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An extension of weighted maxwell-boltzmann distribution: A simulation study

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Abstract
In this paper, we have introduced Mixture of Maxwell-Boltzmann Distribution and Weighted Maxwell-Boltzmann Distribution obtained through convex combination technique and abbreviated it as MWMBD. Different characteristic properties of the introduced distribution have been studied in detail. The estimators are estimated through the technique of MLE using R-software.

Keywords: weighted, maxwell-boltzmann distribution, simulation study

1. Introduction

Maxwell-Boltzmann distribution named after James Clerk Maxwell and Ludwig Boltzmann is a particular probability distribution having its implications in Physics (particularly in Statistical mechanics). The distribution was first derived by Maxwell in 1860 on heuristic grounds and later in 1870's by Boltzmann by making a thorough inquiry into its physical origin. The Maxwell-Boltzmann distribution was first defined and used for describing the speed of gaseous particles (atoms and molecules, assumed to have reached thermodynamic equilibrium) and later provided an explanation to various fundamental properties of gases, including pressure and diffusion. In statistical mechanics, Maxwell-Boltzmann Statistics describes the average distribution of non-interacting material particles over various energy states in thermal equilibrium. Mathematically, this distribution follows chi-distribution with three degrees of freedom with a scale parameter measuring speeds in units proportional to the square root of T/m (T=Temperature and M=Mass of the particle). Maxwell distribution is an excellent approach that is applied fundamentally to the distribution of molecular velocities in a gas. The Maxwell Boltzmann distribution has the pdf as

$$f(x;\theta)=\sqrt{\frac{2}{\pi}}\theta^{\frac{3}{2}}x^2\exp\left(\frac{-\theta x^2}{2}\right);x\geq 0,\theta>0$$

Cdf, Survival function and hazard rate of Maxwell Distribution are respectively given as

$$F(x,\theta)=1-\frac{\Gamma\left(\frac{3}{2},\frac{\theta x^2}{2}\right)}{\Gamma\left(\frac{3}{2}\right)}$$

$$R(x,\theta)=\frac{\Gamma\left(\frac{3}{2},\frac{\theta x^2}{2}\right)}{\Gamma\left(\frac{3}{2}\right)}$$

$$h(x;\theta)=\frac{\theta^{\frac{3}{2}}\, x^{2}\exp\left(\frac{-\theta x^{2}}{2}\right)}{2^{\frac{1}{2}}\,\Gamma\left(\frac{3}{2},\frac{-\theta x^{2}}{2}\right)}$$

A lot of researchers worked on the weighted distributions and carry out research on the same . Dar *et al.* [1] characterized the transmuted weighted Exponential distribution and discussed some of it's application . In ecology, Dennis and Patil [3] used stochastic differential equations to arrive at weighted properties of size-biased Gamma distribution. Rao [5] studied the weighted distributions arising out of method of ascertainment. G.P.Patil and C.R.Rao [8] studied weighted distributions and size biased sampling with applications to wildlife populations and human families. Lappi and Bailey [9] used weighted distributions to analyze HPS diameter increment data. Reshi *et al.* [10] worked on new moment method of estimation of parameters of size-biased classical gamma distribution. K.G. Janardan [12] characterized the weighted Lagrange distributions.

After generalizing it by introducing one more parameter, the new pdf of a random variable X following Weighted Maxwell Boltzmann distribution is given by

$$f(x;\theta,\omega)=\frac{\theta^{\left(\frac{\omega+3}{2}\right)}\, x^{\omega+2}}{2^{\left(\frac{\omega+1}{2}\right)}\,\Gamma\left(\frac{\omega+3}{2}\right)}\exp\left(\frac{-\theta x^{2}}{2}\right);x\geq 0,\theta>0$$

Where θ is the rate parameter and ω is the weight parameter $(\omega>0)$
Cdf, Reliability function and hazard rate of WMD are respectively given as

$$F(x;\theta,\omega)=1-\frac{\Gamma\left(\frac{\omega+3}{2},\frac{\theta x^{2}}{2}\right)}{\Gamma\left(\frac{\omega+3}{2}\right)}$$

$$R(x;\theta,\omega)=\frac{\Gamma\left(\frac{\omega+3}{2},\frac{\theta x^{2}}{2}\right)}{\Gamma\left(\frac{\omega+3}{2}\right)}$$

$$h(x;\theta,\omega)=\frac{\theta^{\left(\frac{\omega+3}{2}\right)}\, x^{\omega+2}\exp\left(\frac{-\theta x^{2}}{2}\right)}{2^{\frac{\omega+1}{2}}\,\Gamma\left(\frac{\omega+3}{2},\frac{\theta x^{2}}{2}\right)}$$

2. Materials and Methods

Let $f(x;\theta)$ and $f(x;\theta,\omega)$ be the pdf and weighted pdf of the random variable X respectively, where $f(x>0,\theta,\omega>0)$ and $0\leq P\leq 1$, then the mixture length biased distribution of X formed by the mixture between $f(x;\theta)$ and $f(x;\theta,\omega)$ in the form of $Pf(x;\theta)+(1-P)\,f(x;\theta,\omega)$ and $f(x;\theta,\omega)$ is as follows

$$f_{M}(x;\theta,\omega)=Pf(x)+(1-P)\,f_{\omega}(x)$$

3. Derivation of Mixture Weighted Maxwell Boltzmann Distribution (MWBMD)

Maxwell (or Maxwell-Boltzmann) distribution finds a lot of applications in Science subjects particularly in Physics and Chemistry. In Physics, the Maxwell distribution forms the basis of the kinetic energy of gases. The Pdf of a random variable X following Maxwell distribution with rate parameter θ is given by

$$f(x;\theta)=\sqrt{\frac{2}{\pi}}\,\theta^{\frac{3}{2}}x^2\exp\left(\frac{-\theta x^2}{2}\right);x\geq 0,\theta>0$$

After considering weighted function w(x) = x(ω) , where ω > 0 is the weight parameter, Then the pdf of Weighted Maxwell Boltzmann Distribution is given by

$$f(x;\theta)=\frac{\theta^{\left(\frac{\omega+3}{2}\right)}x^{\omega+2}}{2^{\left(\frac{\omega+1}{2}\right)}\Gamma\left(\frac{\omega+3}{2}\right)}\exp\left(\frac{-\theta x^2}{2}\right);x\geq 0,\theta>0$$

Where θ is the rate parameter and ω is the weight parameter $(\omega>0)$ After applying the covex combination technique to the above two distributions, the pdf of Mixture of Weighted Maxwell Boltzmann and Maxwell Boltzmann Distribution is given by

$$f_M(x;\theta,\omega)=P\sqrt{\frac{2}{\pi}}\theta^{\frac{3}{2}}x^2\exp\left(\frac{-\theta x^2}{2}\right)+(1-P)\left\{\frac{\theta^{\frac{\omega+3}{2}}}{2^{\frac{\omega+1}{2}}}\frac{x^{\omega+2}}{\Gamma\frac{\omega+3}{2}}\exp\left(\frac{-\theta x^2}{2}\right)\right\}$$

The corresponding cdf is given by

$$F(x;\theta,\omega)=\frac{2P}{\sqrt{\pi}}\Gamma\left(\frac{3}{2},\frac{\theta x^2}{2}\right)+(1-P)\Gamma\frac{\left(\frac{\omega+3}{2},\frac{\theta x^2}{2}\right)}{\Gamma\frac{\omega+3}{2}}$$

4. Reliability measures: Survival function and Hazard Rate function is given by

$$R_M(x;\theta,\omega)=1-\left(\frac{2P}{\sqrt{\pi}}\Gamma\left(\frac{3}{2},\frac{\theta x^2}{2}\right)+(1-P)\Gamma\frac{\left(\frac{\omega+3}{2},\frac{\theta x^2}{2}\right)}{\Gamma\frac{\omega+3}{2}}\right)$$

$$h_M(x;\theta,\omega)=\frac{P\sqrt{\frac{2}{\pi}}\theta^{\frac{3}{2}}x^2\exp\left(\frac{-\theta x^2}{2}\right)+(1-P)\left\{\frac{\theta^{\frac{\omega+3}{2}}}{2^{\frac{\omega+1}{2}}}\frac{x^{\omega+2}}{\Gamma\frac{\omega+3}{2}}\exp\left(\frac{-\theta x^2}{2}\right)\right\}}{1-\left(\frac{2P}{\sqrt{\pi}}\Gamma\left(\frac{3}{2},\frac{\theta x^2}{2}\right)+(1-P)\Gamma\frac{\left(\frac{\omega+3}{2},\frac{\theta x^2}{2}\right)}{\Gamma\frac{\omega+3}{2}}\right)}$$

5. Structural Properties
5.1 Moments

Let X be a random variable follows Mixture Weighted Maxwell Boltzmann Distribution. Then r^{th} moment denoted by μ_r' is given as

$$\mu_r' = \int_0^\infty x^r f_M(x;\theta,\omega)dx$$

$$\mu_r' = \int_0^\infty x^r \left\{ P\sqrt{\frac{2}{\pi}}\theta^{\frac{3}{2}} x^2 \exp\left(\frac{-\theta x^2}{2}\right) + (1-P)\left\{ \frac{\theta^{\frac{\omega+3}{2}}}{2^{\frac{\omega+1}{2}}} \frac{x^{\omega+2}}{\Gamma\frac{\omega+3}{2}} \exp\left(\frac{-\theta x^2}{2}\right)\right\}\right\} dx$$

$$\mu_r' = P\sqrt{\frac{2}{\pi}}\theta^{\frac{3}{2}} \int_0^\infty x^{r+2} \exp\left(\frac{-\theta x^2}{2}\right) dx + (1-P)\frac{\theta^{\frac{\omega+3}{2}}}{2^{\frac{\omega+1}{2}}\Gamma\frac{\omega+3}{2}} \int_0^\infty x^{\omega+r+2} \exp\left(\frac{-\theta x^2}{2}\right)$$

$$put\ x^2 = t \Leftrightarrow 2xdx = dt \Leftrightarrow dx = \frac{dt}{2\sqrt{t}}$$

$$\mu_r' = P\sqrt{\frac{2}{\pi}}\theta^{\frac{3}{2}} \int_0^\infty t^{\frac{r+2}{2}} \exp\left(\frac{-\theta t}{2}\right) \frac{dt}{2\sqrt{t}} + (1-P)\frac{\theta^{\frac{\omega+3}{2}}}{2^{\frac{\omega+1}{2}}\Gamma\frac{\omega+3}{2}} \int_0^\infty t^{\frac{\omega+r+2}{2}} \exp\left(\frac{-\theta t}{2}\right) \frac{dt}{2\sqrt{t}}$$

$$\mu_r' = P\sqrt{\frac{1}{2\pi}}\theta^{\frac{3}{2}} \frac{\Gamma\left(\frac{r+3}{2}\right)}{\left(\frac{\theta}{2}\right)^{\frac{r+3}{2}}} + (1-P)\left(\frac{\theta}{2}\right)^{\frac{\omega+3}{2}} \frac{1}{\Gamma\frac{\omega+3}{2}} \frac{\Gamma\left(\frac{\omega+r+3}{2}\right)}{\left(\frac{\theta}{2}\right)^{\frac{\omega+r+3}{2}}}$$

$$\mu_r' = \left(\frac{2}{\theta}\right)^{\frac{r}{2}} \left\{ \frac{2P}{\sqrt{\pi}}\Gamma\left(\frac{r+3}{2}\right) + (1-P)\frac{\Gamma\left(\frac{\omega+r+3}{2}\right)}{\Gamma\frac{\omega+3}{2}} \right\} \tag{5.1.1}$$

Put r=1,2,3,4 in equation (5.1.1), we get first four moments about origin as given below

$$\mu_1' = \sqrt{\left(\frac{2}{\theta}\right)} \left\{ \frac{2P}{\sqrt{\pi}}\Gamma(2) + (1-P)\frac{\Gamma\left(\frac{\omega+4}{2}\right)}{\Gamma\frac{\omega+3}{2}} \right\}$$

$$\mu_2' = \left(\frac{2}{\theta}\right) \left\{ \frac{2P}{\sqrt{\pi}}\Gamma\left(\frac{5}{2}\right) + (1-P)\frac{\Gamma\left(\frac{\omega+5}{2}\right)}{\Gamma\frac{\omega+3}{2}} \right\}$$

$$\mu_3' = \left(\frac{2}{\theta}\right)^{\frac{3}{2}} \left\{ \frac{2P}{\sqrt{\pi}}\Gamma(3) + (1-P)\frac{\Gamma\left(\frac{\omega+6}{2}\right)}{\Gamma\frac{\omega+3}{2}} \right\}$$

$$\mu_4' = \left(\frac{2}{\theta}\right)^2 \left\{ \frac{2P}{\sqrt{\pi}} \Gamma\left(\frac{7}{2}\right) + (1-P) \frac{\Gamma\left(\frac{\omega+7}{2}\right)}{\Gamma\frac{\omega+3}{2}} \right\}$$

Variance**,** coefficient of variation, Skewness and Kurtosis are given by

$$\sigma^2 = \left(\frac{2}{\theta}\right) \left[\left\{ \frac{2P}{\sqrt{\pi}} \Gamma\left(\frac{5}{2}\right) + (1-P) \frac{\Gamma\left(\frac{\omega+5}{2}\right)}{\Gamma\frac{\omega+3}{2}} \right\} - \left\{ \frac{2P}{\sqrt{\pi}} \Gamma(2) + (1-P) \frac{\Gamma\left(\frac{\omega+4}{2}\right)}{\Gamma\frac{\omega+3}{2}} \right\}^2 \right]$$

The coefficient of variation is given by

$$C.V = \frac{\sqrt{\left\{ \frac{2P}{\sqrt{\pi}} \Gamma\left(\frac{5}{2}\right) + (1-P) \frac{\Gamma\left(\frac{\omega+5}{2}\right)}{\Gamma\frac{\omega+3}{2}} \right\} - \left\{ \frac{2P}{\sqrt{\pi}} \Gamma(2) + (1-P) \frac{\Gamma\left(\frac{\omega+4}{2}\right)}{\Gamma\frac{\omega+3}{2}} \right\}^2}}{\left\{ \frac{2P}{\sqrt{\pi}} \Gamma(2) + (1-P) \frac{\Gamma\left(\frac{\omega+4}{2}\right)}{\Gamma\frac{\omega+3}{2}} \right\}}$$

$$\gamma_1 = \sqrt{\beta_1}$$

Where $\beta_1 = \frac{\mu_3^{\,2}}{\mu_2^{\,3}}$

$$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^3 \text{ and } \mu_2 = \mu_2' - (\mu_1')^2$$

$$\mu_3^{\,2} = \left(\frac{2}{\theta}\right)^3 \left[\begin{array}{l} \left\{ \frac{2P}{\sqrt{\pi}} \Gamma(3) + (1-P) \frac{\Gamma\left(\frac{\omega+6}{2}\right)}{\Gamma\frac{\omega+3}{2}} \right\} - 3\left\{ \frac{2P}{\sqrt{\pi}} \Gamma(\frac{5}{2}) + (1-P) \frac{\Gamma\left(\frac{\omega+5}{2}\right)}{\Gamma\frac{\omega+3}{2}} \right\}\left\{ \frac{2P}{\sqrt{\pi}} \Gamma(2) + (1-P) \frac{\Gamma\left(\frac{\omega+4}{2}\right)}{\Gamma\frac{\omega+3}{2}} \right\} \\ + 2\left\{ \frac{2P}{\sqrt{\pi}} \Gamma(2) + (1-P) \frac{\Gamma\left(\frac{\omega+4}{2}\right)}{\Gamma\frac{\omega+3}{2}} \right\}^3 \end{array} \right]^2$$

$$\beta_1=\frac{\mu_3^{\,2}}{\mu_2^{\,3}}=\frac{\left(\frac{2}{\theta}\right)^3\left[\left\{\frac{2P}{\sqrt{\pi}}\Gamma(3)+(1-P)\frac{\Gamma\left(\frac{\omega+6}{2}\right)}{\Gamma\frac{\omega+3}{2}}\right\}-3\left\{\frac{2P}{\sqrt{\pi}}\Gamma(\frac{5}{2})+(1-P)\frac{\Gamma\left(\frac{\omega+5}{2}\right)}{\Gamma\frac{\omega+3}{2}}\right\}\left\{\frac{2P}{\sqrt{\pi}}\Gamma(2)+(1-P)\frac{\Gamma\left(\frac{\omega+4}{2}\right)}{\Gamma\frac{\omega+3}{2}}\right\}+2\left\{\frac{2P}{\sqrt{\pi}}\Gamma(2)+(1-P)\frac{\Gamma\left(\frac{\omega+4}{2}\right)}{\Gamma\frac{\omega+3}{2}}\right\}^3\right]^2}{\left(\frac{2}{\theta}\right)^3\left[\left\{\frac{2P}{\sqrt{\pi}}\Gamma(\frac{5}{2})+(1-P)\frac{\Gamma\left(\frac{\omega+5}{2}\right)}{\Gamma\frac{\omega+3}{2}}\right\}-\left\{\frac{2P}{\sqrt{\pi}}\Gamma(2)+(1-P)\frac{\Gamma\left(\frac{\omega+4}{2}\right)}{\Gamma\frac{\omega+3}{2}}\right\}^2\right]^3}$$

$\gamma_2=\beta_2-3$ Where $\beta_2=\frac{\mu_4}{\mu_2^{\,2}}$

$$\mu_4=\mu_4^{'}-4\mu_3^{'}\mu_1^{'}+6\mu_2^{'}\mu_1^{'2}-3(\mu_1^{'})^4 \; and \; \mu_2=\mu_2^{'}-(\mu_1^{'})^2$$

$$\mu_4=\left(\frac{2}{\theta}\right)^2\left[\left\{\frac{2P}{\sqrt{\pi}}\Gamma(\frac{7}{2})+(1-P)\frac{\Gamma\left(\frac{\omega+7}{2}\right)}{\Gamma\frac{\omega+3}{2}}\right\}-4\left\{\frac{2P}{\sqrt{\pi}}\Gamma(3)+(1-P)\frac{\Gamma\left(\frac{\omega+6}{2}\right)}{\Gamma\frac{\omega+3}{2}}\right\}\left\{\frac{2P}{\sqrt{\pi}}\Gamma(2)+(1-P)\frac{\Gamma\left(\frac{\omega+4}{2}\right)}{\Gamma\frac{\omega+3}{2}}\right\}+6\left\{\frac{2P}{\sqrt{\pi}}\Gamma(\frac{5}{2})+(1-P)\frac{\Gamma\left(\frac{\omega+5}{2}\right)}{\Gamma\frac{\omega+3}{2}}\right\}\left\{\frac{2P}{\sqrt{\pi}}\Gamma(2)+(1-P)\frac{\Gamma\left(\frac{\omega+4}{2}\right)}{\Gamma\frac{\omega+3}{2}}\right\}^2-3\left\{\frac{2P}{\sqrt{\pi}}\Gamma(2)+(1-P)\frac{\Gamma\left(\frac{\omega+4}{2}\right)}{\Gamma\frac{\omega+3}{2}}\right\}^4\right]$$

And

$$\mu_2^{\,2}=\left(\frac{2}{\theta}\right)^2\left[\left\{\frac{2P}{\sqrt{\pi}}\Gamma(\frac{5}{2})+(1-P)\frac{\Gamma\left(\frac{\omega+5}{2}\right)}{\Gamma\frac{\omega+3}{2}}\right\}-\left\{\frac{2P}{\sqrt{\pi}}\Gamma(2)+(1-P)\frac{\Gamma\left(\frac{\omega+4}{2}\right)}{\Gamma\frac{\omega+3}{2}}\right\}^2\right]$$

Now

$$\gamma_2=\frac{\left[\left\{\frac{2P}{\sqrt{\pi}}\Gamma(\frac{7}{2})+(1-P)\frac{\Gamma\left(\frac{\omega+7}{2}\right)}{\Gamma\frac{\omega+3}{2}}\right\}-4\left\{\frac{2P}{\sqrt{\pi}}\Gamma(3)+(1-P)\frac{\Gamma\left(\frac{\omega+6}{2}\right)}{\Gamma\frac{\omega+3}{2}}\right\}\left\{\frac{2P}{\sqrt{\pi}}\Gamma(2)+(1-P)\frac{\Gamma\left(\frac{\omega+4}{2}\right)}{\Gamma\frac{\omega+3}{2}}\right\}+6\left\{\frac{2P}{\sqrt{\pi}}\Gamma(\frac{5}{2})+(1-P)\frac{\Gamma\left(\frac{\omega+5}{2}\right)}{\Gamma\frac{\omega+3}{2}}\right\}\left\{\frac{2P}{\sqrt{\pi}}\Gamma(2)+(1-P)\frac{\Gamma\left(\frac{\omega+4}{2}\right)}{\Gamma\frac{\omega+3}{2}}\right\}^2-3\left\{\frac{2P}{\sqrt{\pi}}\Gamma(2)+(1-P)\frac{\Gamma\left(\frac{\omega+4}{2}\right)}{\Gamma\frac{\omega+3}{2}}\right\}^4\right]}{\left[\left\{\frac{2P}{\sqrt{\pi}}\Gamma(\frac{5}{2})+(1-P)\frac{\Gamma\left(\frac{\omega+5}{2}\right)}{\Gamma\frac{\omega+3}{2}}\right\}-\left\{\frac{2P}{\sqrt{\pi}}\Gamma(2)+(1-P)\frac{\Gamma\left(\frac{\omega+4}{2}\right)}{\Gamma\frac{\omega+3}{2}}\right\}^2\right]^2}$$

Special case of Mixture Maxwell's Boltzmann Distribution

Parameters			$f_M(x;\theta,\omega)=P\sqrt{\frac{2}{\pi}}\theta^{\frac{3}{2}}x^2\exp\left(\frac{-\theta x^2}{2}\right)$	
ω	θ	P	$+(1-P)\left\{\frac{\theta^{\frac{\omega+3}{2}}}{2^{\frac{\omega+1}{2}}}\frac{x^{\omega+2}}{\Gamma\frac{\omega+3}{2}}\exp\left(\frac{-\theta x^2}{2}\right)\right\}$	Distribution
ω	θ	0	$(1-P)\left\{\frac{\theta^{\frac{\omega+3}{2}}}{2^{\frac{\omega+1}{2}}}\frac{x^{\omega+2}}{\Gamma\frac{\omega+3}{2}}\exp\left(\frac{-\theta x^2}{2}\right)\right\}$	Weighted Maxwell's Boltzmann Distribution
0	θ	1	$P\sqrt{\frac{2}{\pi}}\theta^{\frac{3}{2}}x^2\exp\left(\frac{-\theta x^2}{2}\right)$	Maxwell's Boltzmann Distribution
1	θ	0	$\frac{\theta^2x^3}{2}\exp\left(\frac{-\theta x^2}{2}\right)$	Size Biased Maxwell's Distribution
2	θ	0	$\frac{\theta^{\frac{5}{2}}}{2^{\frac{3}{2}}}\frac{x^4}{\Gamma\left(\frac{5}{2}\right)}\exp\left(\frac{-\theta x^2}{2}\right)$	Area Biased Maxwell's Distribution
1	θ	P	$P\sqrt{\frac{2}{\pi}}\theta^{\frac{3}{2}}x^2\exp\left(\frac{-\theta x^2}{2}\right)+(1-P)\frac{\theta^2x^3}{2}\exp\left(\frac{-\theta x^2}{2}\right)$	Mixture Size Biased Maxwell's Distribution
2	θ	P	$f_M(x;\theta,\omega)=P\sqrt{\frac{2}{\pi}}\theta^{\frac{3}{2}}x^2\exp\left(\frac{-\theta x^2}{2}\right)+(1-P)\frac{\theta^{\frac{5}{2}}}{2^{\frac{3}{2}}}\frac{x^4}{\Gamma\left(\frac{5}{2}\right)}\exp\left(\frac{-\theta x^2}{2}\right)$	Mixture Area Biased Maxwell's Distribution

6. Maximum Likelihood Estimation

Let $(x_1, x_2, x_3 \ldots x_n)$ be random sample of size drawn from Mixture Weighted Maxwell Boltzmann Distribution (MWBMD). Then its likelihood function is given by

$$L_M(x;\theta,\omega)=P^n\left(\frac{2}{\pi}\right)^{\frac{n}{2}}\theta^{\frac{3n}{2}}\prod_{i=1}^{n}x_i^2\exp\left(\frac{-\theta}{2}\sum_{i=1}^{n}x_i^2\right)+(1-P)^n\left[\frac{\theta^{n\left(\frac{\omega+3}{2}\right)}}{2^{n\left(\frac{\omega+1}{2}\right)}}\frac{\prod_{i=1}^{n}x_i^{\omega+2}}{\left(\Gamma\left(\frac{\omega+3}{2}\right)\right)^n}\exp\left(\frac{-\theta}{2}\sum_{i=1}^{n}x_i^2\right)\right]$$

And its log likelihood is given by

$$Log\,L_M(x;\theta,\omega)=\left\{\begin{array}{l} n\log P+\frac{n}{2}\log 2-\frac{n}{2}\log\pi+\frac{3n}{2}\log\theta-\frac{\theta}{2}\sum_{i=1}^{n}x_i^2+n\log(1-P)+ \\ \frac{n(\omega+3)}{2}\log\theta-\frac{n(\omega+1)}{2}\log 2+(\omega+2)\sum_{i=1}^{n}x_i-n\log\Gamma\left(\frac{\omega+3}{2}\right)-\frac{\theta}{2}\sum_{i=1}^{n}x_i^2\end{array}\right\}$$

The partial derivatives of the above equation with respect to parameters are given as

$$\frac{\partial L_M(x;\theta,\omega)}{\partial\theta} = \frac{3n}{2\theta} - \sum_{i=1}^{n} x_i^{\ 2} + \frac{n(\omega+3)}{2\theta}$$

$$\frac{\partial L_M(x;\theta,\omega)}{\partial\theta} = \frac{n}{2}\log\theta - \frac{n}{2}\log 2 + \sum_{i=1}^{n} x_i - n\psi\left(\frac{\omega+3}{2}\right)$$

Where $\psi\left(\frac{\omega+3}{2}\right) = \frac{\Gamma'\left(\frac{\omega+3}{2}\right)}{\Gamma\left(\frac{\omega+3}{2}\right)}$

7. Simulation Study

In this section, we study the performance of ML estimators for different sample sizes (n=25,50,100, 150,400, 600). We have employed the inverse CDF technique for data simulation for mixture of Maxwell-Boltzmann distribution and Weighted Maxwell-Boltzmann distribution using R software. The process was repeated 1000 times for calculation of bias, variance and MSE. For two parameter combinations of mixture of Maxwell-Boltzmann distribution and Weighted Maxwell-Boltzmann Distribution decreasing trend is being observed in average bias, variance and MSE as we increase the sample size. Hence, the performance of ML estimators is quite well, consistent in case of mixture of Maxwell-Boltzmann distribution and Weighted Maxwell-Boltzmann distribution

Table 1: Simulation Study of ML estimators for Mixture of Maxwell-Boltzmann Distribution and Weighted Maxwell-Boltzmann Distribution

Para-meter	n	$\theta=0.5,\ \omega=0.6,\ p=0.1$			$\theta=0.9,\ \omega=1.6,\ p=0.7$		
		Bias	**Variance**	**MSE**	**Bias**	**Variance**	**MSE**
θ	25	0.78208500	0.16616000	0.77781695	0.98407700	0.36105700	1.32946454
ω		1.65445035	0.37125500	3.10846096	1.18666800	0.35101100	1.75919194
p		0.15119240	0.00656430	0.02942344	0.21881700	0.00941859	0.05729947
θ	50	0.56548290	0.12107820	0.44084911	0.45221480	0.23160230	0.43610053
ω		1.25111611	0.24167400	1.80696552	0.70845200	0.34385600	0.84576024
p		0.09121442	0.00501126	0.01333133	0.20759400	0.00783510	0.05093037
θ	100	0.50764982	0.01006800	0.26777634	0.32122123	0.02112468	0.12430776
ω		0.86670167	0.08452890	0.83570068	0.58266300	0.21677566	0.55627183
p		0.07672330	0.00468050	0.01056696	0.19920330	0.00733543	0.04701739
θ	150	0.48589915	0.01007639	0.24617438	0.30259200	0.00732101	0.09888293
ω		0.82230530	0.03962150	0.71580751	0.42699720	0.12658085	0.30890746
p		0.06309250	0.00204800	0.00602866	0.17623400	0.00632127	0.03737969
θ	400	0.44454195	0.00406400	0.20168154	0.26676982	0.00495431	0.07612045
ω		0.76616780	0.02265107	0.60966417	0.25451600	0.05644328	0.12122168
p		0.03617230	0.00133829	0.00264673	0.12406800	0.00376090	0.01915377
θ	600	0.44256541	0.00092800	0.19679214	0.25641378	0.00263700	0.06838503
ω		0.64464830	0.01459861	0.43017004	0.16548969	0.04443695	0.07182379
p		0.01887880	0.00012500	0.00048141	0.09872064	0.00181200	0.01155776

8. Conclusion

From the special cases and from Simulation, it is clear that it is better than the Maxwell-Boltzmann distribution and Weighted Maxwell-Boltzmann distribution. For two parameter combinations of mixture of Maxwell-Boltzmann distribution and Weighted Maxwell-Boltzmann Distribution decreasing trend is being observed in average bias, variance and MSE as we increase the sample size. Hence, the performance of ML estimators is quite well, consistent in case of mixture of Maxwell-Boltzmann distribution and Weighted Maxwell-Boltzmann distribution.

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