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Bhaskar Kalita
Department of Mathematics: T. H. B. College, Sonitpur, Assam, India

Corresponding Author:
Bhaskar Kalita
Department of Mathematics: T. H. B. College, Sonitpur, Assam, India

The transient of viscoelastic MHD fluid through stokes oscillating porous plate: An exact solution

Bhaskar Kalita

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Abstract
The magneto hydrodynamic transient free convection flow of a viscoelastic fluid (Rivlin-Ericksen) caused by the sinusoidal oscillation of a plane flat porous plate has been studied in this paper. The constitutive equations of continuity and mass conservation of viscoelastic fluid are solved by Laplace transform technique. The Velocity profiles of transient and steady-state due to porous plate in presence Magnetic Hartmann Number and porosity of the medium are obtained in exponential forms and Complementary Error Functions. The results got for velocity profiles are shown through graphs and discussed in the concluding section.

Keywords: MHD, transient flow, porous plate, Rivlin-Ericksen fluid, sinusoidal oscillation

Introduction

Stokes first studied the unsteady free convection flow of a viscous incompressible fluid past an impulsively started infinite horizontal plate. The plate oscillates in its own plane. The plate has two natures-one is of impulsively starting in its own plane suddenly set into motion which creates a start - up flow and other one is of oscillating – that oscillates in its own plane. H. Schlichting[*] called the farmer problem as "Stokes first problem" and later one as "Stokes second problem" Stokes presented exact solutions to both the problems. These problems being of fundamental in nature are referred in all the text books of viscous flow. Stokes result for the oscillating plate is the steady - state solution which applies after the effect of any initial velocity profile has died out. But this solution is not a complete solution, since it does not satisfy the initial condition. The complete solution for the problem requires the transient solution as well as steady state solution. And this is given by Panton (7). He presented the solution to transient problem in exact from in terms of standard mathematical functions and velocity distributions for the plate either oscillating as $Sin(T)$ or$-Cos(T)$. Later on, Deka *et al.* [2] studied this problem considering semi – infinite incompressible viscous fluid in the presence of a uniform magnetic field applied transversely to the plate. I determined to extend this paper by considering the fluid as Visco – elastic electrically conducting and the flat plate as Porous. In section 2, the mathematical formulation and a solution to transient component is presented in terms of standard mathematical functions. In section 3, characteristics of the solutions are cited, while in section 4, the problem is concluded with outcome of investigation.

Mathematical Formulation

The constitutive equation of second order Visco – elastic (extended by Rivlin-Ericksen) fluid in tensor notation is as follows –

$$\tau_{ij}=-p\delta_{ij}+\mu_{1}A_{(1)ij}+\mu_{2}A_{(2)ij}+\mu_{3}A_{(1)i\alpha}A_{(1)\alpha j} \qquad (1)$$

Where τ_{ij} is the stress tensor, p is the hydrostatic pressure, δ_{ij} is the Kronecar delta, $A_{(1)}$ and $A_{(2)}$ are Rivlin – Ericksen tensors of order 1 and 2, and $\mu's$ are coefficients of viscosity. Here $A_{(1)}$ and $A_{(2)}$ are given by symmetric tensors and they are defined by Kalita B *et al.* 2012 [4].

$A_{(1)ij=v_{i,j}+v_{j,i}}$ (2)

$A_{(2)ij} = a_{i,j} + a_{j,i} + 2v_{m,i}v_{m,j}$ (3)

$$a_{i,j} = \frac{\partial v_i}{\partial t} + v_j v_{i,j} \ (i,j,m = 1,2,3)$$

v_i = Component of velocity, a_i = component of acceleration.

Here the plate is porous and semi-infinite horizontal. The $X^/ -$ axis is taken along the flat plate while the $Y^/$- axis is taken normal to the plate. Let $u^/ and\ v^/$ be the fluid velocities along $X^/ and\ Y^/$axis, respectively. Then since the plate is semi infinite in extent, the fluid is taken to occupy the upper half plane. $u^/$ is a function of $y^/$ and $t^/$ and $v^/$ is independent of $y^/$. The fluid is electrically conducting and the plate is non- conducting. Let a uniform magnetic field H_0be applied in a direction perpendicular to $X^/$- axis. The fluid is assumed to be of low conductivity; so induced magnetic field is negligible. The Lorentz's force is $-\sigma H_0^2 u^/$. At time $t(\leq 0)$, the plate and fluid are at rest. At t (> 0), the plates start oscillating in its own plane. For boundary condition it is assumed that there is no slip at the wall.
Under these assumptions, we can write the continuity and momentum equations which governs the flow field as –

$$\frac{\partial v^/}{\partial y^/} = 0 \quad (4)$$

$$\frac{\partial u^/}{\partial t^/} + v^/ \frac{\partial u^/}{\partial y^/} = \vartheta^/ \frac{\partial^2 u^/}{\partial y^{/2}} - \frac{\sigma H_0^2 u^/}{\rho} + \frac{K_0}{\rho}\frac{\partial^3 u^/}{\partial t^/ \partial y^{/2}} \quad (5)$$

Where,
ρ = Density of the fluid,
H_0 = Uniform magnetic field applied transversely to the plate,
σ = Electrical conductivity of the fluid,
ϑ = Co-efficient of Kinematic viscosity of the fluid,
K_0 = Coefficient of elasticity.

The initial and boundary conditions are

$$u^/(y^/,0)\text{=0};\ u^/(\infty,t^/) < \infty;\ u^/(0,t^/) = U sin(\omega^/ t^/) \quad (6)$$

The non-dimensional quantities are defined as follows:

$$y = \frac{y^/ U}{\vartheta}, u = \frac{u^/}{U}, t = \frac{t^/ U^2}{\vartheta}, R_c = \frac{K_0 U^2}{\alpha_0}, M = \frac{\sigma H_{0\vartheta}^2}{\rho U^2}, \omega^/ = \frac{U^2}{\vartheta}, V = \frac{v^/}{U}, P = \frac{\alpha_0 C}{K} \quad (7)$$

Hence, the equations of continuity, motion and boundary conditions reduces to-

$$U\frac{\partial V}{\partial y} = 0 \quad (9)$$

$$\frac{\partial u}{\partial t} + V\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} - Mu + R_c \frac{\partial}{\partial t}\left(\frac{\partial^2 u}{\partial y^2}\right), R_c = \frac{K_{0U^2}}{\alpha\vartheta} \quad (10)$$

and

$u(y,0) = 0, u(0.t) = \sin(t), u(\infty,t) < \infty$ (11)

Solution of Equation
Solving equation (9), we obtain
$V = constant$
For constant suction, we consider $V = -V_0$ (12)

where the negative sign indicates that the suction is towards the plate.
Hence the equation (10) reduces to

$$\frac{\partial u}{\partial t} - V_0\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + R_c \frac{\partial}{\partial t}\left(\frac{\partial^2 u}{\partial y^2}\right) - Mu \quad (13)$$

Equation (13) is a 3rd order differential equation due to the presence of the elastic parameter R_c. If the elastic parameter R_c would zero, we would have it as second order differential equation, and thereby the fluid reduces to Newtonian case (viscous fluid). To have the complete solution of (13), we require another boundary condition. But we have only two boundary conditions as specified above. However, we overcome this difficulty by considering the physical condition of the fluid. As, R_c, the elastic parameter is a small quantity based on vanishing memory, it is always $\ll 1$. So we can expand u in powers of R_c as

$$u = u_0 + R_c u_1 \tag{14}$$

Substituting (14) in equation (13), and equating the coefficients of equal powers of R_c, and neglecting those of R_c^2, we have the following equations

$$\frac{\partial u_0}{\partial t} - V_0 \frac{\partial u_0}{\partial y} = \frac{\partial^2 u_0}{\partial y^2} - M u_0 \tag{15}$$

$$\frac{\partial u_1}{\partial t} - V_0 \frac{\partial u_1}{\partial y} = \frac{\partial^2 u_1}{\partial y^2} + \frac{\partial}{\partial t}\left(\frac{\partial^2 u_0}{\partial y^2}\right) - M u_1 \tag{16}$$

The boundary conditions (11) now modified as

$$u_0(y,0) = 0, u_1(y,0) = 0, for\ all\ y, \text{t=0} \tag{17a}$$

$$u_0(\infty,t) < \infty, u_1(\infty,t) < \infty, t > 0 \tag{17b}$$

$$u_0(0,t) = \sin(t), u_1(0,t) = \sin(t), t > 0 \tag{17c}$$

The velocity may be decomposed into steady – state and transient components satisfying equations (15) and (16) as:

$$u_0 = u_0^s + u_0^t \text{ and } u_1 = u_1^s + u_1^t \tag{18}$$

The steady-state components can be derived as:

$$u_0^s = \exp(-ay/\sqrt{2}) sin(t - by/\sqrt{2}) \text{ and } u_1^s = \exp(-ay/\sqrt{2}) sin(t - by/\sqrt{2}) \tag{19}$$

Where $a = \sqrt{M + \sqrt{1 + M^2}}$, $b = {}^1/_a$

The solutions (19) satisfy the boundary conditions (17b) and (17c), but not the initial condition (17a). If the transient solution satisfies the following boundary conditions

$$u_0^t(y,0)[= -e^{-ay/\sqrt{2}} sin(-by/\sqrt{2})] = Im e^{-cy/\sqrt{2}} = u_1^t(y,0) \tag{19a}$$

$$u_0^t(\infty,T) = \infty = u_1^t(\infty,T) \tag{19b}$$

$$u_0^t(0,T) = 0 = u_1^t(0,T) \tag{19c}$$

where $c = a - ib$, then the composition of the both transient and steady – state solutions will completely satisfy eq. (13) or (15) and (16). For the transient solution, we apply Laplace Transform Technique on the transient part of (15) and (16), and on boundary conditions [19(a, b, c)]. The final results are found as –

$$u_0^t(Y,T) = Im\left[\frac{1}{2} e^{\frac{T}{2}(b^2-a^2) - \left(\frac{Y^2}{4T} + \frac{V_0^2 T}{4}\right)} \{w(z_1) - e^{-YV_0} w(z_2)\}\right] \tag{20a}$$

$$u_1^t(Y,T) = \frac{1}{2} Im\left[\begin{array}{l}(M - 1 - i)\left\{e^{\frac{T}{2}(b^2-a^2) - \left(\frac{Y^2}{4T} + \frac{TV_0^2}{4}\right)}\left(w(z_3) + e^{i\frac{Yb}{\sqrt{2}} - \frac{Ya}{\sqrt{2}}} w(z_4)\right)\right\} \\ \qquad -2e^{-\frac{a}{\sqrt{2}}(Y+TV_0)} e^{-i\frac{b}{\sqrt{2}}\left(Y + \frac{\sqrt{2}T}{b} + V_0 T\right)}\end{array}\right] \tag{20b}$$

Where,

$$z_2 = \sqrt{\frac{T}{2}} b + i\left(\frac{Y}{2\sqrt{T}} - \frac{\sqrt{T}}{2} V_0 + \sqrt{\frac{T}{2}} a\right) = z_3$$

$$z_4 = -\sqrt{\frac{T}{2}} b + i\left(\frac{Y}{2\sqrt{T}} - \sqrt{\frac{T}{2}} a + \frac{\sqrt{T}}{2} V_0\right) = z_1$$

The complete steady state and transient solutions are respectively

$$u^t(Y,T) = u_0^t(Y,T) + R_c u_1^t(Y,T) \tag{21}$$

$$u^s(Y,T) = u_0^s(Y,T) + R_c u_0^s(Y,T) \tag{22}$$

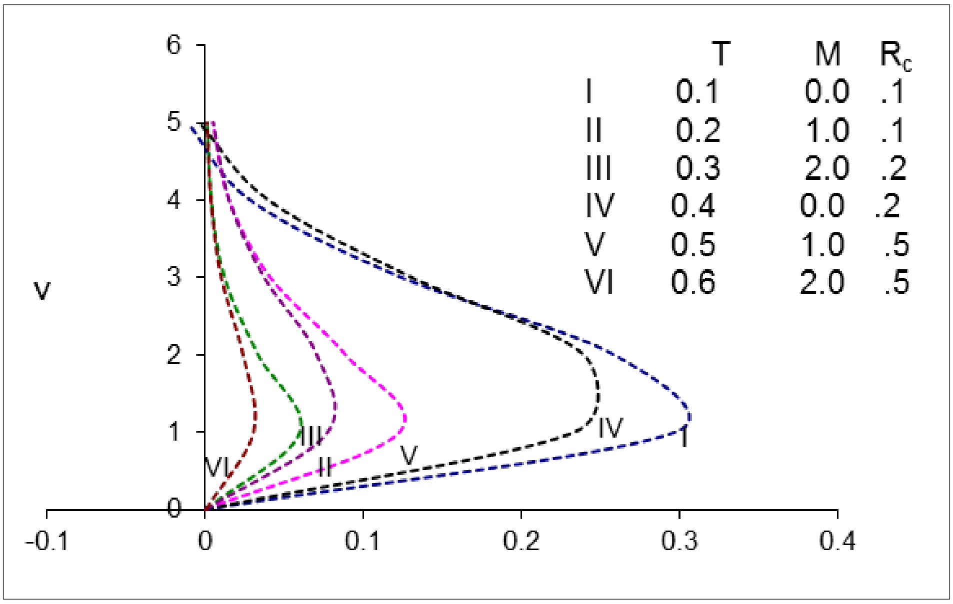

Fig 1: Transient vel. distribution; Plate velocity sin (T)

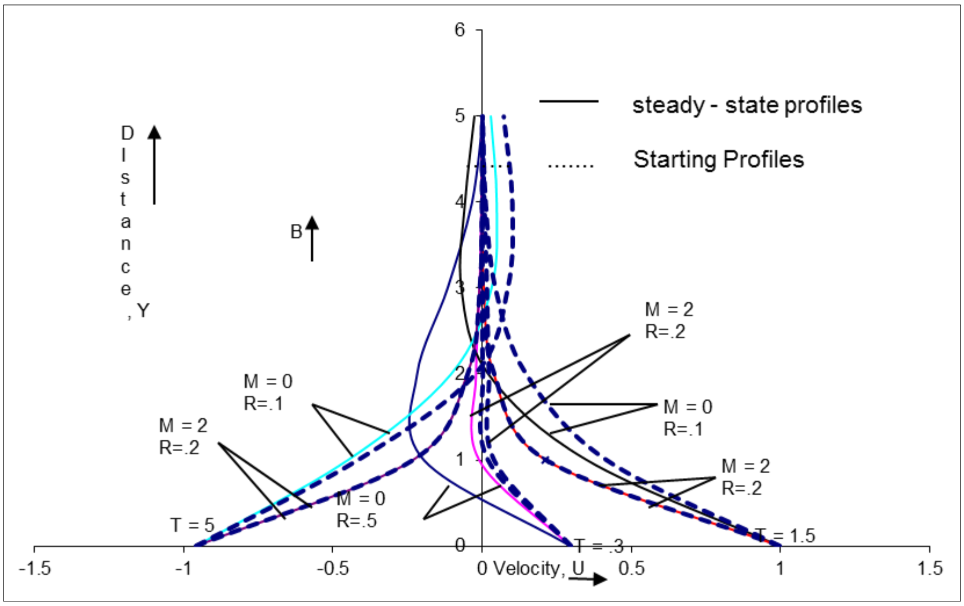

Fig 2: Starting Phase vel. Distribution; Plate velocity sin (T)

Conclusion

If we go through the figures presented by Panton 1968 [7] and Deka *et al.* 1995 [2], in their respective papers with our obtained figures, a clear difference can be seen. We see that the fluid penetrates towards the plate at and near the plate. However, at a far distance from the plate this nature cannot be seen. We feel that this effect is due to the magnetic parameter (M), the porosity (V_0) of the plate on the flow field and due to elastic property of the fluid. However, the effect of Rivlin-Ericksen fluid is very clear.

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