-Jacobi-Bellman equations. A similar idea was developed following a confidence interval approach by Iwarere and Barmish (2010) ^[4].

Trading strategies have been studied for many years and never before have stock investors heavily relied on them. An effective strategy also needs appropriate stocks to work well. In this paper, instead of developing trading strategies, a simple moving average method is implemented to identify stocks in the Standard & Poor's 500 Index that potentially yield high returns. The results show that high returns only exist in long-term trading. In addition, long-term investment can fail in many stocks and so do short-term trading in general.

Moving averages are one of the popular technical indicators in the foreign exchange market (Forex). They have become a staple part of many trading strategies because they are simple to use and apply. While they have been around for a long time, their ability to be easily measured, tested and applied makes them an ideal foundation for modern trading strategies which can incorporate both technical and fundamental analysis (Foreign Exchange Market, 2020) ^[2].

This paper is organized as follows. In §2, we describe the material and method. In particular, we develop an algorithm for maximizing the stock return using a simple moving average approach. In §3, we apply the algorithm for each stock in the S&P 500 Index to identify stocks with high returns and discuss the results. In §4, we conclude the paper by making some remarks.

In the following sections, abbreviations are written in normal texts while their corresponding variables and functions are *italicized*.

2 Material and Method

2.1 Data Description

Standard & Poor's 500 Index (S&P 500) is a market-capitalization-weighted index of the 500 largest U.S. publicly traded companies. The index is widely regarded as the best gauge of large-cap U.S. equities (Kenton, 2018) ^[5]. In this paper, 478 companies' daily adjusted prices of S&P 500, which have available data for the full six years or 1508 daily observations, were collected from Yahoo Finance from January 2nd 2014 through December 27th 2019. These companies' symbols include:

MMM, AOS, ABT, ABBV, ABMD, ACN, ATVI, ADBE, AAP, AMD, AES, AFL, A, APD, AKAM, ALK, ALB, ARE, ALXN, ALGN, ALLE, ADS, LNT, ALL, GOOGL, GOOG, MO, AMZN, AMCR, AEE, AAL, AEP, AXP, AIG, AMT, AWK, AMP, ABC, AME, AMGN, APH, ADI, ANSS, ANTM, AON, APA, AIV, AAPL, AMAT, APTV, ADM, AJG, AIZ, T, ATO, ADSK, ADP, AZO, AVB, AVY, BKR, BLL, BAC, BAX, BDX, BBY, BIIB, BLK, BA, BKNG, BWA, BXP, BSX, BMY, AVGO, BR, CHRW, COG, CDNS, CPB, COF, CAH, KMX, CCL, CAT, CBOE, CBRE, CDW, CE, CNC, CNP, CTL, CERN, CF, SCHW, CHTR, CVX, CMG, CB, CHD, CI, CINF, CTAS, CSCO, C, CTXS, CME, CMS, KO, CTSH, CL, CMCSA, CMA, CAG, CXO, COP, ED, STZ, CPRT, GLW, COST, COTY, CCI, CSX, CMI, CVS, DH, DHR, DRI, DVA, DE, DAL, XRAY, DVN, DXCM, FANG, DLR, DFS, DISCA, DISCK, DISH, DG, DLTR, D, DPZ, DOV, DTE, DUK, DRE, DD, DXC, ETFC, EMN, ETN, EBAY, ECL, EIX, EW, EA, EMR, ETR, EOG, EFX, EQIX, EQR, ESS, EL, RE, EVRG, ES, EXC, EXPE, EXPD, EXR, XOM, FFIV, FB, FAST, FRT, FDX, FIS, FITB, FRC, FE, FISV, FLT, FLIR, FLS, FMC, F, FTNT, FBHS, BEN, FCX, GPS, GRMN, IT, GD, GE, GIS, GM, GPC, GILD, GPN, GL, GS, GWW, HRB, HAL, HBI, HOG, HIG, HAS, HCA, PEAK, HSIC, HES, HLT, HFC, HOLX, HD, HON, HRL, HST, HPQ, HUM, HBAN, HII, IEX, IDXX, ITW, ILMN, INCY, INTC, ICE, IBM, IP, IPG, IFF, INTU, ISRG, IVZ, IGP, IQV, IRM, JBHT, JKHY, J, SJM, JNJ, JCI, JPM, JNPR, KSU, K, KEY, KMB, KIM, KMI, KLAC, KSS, KR, LB, LHX, LH, LRCX, LVS, LEG, LDOS, LEN, LLY, LNC, LIN, LYV, LKQ, LMT, L, LOW, LYB, MTB, MRO, MPC, MKTX, MAR, MMC, MLM, MAS, MA, MXIM, MKC, MCD, MCK, MDT, MRK, MET, MTD, MGM, MCHP, MU, MSFT, MAA, MHK, TAP, MDLZ, MNST, MCO, MS, MSI, MSCI, MYL, NDAQ, NOV, NTAP, NFLX, NWL, NEM, NWSA, NWS, NEE, NLSN, NKE, NI, NBL, JWN, NSC, NTRS, NOC, NCLH, NRG, NUE, NVDA, NVR, ORLY, OXY, ODFL, OMC, OKE, ORCL, PCAR, PKG, PH, PAYX, PNR, PBCT, PEP, PKI, PRGO, PFE, PM, PSX, PNW, PXD, PNC, PPG, PPL, PFG, PG, PGR, PLD, PRU, PEG, PSA, PHM, PVH, QCOM, PWR, DGX, RL, RJF, RTX, O, REG, REGN, RF, RSG, RMD, RHI, ROK, ROL, ROP, ROST, RCL, SPGI, CRM, SBAC, SLB, STX, SEE, SRE, NOW, SHW, SPG, SWKS, SLG, SNA, SO, LUV, SWK, SBUX, STT, STE, SYK, SIVB, SNPS, SYY, TMUS, TROW, TTWO, TPR, TGT, TEL, FTI, TFX, TXN, TXT, BK, CLX, COO, HSY, MOS, TRV, DIS, TMO, TIF, TJX, TSCO, TDG, TFC, TWTR, TSN, USB, UDR, ULTA, UAA, UNP, UAL, UNH, UPS, URI, UHS, UNM, VFC, VLO, VAR, VTR, VRSN, VRSK, VZ, VRTX, V, VNO, VMC, WRB, WAB, WBA, WMT, WM, WAT, WEC, WFC, WELL, WST, WDC, WU, WY, WHR, WMB, WLTW, WYNN, XEL, XRX, XLNX, XYL, YUM, ZBRA, ZBH, ZION, ZTS.

2.2 Simple Moving Average (SMA)

We use simple moving averages to identify trends in stocks. A simple moving average shows the average price over a certain period of time. As prices change, the corresponding simple moving averages generally either increase or decrease. A SMA is calculated by adding prices over a given number of periods, then dividing the sum by the number of periods or length. Each time a new period occurs, the moving average moves forward dropping its first data point and adding the newest one (Learn Price Action, 2020) ^[6]. For a time series of length N periods: $x_1, x_2, \dots, x_N$, a simple moving average of length n or n -day SMA is a time series of the form:

$$SMA_i = \frac{x_{i-n+1} + x_{i-n+2} + \dots + x_i}{n},$$

Where

$$i = n, n+1, \dots, N.$$

In this paper, historical daily adjusted prices of a stock and its corresponding simple moving averages are used together to capture the stock trend directions. When the time series of the stock crosses over its corresponding simple moving average line from below, it indicates an uptrend signal and a buy is initiated. On the other hand, when the time series of the stock crosses its corresponding simple moving average line from above, it indicates a down trend signal and a sell is initiated (Mitchell, 2020) ^[11]. A trading strategy consisting of these buying and selling indicators within a considered time frame is implemented.

The length of a SMA can play a significant role in the effectiveness of the method. A SMA with a short length reacts more quickly to price changes than that with a long length. For example, a 10-day SMA tracks the original prices more closely than a corresponding 100-day SMA does (Kenton, 2018) ^[5]. So it is possible to adjust the lengths so that the SMAs generate more reliable signals for the data. This may help create better future signals. While Forex trading commonly uses moving averages over lengths of 10, 50, 100, and 200 (Lioudis, 2020) ^[7], we use a bigger range of lengths in order to "maximize" the return for each stock. SMAs with lengths of 5, 10, 15, $\dots$, and 500 are employed in this paper.

2.3 Policy and Return

Almost all trading strategies involve optimizing the return: maximizing the return or profit. The returns investors can expect to receive in the future depend on what actions they will take. For a time series x of a stock's daily adjusted prices: $x_1, x_2, \dots, x_N$, where the dates are indexed as $1, 2, \dots, N$, a sequence of buying and selling actions at times $(b_1, s_1), (b_2, s_2), \dots$, within the time frame of the time series, is called a policy π^x . In a policy obtained from a SMA, b_1 (or s_1) is the time where the stock crosses over the SMA from below (or above respectively) for the first time. Similarly, b_n (or s_n) is the time where the stock crosses over the SMA from below (or above respectively) for the n^{th} times, where $b_1 < s_1 < b_2 < s_2 < \dots$.

Buying a stock at time b_i and selling it later at time s_i is called a round-trip trade (RTT).

Given a time series x and an n -day SMA, suppose the corresponding policy $\pi^{x,n}$ contains a sequence of buying and selling actions at times $(b_1, s_1), (b_2, s_2), \dots, (b_K, s_K)$. Then

1. The percentage return (PCR) of the RTT at (b_i, s_i) can be computed by

$$PCR_i^{x,n} = \frac{x_{s_i} - x_{b_i}}{x_{b_i}}$$

2. The mean percentage return (MPCR) of $\pi^{x,n}$ can be computed by

$$MPCR^{x,n} = \frac{PCR_1^{x,n} + PCR_2^{x,n} + \dots + PCR_K^{x,n}}{K}$$

We define $SMA(x, n)$ the time series representing the n -day SMA of the time series x .

We define $MEAN(z)$ the arithmetic mean of z if z is a list of real numbers or the MPCR of z if z is a policy of a time series.

Let P^x be the set of all policies of the time series x . An optimal policy $\pi^{o,x}$, which gives the maximum value of MPCR of the time series, can be defined by

$$\pi^{o,x} = \underset{\pi^x \in P^x}{argmax} MEAN(\pi^x)$$

2.4 Optimal Policy Approximation with an n -day SMA

Pseudocode for obtaining a policy with an n -day SMA is shown in Algorithm 1. The function $GetPolicy(x, n)$ is assumed to take a time series x and a length n of a SMA, and return the corresponding policy $\pi^{x,n}$.

Algorithm 1: $GetPolicy(x, n)$ for obtaining a policy with an n -day SMA

Input: x : a time series of length N . n : the length of a simple moving average.	
Output: The policy $\pi^{x,n}$ containing a list of buying and selling times.	
1	$y \leftarrow SMA(x, n)$
2	for i from n to N do
3	if $x_i > y_{i-n+1}$ then
4	$sig_i = 1$
5	else if $x_i < y_{i-n+1}$ then
6	$sig_i = -1$
7	else
8	$sig_i = 0$
9	$b \leftarrow$ list of indices i where $(sig_i = 0 \text{ and } sig_{i-1} = -1)$
10	$s \leftarrow$ list of indices i where $(sig_i = 0 \text{ and } sig_{i-1} = 1)$
11	$\pi^{x,n} \leftarrow$ the policy containing b and s

Explanation of Algorithm 1:

- Line 1: Calculate the n -day SMA for the time series x and store the result in y .
- Line 2: Loop through all the time indices to compare the values of the n -day SMA and their corresponding values of x at each time. Note that an n -day SMA starts at time n .
- Line 3 and 4: If the value of x is above that of its simple moving average at time i then assign the signal variable sig_i to 1.
- Line 5 and 6: If the value of x is below that of its simple moving average at time i then assign the signal variable sig_i to -1.
- Line 7 and 8: If the value of x is equal to that of its simple moving average at time i then assign the signal variable sig_i to 0 (the two lines intersect at i).
- Line 9: Obtain all the time indices where x crosses over its simple moving average from below and store them in b . These are the buy signals.
- Line 10: Obtain all the time indices where x crosses over its simple moving average from above and store them in s . These are the sell signals.
- Line 11: Return the policy of x corresponding to the n -day SMA.

Next pseudocode for an optimal policy approximation with n -day SMAs is presented in Algorithm 2. The algorithm is assumed to take the set of stocks and the set of lengths used in SMAs, and returns an optimal policy approximation $\pi^{o,x}$ for each stock x and the corresponding SMA's length $n^{o,x}$ and $MPCR^{o,x}$.

Algorithm 2: Optimal Policy Approximation with n -day SMAs

Input: S : the set of stocks. L : the set of lengths used in SMAs.	
Output: $\pi^{o,x}$: An optimal policy approximation for each stock x in S . $n^{o,x}$: The length used in a SMA of the policy $\pi^{o,x}$. $MPCR^{o,x}$: The average percentage return per RTT in the policy $\pi^{o,x}$.	
1	for x in S do
2	$\pi^{o,x} \leftarrow$ empty list
3	$n^{o,x} \leftarrow 0$
4	$MPCR^{o,x} \leftarrow -1000$
5	for n in L do
6	$\pi^{x,n} = \text{GetPolicy}(x, n)$
7	$b \leftarrow b_1, b_2, \dots, b_{K1}$
8	$s \leftarrow s_1, s_2, \dots, s_{K2}$
9	if $K1 > 0$ and $K2 > 0$ then
10	$K = K1$
11	if $K1 > K2$ then
12	$K = K2$
13	$b \leftarrow b_1, b_2, \dots, b_K$
14	else if $K1 < K2$ then
15	$s \leftarrow s_2, s_3, \dots, s_{K2}$
16	$PCR^{x,n} \leftarrow (x_s - x_b)/x_b$
17	$MPCR^{x,n} \leftarrow \text{MEAN}(PCR^{x,n})$
18	if $MPCR^{o,x} < MPCR^{x,n}$ then
19	$\pi^{o,x} \leftarrow b_1, s_1, b_2, s_2, \dots, b_K, s_K$
20	$n^{o,x} \leftarrow n$
21	$MPCR^{o,x} \leftarrow MPCR^{x,n}$

Explanation of Algorithm 2:

- Line 1: Loop through each stock x in S .
- Line 2, 3, and 4: Initiate the values for the optimal policy for each stock x . $MPCR^{o,x}$ is assigned to a small value so that it can be updated later.
- Line 5: For each stock x , loop through the list of SMAs' lengths to search for a n -day SMA which gives a "best" approximation of the optimal policy.
- Line 6, 7, and 8: Obtain the policy for x corresponding to the n -day SMA, and store the buy signals in b and the sell signals in s . Note that b and s may have different lengths at this point.
- Line 9: Check if there is any round-trip trade to calculate the $MPCR^{x,n}$.
- Line 10 through 15: Equalize the lengths of b and s , and store the common length in K . If there are more buy signals than sell signals, the last buy signal will be dropped. On the other hand, if there are more sell signals than buy signals, the first sell signal will be dropped. Note that b and s are adjusted to have the same indices: $1, 2, \dots, K$.
- Line 16: Calculate each percentage return $(x_{si} - x_{bi})/x_{bi}$ for each round-trip trade, and store the values in the list $PCR^{x,n}$.
- Line 17: Calculate the mean percentage return for the current n -day SMA.
- Line 18 through 21: Update the approximation of the optimal policy for x if a better policy is found.

3. Result and Discussion

The algorithm Optimal Policy Approximation with n -day SMAs are implemented using the R programming language with the following input:

S = the set of all 478 stocks of S&P 500 as described above.

$L = \{5, 10, 15, \dots, 500\}$: the set of SMA lengths.

The output of the algorithm is the approximation of the optimal policy for each stock in S . Each row of Table 3 and Table 4 lists the stock and its summary of the optimal policy approximation containing the corresponding length of the simple moving average (SMA length), the number of round-trip trades (# RTTs), and the mean percentage return (MPCR).

As can be seen from Table 1 and Table 2, it is possible to gain 43.5% per round-trip trade on average for the top 50 stocks, and there are approximately 4 round-trip trades on average over the six-year period. Moreover, the result is very impressive for the top 10 stocks. In particular, it is possible to gain up to 100.6% per round-trip trade on average and there are approximately 2 round-trip trades over the six-year period.

Table 1: Results of the Optimal Policy Approximations for the 50 stocks with highest MPCRs, arranged in descending order of the MPCR

	Symbols	SMA Length	# RTTs	MPCR		Symbols	SMA Length	# RTTs	MPCR
1	NVDA	195	5	2.0854	26	DRI	370	3	0.3149
2	DXC	435	2	1.184	27	ALGN	425	10	0.2969
3	NRG	475	1	1.1664	28	FCX	400	1	0.2569
4	AMD	420	3	1.0041	29	STT	275	2	0.2551
5	ISRG	440	2	0.9681	30	IEX	425	3	0.2516
6	ALB	435	1	0.8244	31	VRSN	270	6	0.2511
7	OKE	375	1	0.8003	32	GRMN	290	3	0.2426
8	CPRT	415	2	0.7421	33	KLAC	375	3	0.2406

9	GE	445	2	0.6607	34	ATVI	240	6	0.2271
10	CSX	500	2	0.6265	35	DOV	445	2	0.2254
11	MU	270	4	0.5284	36	PGR	240	5	0.2172
12	AXP	290	1	0.5247	37	AEE	420	2	0.2123
13	ABMD	300	6	0.5099	38	ZBRA	205	8	0.2035
14	URI	390	2	0.4668	39	CMG	395	2	0.2027
15	DD	460	1	0.4604	40	NEM	475	3	0.2001
16	SPGI	440	2	0.4574	41	UNP	365	3	0.1998
17	MSFT	325	3	0.4228	42	AMZN	195	11	0.1981
18	NTAP	470	2	0.4124	43	BWA	390	1	0.1967
19	J	490	1	0.3824	44	LDOS	395	5	0.194
20	SY	385	2	0.3795	45	ADSK	445	6	0.1881
21	NFLX	450	5	0.3708	46	XYL	415	5	0.1875
22	PLD	420	2	0.3377	47	AFL	415	3	0.1809
23	AMAT	385	4	0.3348	48	AJG	265	4	0.1793
24	PRGO	315	4	0.3335	49	IDXX	395	9	0.1674
25	HAL	185	5	0.3303	50	VLO	235	5	0.1674

Table 2: Summary of the top 50 and top 10 stocks

	SMA Length	# TTs	MPCR
Avg. Top 50	369.5	3.52	0.43542
Avg. Top 10	413.5	2.1	1.0062

Note: The values in the first and second rows are the averages of each column of SMA length, #RTTs, and MPCR using all 50 rows and the first 10 rows from Table 3 respectively.

Table 3 and Table 4 show that one can gain at most 0.2% per round-trip trade on average for the bottom 50 stocks in Table 3, and there are approximately 107 round-trip trades on average over the six-year period. Moreover, the performance is very poor for the bottom 10 stocks. Specifically, one can lose at least 0.1% per round-trip trade on average and there are approximately 193 round-trip trades over the six-year period.

These results suggest that long-term investment (small number of round-trip trades) performs better. Moreover, the top 10 stocks, where the numbers of round-trip trades are between 1 and 5, can possibly yield high returns in the long run.

Table 3: Results of the Optimal Policy Approximations for the 50 stocks with lowest MPCRs, arranged in descending order of the MPCR

	Symbols	SMA Length	# RTTs	MPCR		Symbols	SMA Length	# RTTs	MPCR
429	SEE	15	88	0.004	466	CL	35	64	0.0002
430	IPG	420	15	0.004	467	BXP	20	89	0.0001
431	EXPE	145	17	0.0039	468	EIX	5	201	0.0001
432	COF	5	182	0.0039	469	COG	15	114	-0.0005
433	K	365	11	0.0038	470	HBI	5	204	-0.0006
434	RE	25	83	0.0038	471	CAH	5	213	-0.0006
435	T	50	46	0.0038	472	KSU	5	197	-0.0009
436	RSG	25	82	0.0037	473	JNPR	5	205	-0.001
437	PRU	25	78	0.0037	474	ALXN	5	203	-0.001
438	AIV	25	78	0.0035	475	OMC	5	213	-0.0013
.	.	.	.	.	476	SWKS	5	179	-0.0023
.	.	.	.	.	477	ARE	5	196	-0.0024
.	.	.	.	.	478	SRE	5	208	-0.003

Table 4: Summary of the bottom 50 and bottom 10 stocks

	SMA Length	# TTs	MPCR
Avg. bottom 50	54.8	107.28	0.001502
Avg. bottom 10	6	193.2	-0.00136

Note: The values in the first and second rows are the averages of each column of SMA length, # RTTs, and MPCR using all 50 rows and the last 10 rows from Exhibit 8 respectively.

However, not all long-term investments can generate good results. For example, Table 5 lists stocks with small numbers of RTTs (at most 6), yet the corresponding MPCRs are less than 10%.

Table 5: Results of the Optimal Policy Approximations for stocks with the number of RTTs < 7 and MPCR < 0.1, arranged in ascending order of the MPCR

Symbols	SMA Length	#RTTs	MPCR	Symbols	SMA Length	# RTTs	MPCR
NLSN	445	2	0.0008	TPR	500	4	0.0566
BEN	295	6	0.0058	TXT	495	5	0.0604
ETR	490	6	0.0071	TSCO	420	3	0.0612
SPG	400	6	0.0146	TAP	410	4	0.0633
EMN	405	6	0.0251	LIN	355	5	0.0682

PVH	480	4	0.0341	UNM	320	6	0.0706
AMP	495	5	0.0365	TIF	455	3	0.0735
ECL	420	5	0.0423	WEC	375	5	0.0798
DUK	480	4	0.0429	HON	435	6	0.0863
GIS	460	3	0.0456	EXC	435	5	0.0880
EA	450	1	0.0463	ULTA	265	5	0.0883

The scatter plot in Figure 1 confirms that high returns only exist in long-term investing in S&P 500, while short-term investing (large number of round-trip trades) generally yields poor result. As the number of round-trip trades increases, the MPCR is very close to zero.

Note: Large numbers of RTTs corresponds to small MPCRs.

Fig 1: Scatter plot MPCR vs # RTTs in the Optimal Policy Approximation for all 478 stocks in S&P 500

4. Conclusion

With the configuration used in the paper, the Optimal Policy Approximation using n -day SMAs finds ten stocks of the S&P 500 Index that potentially generate high returns for long-term investing. These stocks include NVDA, DXC, NRG, AMD, ISRG, ALB, OKE, CPRT, GE, and CSX with the MPCRs ranging from 62.7% to 208.5% and the number of RTTs ranging from 1 to 5. However, not all long-term investments are satisfactory. For instance, NLSN generates an MPCR of at most 0.08% with only 2 RTTs over the six-year period. Likewise, short-term trading can hardly generate positive profits.

With the convenience of the R programming language, simplicity of the algorithm, and availability of the up-to-date data, extensive configurations of the algorithm can be easily implemented to obtain practical and more accurate results. Note that the bigger set of SMA lengths one uses, the more accurate the result is. The algorithm can also be applied to Dow Jones, Nasdaq, or many other sectors including technology, healthcare, energy, etc. instead of S&P 500.

The implementation in the paper is not accounted for transaction costs. Since a policy of short-term investing utilizes more RTTs, the transaction costs will reduce its MPCR more than that of a long-term one. Hence, the conclusion that favors long-term investing becomes even more convincing with the presence of transaction costs.

5. Conflict of interest: The authors declared that there is no conflict of interest.

6. References

1. Dai M, Zhang Q, Zhu QJ. Trend following trading under a regime switching model. *SIAM Journal on Financial Mathematics*. 2010;1(1):780-810, doi: 10.1137/090770552. URL <https://doi.org/10.1137/090770552>.
2. Foreign Exchange Market. Technical analysis: Moving average crossover. Available at <https://www.forex.com/en-us/education/education-themes/technical-analysis/moving-average-crossover/>, 2020. Retrieved Aug 2, 2020.
3. Guo X, Zhang Q. Optimal selling rules in a regime switching model. *IEEE Transactions on Automatic Control*. 2005;50(9):1450-1455, doi: 10.1109/TAC.2005.854657. URL <https://ieeexplore.ieee.org/document/1506961>.
4. S. Iwarere and B. R. Barmish. A confidence interval triggering method for stock trading via feedback control. In *Proceedings of the American Control Conference*, 2010, 6910-6916, 2010. doi: 10.1.1.303.6430. URL <http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.303.6430&rep=rep1&type=pdf>.
5. Will Kenton. S&p 500 index - standard & poor's 500 index. Available at <https://www.investopedia.com/terms/s/sp500.asp>, 2018, 7. Retrieved Aug 2, 2020.
6. Learn Price Action. Moving average: How to use in stock and forex markets. Available at <https://learnpriceaction.com/moving-average/>, 2020. Retrieved Aug 2, 2020.
7. Nick Lioudis. Moving average strategies for forex trading. Available at <https://www.investopedia.com/ask/answers/122314/how-do-i-use-moving-average-ma-create-forex-trading-strategy.asp>, 2020 3. Retrieved Aug 2, 2020.

8. Phong Luu, Jingzhi Tie, Qing Zhang. A Threshold Type Policy for Trading a Mean-Reverting Asset with Fixed Transaction Costs. *Risks* 2018a;6(4):1-15. September. doi:10.3390/risks6040107. URL <https://www.mdpi.com/2227-9091/6/4/107>.
9. Phong Luu, Jingzhi Tie, Qing Zhang. Pairs trading. *Applied Analysis and Optimization* 2018b;2(1):117-135 ISSN 2189-1664. URL <http://www.ybook.co.jp/online2/opaa/vol2/p117.html>.
10. Amal Merhi, Mihail Zervos. A model for reversible investment capacity expansion. *SIAM Journal on Control and Optimization* 2007;46(3):839-876. doi: 10.1137/050640758. URL <https://doi.org/10.1137/050640758>.
11. Cory Mitchell. How to use a moving average to buy stocks. Available at <https://www.investopedia.com/articles/active-trading/052014/how-use-moving-average-buy-stocks.asp> 2020, 5. Retrieved Aug 2