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Lomax gumbel type two distributions with applications to lifetime data

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Abstract
The Gumbel Type-Two (GTT) distribution among many classical distributions has been found to lack the capacity to adequately fit some random phenomenon due to its monotonic failure rates thereby limiting its application. The Lomax-Generator was employed in this study to generalize the GTT distribution in order to derive the Lomax Gumbel Type-Two (LGTT) distribution capable of providing better modeling fits to real dataset. The new distribution has the Lomax Inverse Exponential distribution as a special case; the reliability and hazard rates functions were investigated. The distribution is unimodal, positively skewed and close to bell shape depending on parameter values. The quantile, median and order statistics were derived while the method of maximum likelihood estimation was used for estimating the parameters of the distribution. The proposed distribution demonstrated its potentials for modeling events whose distributions tends to be platykurtic, leptokurtic and approximately symmetric. Two lifetime survival datasets were analyzed using the distribution and results revealed the importance of application of LGTT distribution to the datasets with superior modeling fits than other four generalizations of GTT distributions existing in literatures derived using other generator. Three additional real life datasets were also used to compare the performance of LGTT with some distributions derived using Lomax as generator, results of analysis revealed that LGTT exhibits greater flexible potentials for modelling the real life datasets.

Keywords: Lomax-generator, gumbel type-two distribution, monotonic failure rates, lomax gumbel type-two distribution, platykurtic, leptokurtic

1. Introduction

The Gumbel distribution identified with Gumbel (1954) [6] is one of the extreme value distributions with wide application in diverse areas like engineering, meteorology, reliability theory and hydrology. However the Gumbel Type-Two distribution among many classical distributions has been found to lack the capacity to adequately fit some random phenomenon. The Lomax distribution on the other hand is characterized with a heavy tail and is widely used in many fields like insurance and actuaries, economics, medical statistics and engineering for modeling and data analysis. It started its journey in literatures from Lomax (1954) [12] and has been generalized by various statisticians. Cordeiro *et al*. (2014) [3] introduced the use of Lomax distribution as a generator to propose a new family of distribution called Lomax-G distribution.

Gupta, Grag and Gupta (2016) [8] introduced Lomax-Gumbel distribution from the combination of cumulative distribution functions of Lomax and Gumbel distributions. The researchers used the Gumbel distribution for modeling maximum of random samples called the extreme value distribution type-1. Some of the Lomax generalized model existing in literatures includes Lomax-Weibull distribution by Gupta and Grag (2018) [7]. After the introduction of Lomax-G technique in 2014, the usefulness has recently received major interests and wide applications as witnessed by the introduction of various Lomax generalized convoluted distributions in literatures by the following authors; Ieren and Kuje (2018) [10] introduced Lomax-Exponential distribution; Venegas, Iriarte and Astorg (2019) [9] introduced Lomax Rayleigh distribution while Omale, Yahaya and Asiribo (2019) [16] introduced Lomax-Gompertz distribution. Yassmen (2019) [19] developed the Lomax-Lindley distribution while Leren *et al*. (2019) [9] introduced the Lomax-Inverse Lindley distribution.

Ijaz, Asim and Alamgir (2019) [11] introduced Lomax exponential distribution and Lomax inverse exponential by Abdulkadir, Jerry and leren (2020) [1].

The Gumbel Type-Two (GTT) distribution has received some further studies in literatures and some of the recent studies are presented in Table1.

Table 1: Extensions of Gumbel Type-two (GTT) Distribution and Authors

Acronyms	Probability Distributions	Authors	Year
LGTT	Lomax Gumbel Type-two	Proposed	2022
GGTT	Gompertz Gumbel Type-two	Ogunde, Olalude, Omosigho,	2021 [14]
ETGTT	Exponentiated Transformation of Gumbel Type-II	Sindhu, Shafiq and Al-Mdalla	2021
EGTT	Extended Gumbel Type-II	Ogunde, Fayose, Ajayi, Omosigho	2020 [13]
ExGTT	Exponentiated Gumbel Type-two	Okorie Akpanta and Ohakwe	2016

This present study is motivated by the expected benefits inherent in Lomax-G family of distribution revealed by recent studies of some researchers that have generalized flexible distributions with the aid of the Lomax-Generator. This research will contribute to knowledge from a further extension of Gumbel Type-Two (GTT) distribution and enhance its wide applications and popularity among researchers. The remaining part of the study is organized as follows; section 2 contains the resources for constructing the LGTT distribution, the definitions and presentations of the lifetime distribution. Some of the properties of the distribution were investigated in section 3 with various graphical description of the distribution while section 4 was devoted for estimating the parameters using the method of maximum likelihood estimation. Section 5 contains several applications for testing the usefulness of the distribution and the study was concluded in section 6.

2. Materials and Methods

The Gumbel Type-two distribution is to be extended using the Lomax-G technique as follows;

Let X be a random variable from the Gumbel Type-2 distribution; then the cdf is defined as;

$$G(x) = e^{-\theta x^{-k}} \tag{1}$$

And the associated density function is the derivative of equation (1) given as;

$$g(x) = \theta k x^{-k-1} e^{-\theta x^{-k}} \;; x > 0, k, \theta > 0 \tag{2}$$

Where k is scale and θ the shape parameters of the distribution. The CDF of Lomax-G distribution as defined and studied by Cordeiro *et al.* (2014) [3] is as follows;

$$F(x) = \int_0^{-\log(1-G(x))} \alpha\beta^\alpha \frac{dt}{(\beta+t)^{\alpha+1}} \;; \alpha, \beta > 0$$

$$F(x) = 1 - \left\{\frac{\beta}{\beta - \log(1-G(x))}\right\}^\alpha \tag{3}$$

The pdf associated with equation (3) is given by;

$$f(x) = \frac{\alpha\beta^\alpha g(x)}{[1-G(x)]}\left[\beta - log[1 - G(x)]\right]^{-(\alpha+1)} \tag{4}$$

G(x) is the cumulative distribution function (cdf) and g(x) is the probability density function (pdf) of the baseline distribution while α, β are shape parameters.

2.1 The Lomax Gumbel Type -Two (LGTT) Distribution

The new distribution is derived by substituting the cdf in equation (1) into Lomax-G cdf in equation (3) .The new distribution has cdf derived and presented as;

$$F(x) = 1 - \beta^\alpha\left(\beta - log\left[1 - e^{-\theta x^{-k}}\right]\right)^{-\alpha} \tag{5}$$

The density function corresponding to the cdf is given as;

$$f(x) = \frac{\alpha\beta^\alpha \theta k x^{-k-1} e^{-\theta x^{-k}}}{\left[1-e^{-\theta x^{-k}}\right]}\left[\beta - log\left[1 - e^{-\theta x^{-k}}\right]\right]^{-(\alpha+1)} \quad x > 0;\ \alpha, \beta, k, \theta > 0 \tag{6}$$

Sub models of LGTT distributions are obtained as follows;

i. The Lomax Inverse Exponential distribution (LOMINEXD) defined and studied by Abdulkadir *et al.* (2020) is a special case of (LGTT) when parameter $k = 1$

Plots of the density function and cdf for various parameter values is presented in Figure 1.

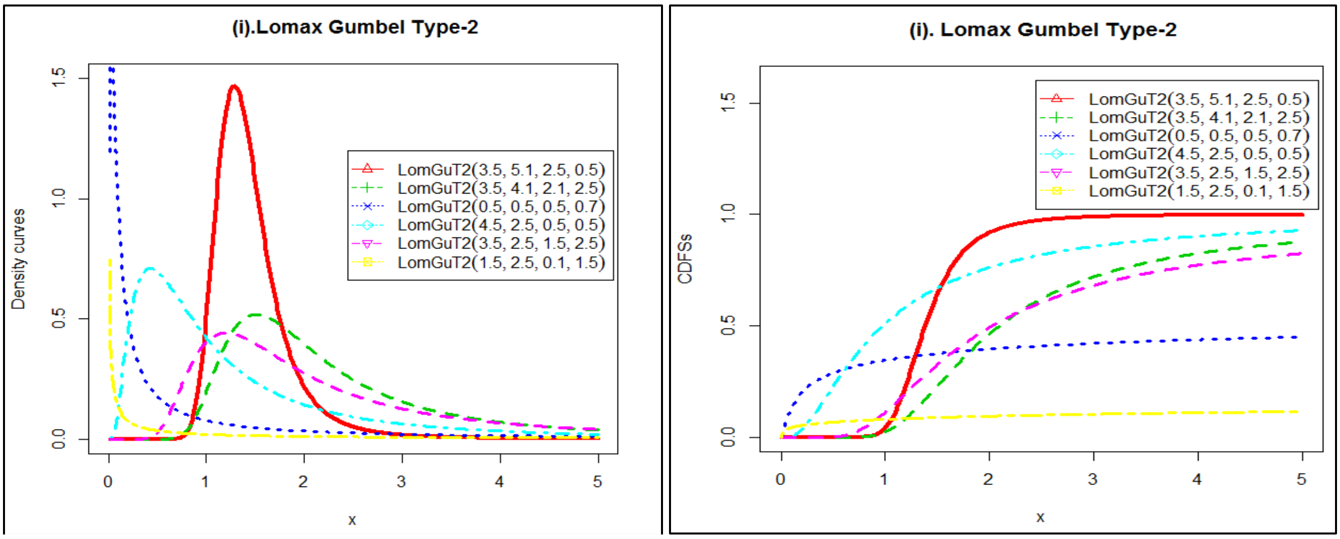

Fig 1: Density function and CDF of LGTT distribution for some values of parameters.

2.2. Some Properties of Lomax Gumbel Type-Two Distributions

2.2.1 Reliability Analysis

The Survival Function is given as;

$$S(x) = 1 - F(x)$$

The survival function for the LomGumT2 is derived and given as;

$$S(x) = \beta^{\alpha}\left(\beta - log\left[1 - e^{-\theta x^{-k}}\right]\right)^{-\alpha} \; ; \alpha, \beta, k, \theta > 0 \quad (7)$$

The Hazard Function (Failure Rate) is given as;

$$h(x) = \frac{f(x)}{s(x)} = \frac{\alpha\beta^{\alpha}\theta k x^{-k-1} e^{-\theta x^{-k}}}{\left[1 - e^{-\theta x^{-k}}\right]}\left[\beta - log\left[1 - e^{-\theta x^{-k}}\right]\right]^{-1} \quad (8)$$

$\alpha, \beta, k, \theta > 0$

Plots of the survival and hazard rate functions are as shown in figure 2 for some values of parameters of the distribution.

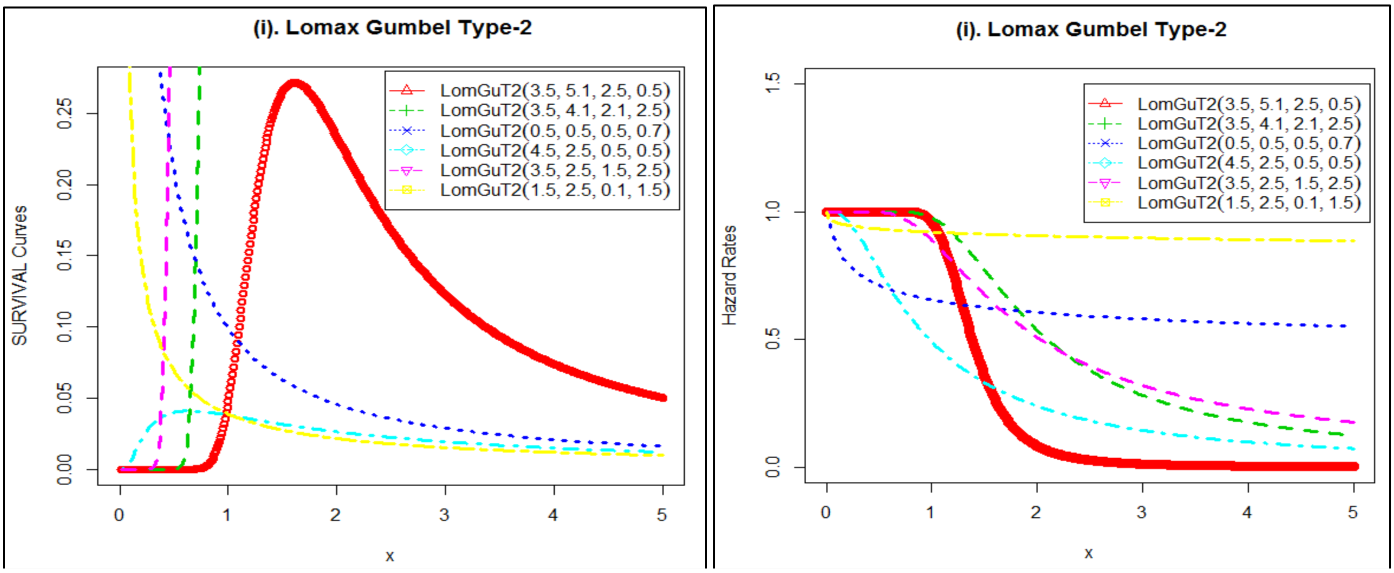

Fig 2: Survival and Hazard Rates functions of LGTT for some values of parameters.

2.2.2 Asymptotic Behavior of Lomax Gumbel Type-Two Distribution

The Asymptotic properties of the LGTT distribution is investigated by taking limits of the density function, and hazard rate function as $x \to \infty$ *and as* $x \to 0$ using theorems (1 to 2)

Theorem 1: Limit of Lomax Gumbel Type -2 density function as $x \to \infty$ *is* 0 and as $\to 0$ *is* ∞

Proof:

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{\alpha\beta^{\alpha}\theta k x^{-k-1} e^{-\theta x^{-k}}}{\left[1-e^{-\theta x^{-k}}\right]} \left[\beta - log\left[1-e^{-\theta x^{-k}}\right]\right]^{-(\alpha+1)} = 0$$

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{\alpha\beta^{\alpha}\theta k x^{-k-1} e^{-\theta x^{-k}}}{\left[1-e^{-\theta x^{-k}}\right]} \left[\beta - log\left[1-e^{-\theta x^{-k}}\right]\right]^{-(\alpha+1)} = \infty$$

Theorem 2: The limit of LGTT hazard rate function as $x \to \infty$ is 0 and as $x \to 0$ is ∞
Proof:
The hazard rate function is given as,

$$h(x) = \frac{\alpha\beta^{\alpha}\theta k x^{-k-1} e^{-\theta x^{-k}}}{\left[1-e^{-\theta x^{-k}}\right]} \left[\beta - log\left[1-e^{-\theta x^{-k}}\right]\right]^{-1}$$

Asymptotes of hazard rate function as $x \to \infty$

$$\lim_{x\to\infty} h(x) = 0$$

Asymptotes of hazard rate function as $x \to 0$

$$\lim_{x\to 0} h(x) = \infty$$

Theorem 3: Let $f(x)$ and $h(x)$ be the probability density and the hazard function of Lomax Gumbel Type-2 distribution respectively, then the asymptotes for the functions as $x \to 0$ and $x \to \infty$ implies $f(.) = h(.)$

Proof:
Results follow from proofs of theorems (1) and (2).

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} h(x) = 0$$

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} h(x) = \infty$$

The Asymptotes of Lomax Gumbel Type-2 distribution hazard rates h(x) is constant and the distribution also exhibits decreasing failure rates as shown in figure 3. The reliability function revealed a decreasing and upside bath tub shapes. The plots of the hazard also revealed constant and decreasing failure rates.

2.2.3 The Quantile Function and Median
Let X be a random variable from the Lomax-Gumbel Type-two distribution having cdf $F(x;\alpha,\beta,k,\theta)$; the quantile function is defined by; $u = P(X \le x_u) = F(x)$, where U is a uniform random variable on U(0,1). The Quantile for the proposed Lomax-Gumbel Type -2 distribution is obtained as;

$$Q(u) = \left(\frac{\theta}{\beta - \beta(1-u)^{-\frac{1}{\alpha}}}\right)^{\frac{1}{k}} \tag{9}$$

Simulation is achievable by generating random samples from the Lomax-Gumbel Type-Two distribution using the random variable X as follows;

$$X = \left(\frac{\theta}{\beta - \beta(1-u)^{-\frac{1}{\alpha}}}\right)^{\frac{1}{k}}$$

The median is obtained by substituting $u = 0.5$ into $Q(u)$, the median of Lomax-Gumbel Type-Two distribution is given by

$$Q(0.5) = \left(\frac{\theta}{\beta - 0.5^{-\frac{1}{\alpha}}}\right)^{\frac{1}{k}} \tag{10}$$

2.2.4 Order Statistics: Mathematical expression for the order statistics is presented in this section. Suppose we have $X_1, X_2, \dots, X_n$ as random sample of size n from the Lomax Gumbel Type-two distribution with the CDF and PDF given as

$F(x)$ and $f(x)$ respectively, let $X_{(1)}, X_{(2)}, \dots, X_{(n)}$ be the associated order statistics from the random variables in order of magnitude, then the probability density function of the order statistics $X_{(s)}$ of observation in the s^{th} position is given by;

$$f_{X\,s:n}(x) = \frac{n}{(s-1)!(n-s)!} G(x)^{s-1}(1-G(x))^{n-s}G'(x) \tag{11}$$

Using the pdf $f(x)$ in equation (6) and the corresponding cdf; the distribution of the s^{th}order statistics for Lomax Gumbel Type-two distribution is obtained as;

$$f_{X\,s:n}(x) = \frac{n\alpha\beta^{\alpha}\theta k x^{-k-1}e^{-\theta x^{-k}}\left[1-e^{-\theta x^{-k}}\right]^{-1}}{(s-1)!\,(n-s)!\left[\beta - log\left[1-e^{-\theta x^{-k}}\right]\right]^{(\alpha+1)}}\left[1-\beta^{\alpha}\left(\beta - log\left[1-e^{-\theta x^{-k}}\right]\right)^{-\alpha}\right]^{s-1} \times \left[\beta^{\alpha}\left(\beta - log\left[1-e^{-\theta x^{-k}}\right]\right)^{-\alpha}\right]^{n-s}$$

$x > 0;\ \alpha, \beta, k, \theta > 0$

The order statistics for smallest observation is derived when $s = 1$ and is given as;

$$f_{X\,1:n}(x) = \frac{n\alpha\beta^{\alpha}\theta k x^{-k-1}e^{-\theta x^{-k}}\left[1-e^{-\theta x^{-k}}\right]^{-1}}{\left[\beta - log\left[1-e^{-\theta x^{-k}}\right]\right]^{(\alpha+1)}}\left[\beta^{\alpha}\left(\beta - log\left[1-e^{-\theta x^{-k}}\right]\right)^{-\alpha}\right]^{n-1}$$

$x > 0;\ \alpha, \beta, k, \theta > 0$

The largest order statistics is derived when $s = n$

$$f_{X\,n:n}(x) = \frac{n\alpha\beta^{\alpha}\theta k x^{-k-1}e^{-\theta x^{-k}}\left[1-e^{-\theta x^{-k}}\right]^{-1}}{\left[\beta - log\left[1-e^{-\theta x^{-k}}\right]\right]^{(\alpha+1)}}\left[1-\beta^{\alpha}\left(\beta - log\left[1-e^{-\theta x^{-k}}\right]\right)^{-\alpha}\right]^{n-1}$$

$x > 0;\ \alpha, \beta, k, \theta > 0$

2.2.5. Estimation of Parameters of LGTT Distribution

The method of maximum likelihood estimation (MLE) is utilized for the estimation of parameters of the distribution. Let $X_1, X_2, \dots, X_n$ be random sample of size n from the Lomax Gumbel Type-Two distribution with density function $f(x)$ as given in equation (6) with set of parameters $\varphi = (\alpha, \beta, k, \theta)$.

$$f(x;\varphi) = \frac{\alpha\beta^{\alpha}\theta k x^{-k-1}e^{-\theta x^{-k}}}{\left[1-e^{-\theta x^{-k}}\right]}\left[\beta - log\left[1-e^{-\theta x^{-k}}\right]\right]^{-(\alpha+1)} \quad x > 0;\ \alpha, \beta, k, \theta > 0$$

The likelihood function of the distribution is obtained as;

$$Lik[f(x,\varphi)] = \prod_{i=1}^{n}\left[\frac{\alpha\beta^{\alpha}\theta k x^{-k-1}e^{-\theta x^{-k}}}{\left[1-e^{-\theta x^{-k}}\right]}\left[\beta - log\left[1-e^{-\theta x^{-k}}\right]\right]^{-(\alpha+1)}\right]$$

The log Likelihood function $logLik[f(x,\varphi)]$ denoted as $LogL$ is

$$LogL[X/\alpha,\beta,k,\theta] = nlog\alpha + n\alpha log\beta + nlog\theta + nlogk - (k+1)\textstyle\sum_1^n logx_i - \theta\sum_1^n x^{-k}$$

$$-\sum_1^n log\left[1-e^{-\theta x^{-k}}\right] - (\alpha+1)\sum_1^n log\left[\beta - log\left[1-e^{-\theta x^{-k}}\right]\right]$$

The next step is to generate derivatives of $LogL[X/\alpha,\beta,k,\theta]$ with respect to the parameters;

$$\frac{dLogL}{d\alpha} = \frac{n}{\alpha} + nlog\beta - \sum_1^n log\left[\beta - log\left[1-e^{-\theta x^{-k}}\right]\right]$$

$$\frac{dLogL}{d\beta} = \frac{n\alpha}{\beta} - (\alpha+1)\sum_1^n \frac{1}{\left[\beta - log\left[1-e^{-\theta x^{-k}}\right]\right]}$$

$$\frac{dLogL}{d\theta}=\frac{n}{\theta}-\sum_{1}^{n}x^{-k}-\sum_{1}^{n}\frac{\theta k\,x^{-k-1}e^{-\theta x^{-k}}}{\left[1-e^{-\theta x^{-k}}\right]}-(\alpha+1)\sum_{1}^{n}\frac{\theta k\,x^{-k-1}e^{-\theta x^{-k}}}{\left[\beta-log\left[1-e^{-\theta x^{-k}}\right]\right]\left[1-e^{-\theta x^{-k}}\right]}$$

$$\frac{dLogL}{dk}=\frac{n}{k}-\sum_{1}^{n}log\,x_i-\theta\sum_{1}^{n}x^{-k}\log x-\sum_{1}^{n}\frac{\theta x^{-k}e^{-\theta x^{-k}}\log x}{\left[1-e^{-\theta x^{-k}}\right]}\left\{1-\frac{(\alpha+1)}{\left[\beta-log\left[1-e^{-\theta x^{-k}}\right]\right]}\right\}$$

By setting $\frac{dLogL}{d\alpha}=\frac{dLogL}{d\beta}=\frac{dLogL}{d\theta}=\frac{dLogL}{dk}=0$;

The parameters can be obtained but the complexity of the functions will require solutions to the above equations to be estimated through statistical software. Fisher information matrix for generating variances for the confidence intervals for estimated parameters $\hat{\varphi}=(\hat{\alpha}\;\hat{\beta}\;\hat{\theta}\;\hat{k})$ can be obtained after second derivates of the normal equations

The 100 $(1-\epsilon)$% confidence intervals for parameters of Lomax Gumbel Type-2 distribution are provided as follows;

$\hat{\alpha}\pm Z_{\epsilon/2}\sqrt{I^{-1}{}_{\alpha\alpha}(\hat{\Theta})}$, $\hat{\beta}\pm Z_{\epsilon/2}\sqrt{I^{-1}{}_{\beta\beta}(\hat{\Theta})}$, $\hat{\theta}\pm Z_{\epsilon/2}\sqrt{I^{-1}{}_{\theta\theta}(\hat{\Theta})}$ and $\hat{k}\pm Z_{\epsilon/2}\sqrt{I^{-1}{}_{kk}(\hat{\Theta})}$

Where Z_{ϵ} is used as the 100 $(1-\epsilon)$% upper percentile of the standard normal distribution.

3. Results and Discussions from Application to Real Dataset

Five real life data sets were analyzed in this section to investigate the performance of the LGTT distribution; descriptive statistics of the five datasets are presented in Table 1.

Table 2: Descriptive statistics of datasets

Data	n	Min	Q_1	median	Q_3	Mean	Max	Skew	Kurt.
Data1	20	1.10	1.475	1.700	2.05	1.90	4.10	1.719	5.924
Data2	31	18.83	25.51	29.90	35.83	30.81	45.38	0.405	2.287
Data3	38	2.00	2. 00	5.00	13.50	9.50	43.00	1.503	4.369
Data4	72	0.10	1.08	1.49	2.24	1.77	5.55	1.314	1.853
Data5	128	0.08	3.35	6.40	11.84	9.37	79.05	3.248	15.195

LGTT distribution will be compared with GGTT, ExGTT, EGTT and ETGTT which are some notable generalizations of the GTT distribution. LGTT distribution will also be compared with some related distributions developed using the Lomax generalized families of distribution. MLE of the model parameters are obtained using R statistical software. The goodness of fit measures for selection of the best model to the data are reported for the Akaike Information Criteria (AIC), Bayesian Information Criteria (BIC), the P- values, Kolgomorov Smirnoff (K-S) statistics and the Log-Likelihhod (LL) estimates.

Data 1: Relief Times of Patients on Analgesic

This dataset represents the lifetime's data relating to relief times (in minutes) of 20 patients receiving an analgesic and reported by Gross and Clark (1975). The data was recently analyzed by Ieren *et al*. (2019) [9] using LOMINLIND distribution with Log Likelihood (LL) estimates LL=16.3968. The flexibility of LGTT distribution will be assessed by comparing with LOMINLIND, Lomax–Exponential (LomExp) and Lomax-Gompertz (LomGo) distributions.

.**Table 3:** Maximum Likelihood Estimates and Criteria for Model Selection- Data1

Models	Estimated Model Parameters				LL	AIC	BIC	K-S	P-value
	α	β	θ	K					
LGTT	9.2629	7.7862	6.7062	3.9373	15.5287	39.0587	43.0403	0.0913	0.9963
LomGo	0.3259	6.9501	0.0163	5.6039	16.1633	40.3265	44.3095	0.1072	0.9755
LomExp	2.2555	29.6115	-	-	21.6748	47.3495	49.3409	0.2123	0.3282

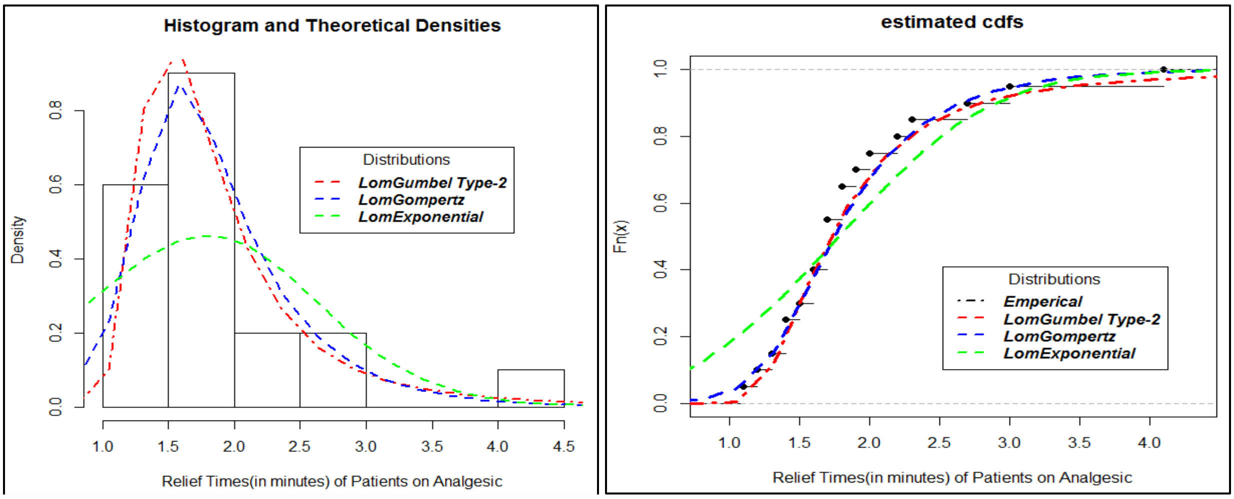

Fig 3: Plots of density functions and estimated cdfs fitted to Patients on Analgesic

Results of analysis on Table 3 and graphical plots of density function with the estimated cdfs displayed in Figure 3 shows that LGTT distribution can be considered as better model for the data set than the Lomax Gompertz, Lomax Exponential and LOMINLIND distributions.

Data 2: Strength of Glass of Aircraft Windows

This second dataset represent the strength data of glass of the aircraft window reported by Fuller *et al*. (1994) and recently applied using Lomax-Gompertz distribution by Omale *et al*. (2019) [16].

Table 4: Maximum Likelihood Estimates and Criteria for Model Selection- Data2

Models	Estimated Model Parameters				LL	AIC	BIC	K-S	P-value
	α	β	θ	k					
LGTT	31.7086	0.4549	89.5227	0.8379	104.8179	217.6358	223.3717	0.1076	0.8281
LomGo	0.2952	3.7704	0.0005	0.2523	105.7285	219.4571	225.1930	0.1196	0.7222

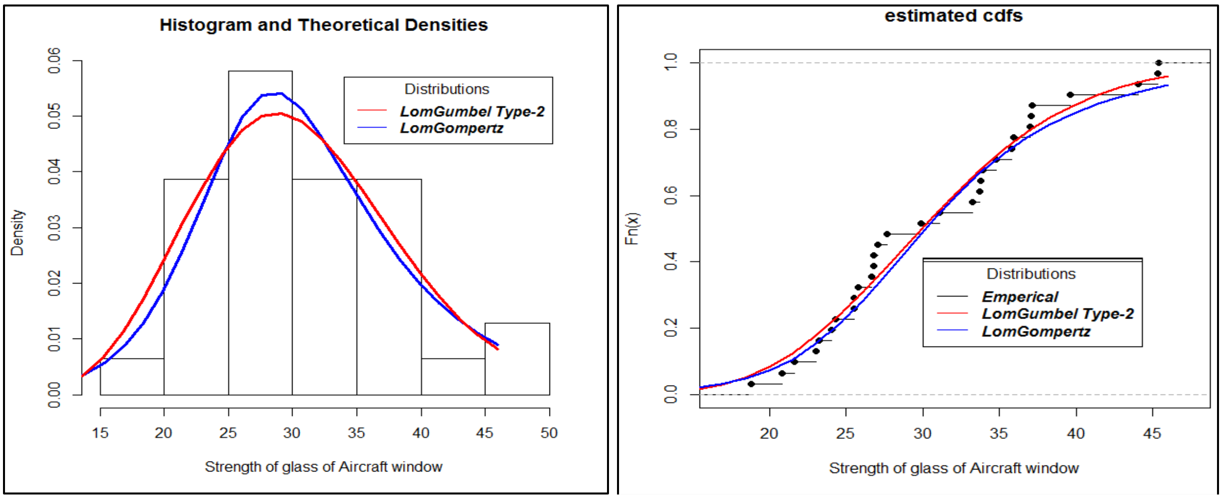

Fig 4: Plots of pdf and estimated cdfs fitted to strength of glass of Aircraft windows

Results of analysis on Table 4 and graphical plots of density function with the estimated cdfs displayed in Figure 4 shows that LGTT distribution can be considered as better model for the data set than the Lomax Gompertz distribution.

Data 3: Survival times in month's of 20 acute myeloid leukemia patients

The third data represents survival times in months of 20 acute myeloid leukemia patients which was used by Okorie *et al*. (2016) [15] for testing the performance of Exponentiated Gumbel (ExGTT) distribution.

Table 5: MLEs and Criteria for Model Selection- leukemia patients

Models	Estimated Model Parameters				LL	AIC	K-S	P-value
	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\theta}$	$\hat{k}$				
LGTT	24.755	0.0018	12.6196	0.3157	24.4949	56.9899	0.1400	0.7779
EGTT	8.3219	0.7369	68.3513	0.6747	24.9116	57.8232	0.1514	0.6733
ExGTT	6.7500	-	38.0888	0.6781	25.2857	56.5713	0.1619	0.6138
GTT	2.5583	-	-	2.0589	29.0895	62.1790	0.2766	0.0763

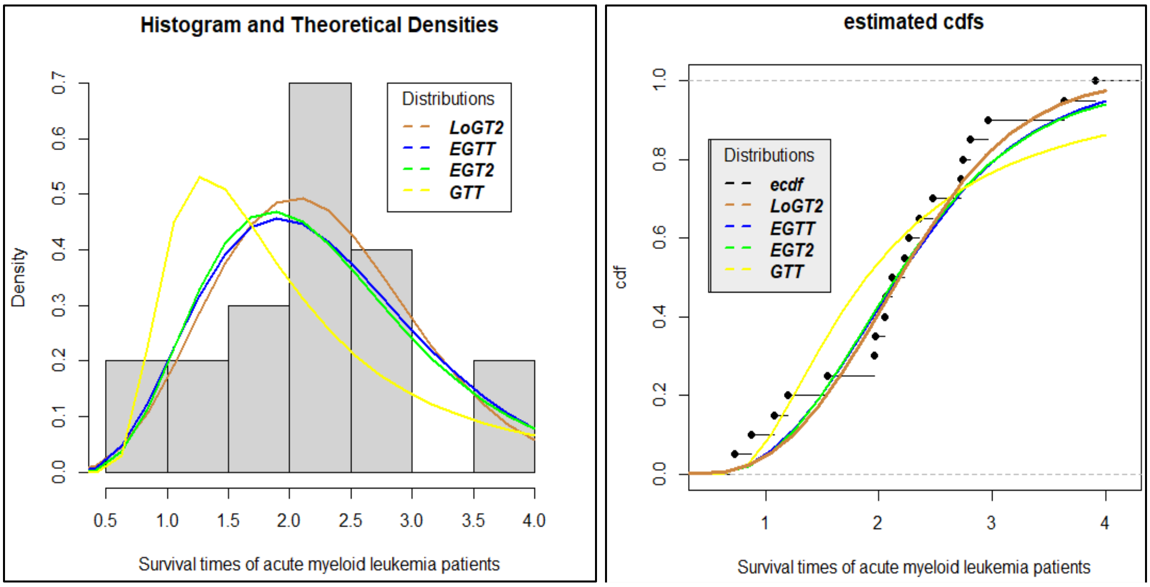

Fig 5: Plots of density functions and estimated cdfs to leukemia patients

Results of analysis on Table 5 and graphical plots of density function with the estimated cdfs displayed in Figure 5 shows that LGTT distribution can be considered as better model for the data set than other extensions of the GTT distributions.

Data 4: Guinea pigs data

The fourth data represents survival times (in days) of 72 guinea pigs infected with virulent tubercle bacilli, reported by Bjerkedal (1960) [2]. It has received several applications and recently by Ogunde *et al*. (2020) [13] for testing the flexibility of Extended Gumbel Type-two (EGTT) and Ogunde *et al*. (2021) [14] in Gompertz Gumbel Type-two (GGTT) distributions.

Table 6: MLEs and Criteria for Model Selection for Guinea pig dataset

Models	Estimated Model Parameters				LL	AIC	K-S	P-value
	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\theta}$	$\hat{k}$				
LGTT	9.7530	0.0001	13.2058	0.1708	94.4005	196.8009	0.0879	0.6339
GGTT	38.1632	4.9660	4.8914	0.4174	95.6619	199.3238	0.1067	0.3854
EGTT	7.7750	0.3862	72.3458	0.6130	96.9036	201.8073	0.1097	0.3519
ExGTT	3.3517	-	9.5205	0.6327	99.1929	204.3858	0.1351	0.1446
ETGTT	5.5752	-	1.5879	0.19363	105.0777	216.1551	0.1398	0.1198
GTT	1.1359	-	-	1.1470	118.3037	240.6074	0.1840	0.0153

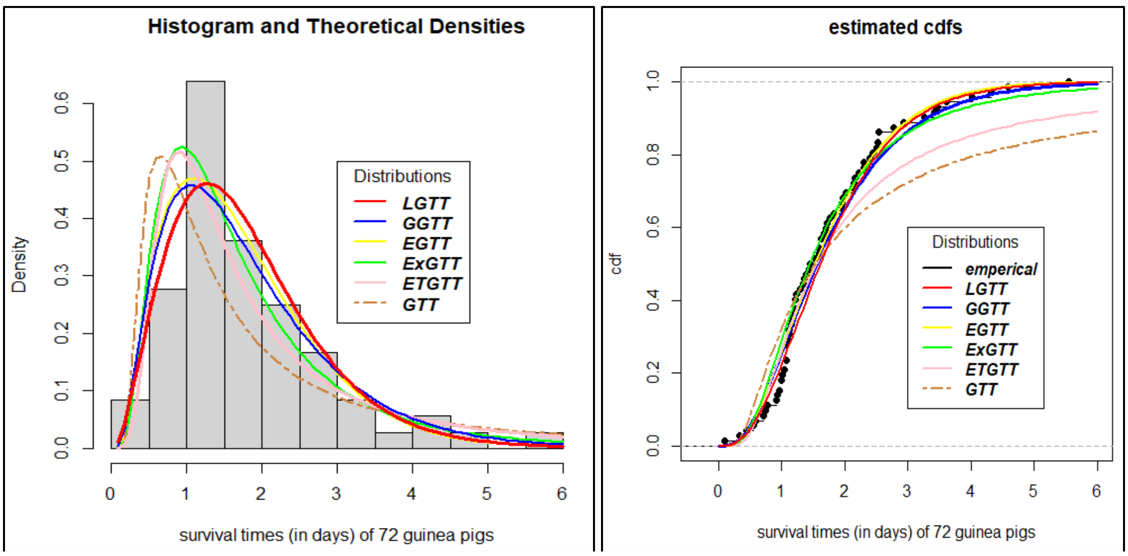

Fig 6: Plots of density functions and estimated cdfs to Guinea pig dataset

Results of analysis on Table 6 and graphical plots of density function with the estimated cdfs displayed in Figure 6 shows that LGTT distribution can be considered as better model for the data set than all the competitive distributions derived by extensions of the GTT distribution.

Data 5: Bladder cancer data

This dataset is the remission times (in months) of a random sample of 128 patients with bladder cancer. It has been widely applied by notable researchers and recently by Ieren *et al.* (2019) [9], Ogunde *et al.* (2020) [13] using EGTT and Ogunde *et al.* (2021) [14] in the study of GGTT distributions. The LGTT distribution is applied to the data and compared with some notable generalizations of the Gumbel Type-two distribution GGTT, EGTT, ExGTT, ETGTT, and GTT distributions.

Table 7: MLEs and Criteria for Model Selection for bladder cancer dataset

Models	Estimated Model Parameters				LL	AIC	K-S	P-value
	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\theta}$	$\hat{k}$				
LGTT	46.8822	0.3205	8.0017	0.2189	411.75	831.5074	0.0355	0.9970
GGTT	24.1803	0.5783	6.1335	0.3086	413.59	835.1758	0.0579	0.7837
EGTT	6.0887	1.7335	76.0650	0.1893	413.81	835.6189	0.0725	0.5110
ExGTT	5.4315	-	12.2088	0.3558	415.79	837.5569	0.0598	0.7502
ETGTT	6.6668	-	1.0153	0.6163	427.17	860.3483	0.0899	0.2517
GTT	2.5254	-	-	0.7942	444.52	893.0436	0.1371	0.0163

Results of analysis on Table 7 and graphical plots of density function with the estimated cdfs displayed in Figure 7 shows that LGTT distribution can be considered as better model for the bladder cancer data than other extensions of the GTT distributions already used for the dataset.

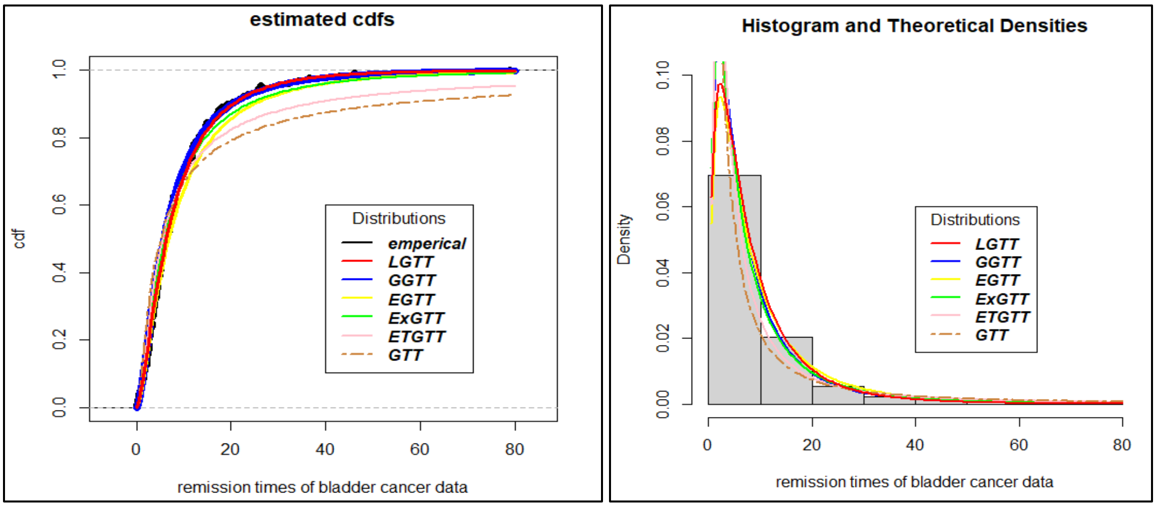

Fig 7: Plots of density functions and estimated cdfs to bladder cancer dataset

4. Conclusion of the Study

The Gumbel Type-Two (GTT) was generalized in this study to address the monotonic failure rates and enhance the potential for modeling various types of positively skewed data. The new distribution called Lomax Gumbel Type-Two (LGTT) is positively skewed and close to a bell shape depending on values of parameters. The reliability and hazard rate functions were derived while expressions were obtained for the Quantile function and the order statistics. The new distribution has the Lomax Inverse Exponential distribution as a sub-model. Extensive applications to diverse lifetime datasets were carried out to test the potentials of LGTT. It was compared with (GGTT, EGTT, ExGTT and ETGTT) some popular distributions existing in literatures that were also developed by extension of the GTT as baseline distribution using different methodologies. Results have proved that extension of GTT using the Lomax-G method produced superior performance than the competitive distributions. In addition, the LGTT also shows greater flexibility than some other distribution developed using the Lomax-G.

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