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Fuzzy gb* - Closed Set

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Abstract
In 2017 S. Sekar and S. Loganayagi [7] introduced the concepts of gb^*- closed set in general topology. The present paper extends the concepts of gb^*-closed sets in Fuzzy topology and explore their study.

Keywords: fuzzy set, fuzzy closed set, fuzzy open set, fuzzy topological space, fuzzy gb* - closed set, fuzzy gb* - open set

Introduction

Let X be a non-empty set and I=[0,1]. A fuzzy set on X is a mapping from X to I. The null fuzzy set 0 on X into I which assume only the values 0 and the whole fuzzy set 1 is a mapping from X onto I which takes the values 1 only. The union (res. intersection) of family $\{A_a : a \in \wedge\}$ of fuzzy set of X is defined to be the mapping sup A_a {resp. inf A_a }. A fuzzy set A of X is contained in fuzzy set B of X if $A(\mathrm{x}) \le B(\mathrm{x})$ for each $\mathrm{x} \in X$. A fuzzy point X_β in X is a fuzzy set defined by $\mathrm{x}_\beta(y) = \beta$ for $y = \mathrm{x}$ and x(y)=0 for $y \ne \mathrm{x}, \beta \in [0,1]$ and $y \in X$ A fuzzy point X_β is said to be quasi-coincident with the fuzzy set A denoted by $X_{\beta q} A$ if and only if $\beta + A(\mathrm{x}) > 1$. A fuzzy set A is quasi coincident with a fuzzy set B is denoted by $A_q B$ if and only if there exists a point $\mathrm{x} \in X$ such that $A(\mathrm{x}) + B(\mathrm{x}) > 1$, $A \le B$ if and only if $\neg A_q B^c$. [6]

A family τ of fuzzy set of X is called the fuzzy topology [4] on X if 0 to 1 belongs to τ and τ is closed with respect to arbitrary union and finite intersection. The member of τ are called fuzzy open sets and their compliment are fuzzy closed sets. For a fuzzy set A of X the closure of A (denoted by cl(A)) is the intersection of all the fuzzy closed superset of A and the interior of A (denoted by int(A)) is the union of all fuzzy open subsets of A.

Definition: 1.1
A fuzzy set A of a fuzzy topological space (X, τ) is called

(a) Fuzzy gb-closed if $bcl\,(A) \le G$ whenever $(A) \le G$ and $G \in \tau$ [7]
(b) Fuzzy gb-open if 1-A is fuzzy gb^*-closed[7]

Defention: 1.2
A mapping $f : (X, \tau) \to (y, \Gamma)$ is said to be gb-irresolute f (O) is fuzzy gb-closed in X for every fuzzy gb-closed set O in Y [8].

Fuzzy gb*-Closed sets

Definition: 2.1
A fuzzy set A of a fuzzy topological space (X, χ) is called fuzzy gb^*-closed if $bcl\,(A) \le O$ whenever $(A) \le O$ and O is fuzzy g*-open.

Remark: 2.1

Every fuzzy closed set is fuzzy gb^*-closed and every fuzzy gb^*-closed set is fuzzy gb-closed but the converses may not be true.

Example: 2.1

Let X={a,b} and $\chi = \{0,1,U\}$ be an fuzzy topology on X, where U(a)=0.5, U(b)=0.6. Then the fuzzy set defined by A(a)=0.3, A(b)=0.4 is gb^*-closed and the fuzzy set B defined by B(a)=0.6, B(b)=0.6 is fuzzy gb-closed set, but it is not fuzzy gb^*-closed.

Theorem: 2.1

Let (X,χ) be a fuzzy topological space and A is a fuzzy set of X. Then A is fuzzy gb^*-closed if and only if $\neg(A_q F) \Rightarrow \neg(bcl(A)_q F)$ for every fuzzy gb-closed set F of X.

Proof

Necessity

Let F be an fuzzy gb-closed subset of X, and $\neg(A_q F)$. Then by definition (1.1), $A \leq 1-F$ and 1-F fuzzy gb-open in X. Therefore bcl(A)≤1-Fbecause A is fuzzy gb^*-closed. Hence by $\neg(bcl(A)_q F)$.

Sufficiency

Let O be a fuzzy gb-open set of X such that A ≤ O. Then $\neg(A_q(1-O))$ and 1-O is an fuzzy gb-closed set in X. Hence by hypothesis $\neg(bcl(A)_q(1-O))$. Therefore $bcl(A) \leq O$. Hence A is fuzzy gb^*-closed in X.

Theorem: 2.2

Let A and B are two fuzzy gb^*-closed sets in a fuzzy topological space (X,χ), then $A \cup B$ is fuzzy gb^*-closed.

Proof

Let O be and fuzzy gb-open set in X, such that $A \cup B \leq O$. Then A ≤ O and A ≤ O and $bcl(A) \leq O$ and $bcl(B) \leq O$ therefore $bcl(A) \cup bcl(B) = bcl(A \cup B) \leq O$. Hence $A \cup B$ is fuzzy gb^*-closed.

Remark: 2.2

The intersection of two fuzzy gb^*-closed sets in a fuzzy topological space (X,χ) may not be fuzzy gb^*-closed. For

Example: 2.2

Let X={a,b} and U, A and B be the fuzzy set of X defined as follows

U(a)=0.7	U(b)=0.6
A(a)=0.6	A(b)=0.7
B(a)=0.8	B(b)=0.5

Let $\chi = \{0,1,U\}$ be an fuzzy topology on X. Then A and B are fuzzy gb^*-closed in (X,χ) but $A \cap B$ is not fuzzy gb-closed.

Theorem: 2.3

Let A be a fuzzy gb^*-closed set in fuzzy topological space (X,χ) and $A \leq B \leq bcl(B)$. Then B is fuzzy gb^*-closed in X.

Proof:

Let O be fuzzy gb-open set such that $B \leq O.$ Then $A \leq O$. Since A is fuzzy gb^*-closed, $bcl(A) \leq O$ now $B \leq bcl(A) \Rightarrow bcl(B) \leq bcl(A) \leq O$. Consequently B is fuzzy gb^*-closed.

Theorem: 2.4

Let (Y,χ_Y) be a subspace of a fuzzy topological space (X,χ) and A be fuzzy set in Y. If A is fuzzy gb^*-closed in X then A is fuzzy gb^*-closed in X then A is fuzzy gb^*-closed in Y.

Proof:

Let $A \le O_Y$ where O_Y is fuzzy gb-open in Y. Then there exists fuzzy gb-open set O in X such that $O_Y = O \cap Y$. Therefore $A \subseteq O$ and since A is fuzzy gb^*-closed in X, $bcl(A) \le O$. It follows that $bcl_Y(A) = bcl(A) \cap Y \le O \cap Y = O_Y$. Hence A is fuzzy gb^*-closed in Y.

Definition: 2.2

A fuzzy set A of fuzzy topological space (X, χ) is called fuzzy gb^*-open if it's complement 1-A is fuzzy gb^*-closed.

Remark: 2.3

Every fuzzy open set is fuzzy gb*-open and every fuzzy gb*-open set is fuzzy gb-open. But the converses may not be true.

Remark: 2.4

An fuzzy set A of a fuzzy topological space (X, χ) is fuzzy gb^*-open if and only if $F \le \text{int}(A)$ whenever F is fuzzy gb-closed and $F \le A$. (Proof: obvious)

Theorem: 2.5

Let A and B are q-separated fuzzy gb^*-open subsets of a fuzzy topological space (X, χ), then $A \cup B$ is fuzzy gb^*-open.

Proof:

Let F be a fuzzy g-closed subset of X and $F \le A \cup B$.

Then $F \cap bcl(A) \le A \cup B \cap bcl(A) = (A \cap bcl(A)) \cup (B \cap bcl(A)) \in Int(A)$

Similarly $F \cap bcl(B) \in Int(B)$

Now, $F = F \cap (A \cup B) \le (F \cap bcl(A)) \cup (F \cap bcl(B)) \in Int(A) \cup Int(B) \le (A \cup B)$

Hence $F \le \text{int}(A \cup B)$ and byRemark:2.4 $A \cup B$ is fuzzy gb^*-open.

Theorem: 2.6

Let A and B be two fuzzy gb^*-closed sets of a fuzzy topological space (X, χ) and suppose that 1-A and 1-B are q-separated, then $A \cap B$ is fuzzy gb^*-closed.

Proof:

Since A^c and B^c are q-separated fuzzy gb^*-open sets, by theorem 2.5, $1-(A \cap B) = 1 - A \cup (1-B)$ is fuzzy gb*-open. Hence $A \cap B$ is fuzzy gb*-closed.

Theorem: 2.7

Let A be fuzzy gb*-open set of fuzzy topological space (X, χ) and $Int(A) \le B \le A$. Then B is fuzzy gb*-open.

Proof:

Since $1 - A \le 1 - B \le bcl(1-A)$ and 1-A is fuzzy gb*-closed it follos from theorem 2.3 that 1-B is fuzzy gb-closed. Hence B is fuzzy gb*-open.

Definition: 2.3

A fuzzy topological space (X, χ) is said to be fuzzy gb*-compact if every fuzzy gb*-open covers of X has a finite subcover.

Theorem: 2.8

Let (X, χ) be a fuzzy gb-compact space and suppose that Y is fuzzy gb*-closed crisp subset of X, then (X, χ_Y) is gb-compact.

Proof

Let V be a χ_Y fuzzy gb*-open covering of Y and let $G = \{V \in \chi, V \cap Y \in V\}$ Then $Y \in UG$. Since Y is fuzzy gb*-closed $bcl(y) \le UG$. Therefore $G \cup 1-(bcl(y))$ is a χ-fuzzy gb-open cover of X. Since X is fuzzy gb-compact, $G \cup 1-(bcl(y))$ has a finite sub cover $\{v_1, v_2, ..., v_n, 1-bcl(y)\}$ but then $\{v_1 \cap y, v_2 \cap y, ..., v_n \cap y\}$ is a finite sub cover of V.

Theorem: 2.9

Let A be a fuzzy gb-closed set in fuzzy topological space (X,χ) and $f:(X,\chi)\rightarrow(y,\chi^{*})$ is fuzzy gb-irresolute and fuzzy closed mapping then f(A) is an gb-closed set in y.

Proof

If $f(A)\leq G$ where G is fuzzy gb-open in y then $A\leq f^{-1}(G)$ and hence $bcl(A)\leq f^{-1}(G)$. Thus $f(bcl(A))\leq G$ and $f(bcl(A))$ is fuzzy closed set. It follows that $bcl(A)\leq bcl(f(bcl(A)))=f(bcl(A))\leq G$. Hence $bcl(f(A))\leq G$ and $f(A)$ is fuzzy gb*-closed.

Conclusion

The fuzzification of Topology primarily the set gb*-closed set and gb*-open set yield a new class of fuzzy sets namely fuzzy gb*-closed set and fuzzy gb*-open set. Properties of the aforementioned new fuzzy sets in fuzzy topology were established.

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