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Availability analysis considering two subsystem failures simultaneously via Markov model

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Abstract
This paper discusses the performance analysis of the alloy wheel plant (AWP). Till now researchers (1, 5, 9, 14, and 25) took the assumption that only one failure at one time but this paper considered two failures at one time. The AWP mainly consists of four subsystems in series (Gdc Machine, Cutting Machine, Heat Treatment Machine and Shot Blasting Machine). The failure and repair rates are implicit to be constant and statistically independent. A transition diagram developed form the interrelationship among the working and failed state. Mathematical modelling of this system is done by applying markov- birth –death processes. These equations are solved by recursive method and optimized reliability is calculated for failure and repair rates. The obtaining result is helpful to improve the overall performance of alloy wheel plant.

Keywords: Availability, differential equation

Introduction

In today's scenario changes in technology are very rapid, every industry trying to make a more reliable product, which is having better life, better performance. In this paper authors try to improve reliability of AWP and for optimizing reliability failure and repair rates are given. Singh and Goyal (2013) [2] discussed the Behavior analysis of a biscuit making plant using Markov Regenerative modeling. The methodology of the system based on Markov model which is used to find the availability of the system and transient behavior of repairable mechanical biscuit shaping system pertaining to a biscuit manufacturing plant.

Garg, Kumar and Singh (2009) [2] discussed about the Availability analysis of a cattle feed plant using matrix method. This plant contains seven subsystems arrange in series. The mathematical model has been developed by using Markov birth- death process and makes a transition diagram. Tewari and Kumar (2016) [3] the paper describes the availability analysis of milling system in a rice milling plant. The failure and repairs rates are assumed to be exponential distribution and problem is formulated using Markov birth- death process. Kumar (2014) [4] discussed the availability analysis of a thermal power plant boiler air circulation system using Markov approach. In this plant the system contains four subsystems with full working state, reduced capacity state and failed state.

Rajbala & Garg (2019) [6] discussed about the steady state and time dependent availability analysis of a manufacturing plant. Rajbala & Garg (2019) [6] discussed about the Behaviour analysis of alloy wheel plant. Rajbala & Kumar (2021) [7] discussed about an article on the system reliability and availability analysis using RPGT-A general approach. Kumar and Garg (2019) [19] have discussed the reliability technology theory and its applications. Kumar *et al*. (2018) [10] have studied behaviour analysis of a bread making system. Kumar *et al*. (2019) [19] analyzed sensitivity analysis of a cold standby framework with priority for preventive maintenance consist two identical units with server failure utilizing RPGT. Present paper consists two units one of which is online while other is kept is cold standby mode. Online & cold standby unit are indistinguishable in nature & have just two modes one is good and other is totally failed. Rajbala, *et al*. (2019) [6] have studied the system modeling and analysis: a case study EAEP manufacturing plant. Kumar *et al*. (2017) [14] have studied behavior analysis in the urea fertilizer industry.

Kumar *et al*. (2017) [14] have examined the mathematical modeling & profit analysis of an edible oil refinery plant. Kumar *et al*. (2019) [13] studied mathematical modeling & behavioral analysis of a washing unit in paper mill.
Kumar *et al*. (2018) [10] paper analyzed sensitivity analysis of 3:4: good system plant. Priya *et al*. (2021) [15] Vedic mathematics in derivatives and integration, differential equations and partial differential equations. Kumari *et al*. (2021) [19] studied the constrained problems using PSO. Kumari *et al*. (2021) [17] discussed the profit analysis of an agriculture thresher plant in steady state using RPGT. Anchal *et al*. (2021) [18] discussed the SRGM model using differential equation has been proposed, in which two categories of faults: simple and hard with respect to time in which these occur for isolation and removal after their detection has been presented.

System Description
The description of the following subsystems are described as below:

- **Gdc Machine**: In the Gravity die casting machine, casting is produced with the help of gravity pressure.
- **Cutting Machine**: Cutting machine is used to cut the riser.
- **Heat Treatment Machine**: In heat treatment machine hardness of casting is improved. In heat treatment machine there are two types of hot chamber in which wheel is hold for 10 hours.

i. Solutionizing Chamber- In this chamber wheel is hold for 6 hours and then wheel is cooled quickely with the help of water.
ii. Azing Chamber- After solutiuonizing wheel is forward to azing chamebr for 4 hours.

- **Shotblasting Machine**: In this machine small steel shots are bumbared on casting to make it rough. Steel balls are bumbarded with the help of high air pressure.

Transition Diagram
The transition diagram is used to express the behaviour of the all the subsystems.

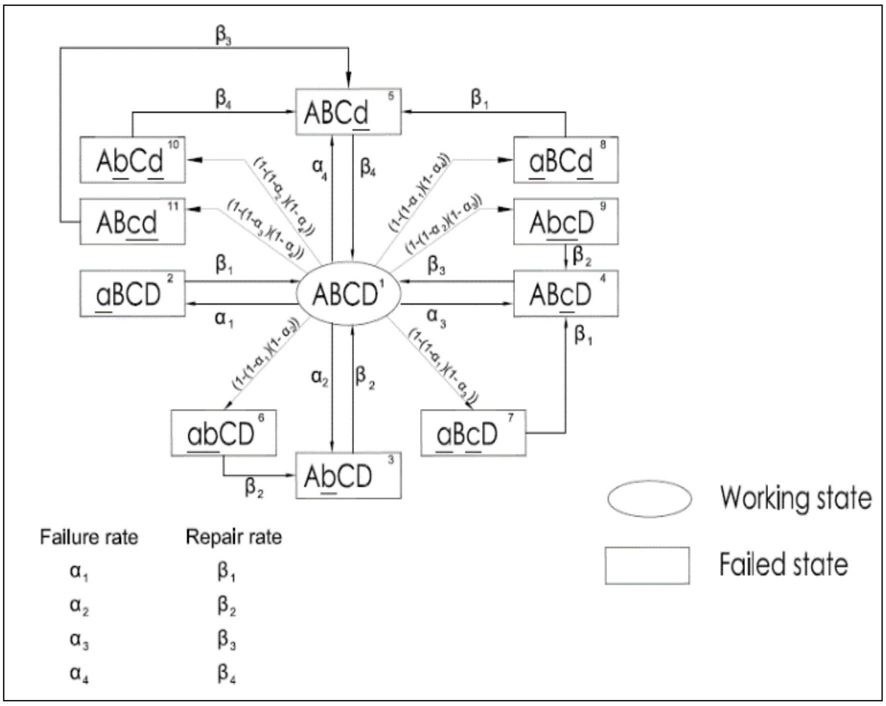

Fig 1: Transition diagram of the states

Symbols and Notations:

- A, B, C, and D Indicate that subsystem are in working state
- A, B, C, and D Indicate that subsystem is in failed state
- α_i, indicate failure rate of the subsystems where i=1,2,3,4.
- β_i, indicate repair rate of the subsystem where i=1,2,3,4.
- $p'_I(t)$, Indicate the differential of probability function $p_I(t)$.
- $p_I(t)$, probability of the system is in i^{th} state at time

Differential equation of transition diagram

$P_1'(t)+ (\alpha_1+\alpha_2+\alpha_3+\alpha_4 + (1-(1-\alpha_1)(1-\alpha_2)) + (1-(1-\alpha_1)(1-\alpha_3)) + (1-(1-\alpha_1)(1-\alpha_4)) + (1-(1-\alpha_2)(1-\alpha_3)) + (1-(1-\alpha_2)(1-\alpha_4)) + (1-(1-\alpha_3)(1-\alpha_4))P_1(t) = \beta_1 P_2(t) + \beta_2 P_3(t) + \beta_3 P_4(t) + \beta_4 P_5(t)$.

After simplification this equation can be written as:

$p_1'(t) + (4(\alpha_1+\alpha_2+\alpha_3+\alpha_4) - (\alpha_1\alpha_2+\alpha_1\alpha_3+\alpha_1\alpha_4+\alpha_2\alpha_3+\alpha_2\alpha_4+\alpha_3\alpha_4)) p_1(t) = \beta_1 p_2(t) + \beta_2 p_3(t) + \beta_3 p_4(t) + \beta_4 p_5(t)$

$p_2'(t) + \beta_1 p_2(t) = \alpha_1 p_1(t)$

$p_3'(t) + \beta_2 p_3(t) = \alpha_2 p_1(t) + \beta_2 p_6(t)$

$p_4'(t) + \beta_3 p_4(t) = \alpha_3 p_1(t) + \beta_1 p_7(t) + \beta_2 p_9(t)$

$p_5'(t) + \beta_4 p_5(t) = \alpha_4 p_1(t) + \beta_1 p_8(t) + \beta_4 p_{10}(t) + \beta_3 p_{11}(t)$

$p_6'(t) + \beta_2 p_6(t) = (1-(1-\alpha_1)(1-\alpha_2)) p_1(t)$

$p_7'(t) + \beta_1 p_7(t) = (1-(1-\alpha_1)(1-\alpha_3)) p_1(t)$

$p_8'(t) + \beta_1 p_8(t) = (1-(1-\alpha_1)(1-\alpha_4)) p_1(t)$

$p_9'(t) + \beta_2 p_9(t) = (1-(1-\alpha_2)(1-\alpha_3)) p_1(t)$

$p_{10}'(t) + \beta_4 p_{10}(t) = (1-(1-\alpha_2)(1-\alpha_4)) p_1(t)$

$p_{11}'(t) + \beta_3 p_{11}(t) = (1-(1-\alpha_3)(1-\alpha_4)) p_1(t)$

After solving these equations, we get the following equations

$p_2(t) = z_0 p_1(t)$; Where $z_0 = \alpha_1/\beta_1$

$p_3(t) = z_1 p_1(t)$; Where $z_1 = (2\alpha_2+\alpha_1-\alpha_2\alpha_1 /\beta_2)$

$p_4(t) = z_2 p_1(t)$; Where $z_2 = (3\alpha_3+\alpha_1+\alpha_2-\alpha_3\alpha_1-\alpha_3\alpha_2 / \beta_3)$

$p_5(t) = z_3 p_1(t)$; Where $z_3 = (4\alpha_4+\alpha_1+\alpha_2+\alpha_3-\alpha_4\alpha_1-\alpha_4\alpha_2-\alpha_4\alpha_3/\beta_4)$

$p_6(t) = z_4 p_1(t)$; Where $z_4 = (\alpha_2+\alpha_1-\alpha_1\alpha_2/\beta_2)$

$p_7(t) = z_5 p_1(t)$; Where $z_5 = (\alpha_3+\alpha_1-\alpha_3\alpha_1/\beta_1)$

$p_8(t) = z_6 p_1(t)$; Where $z_6 = (\alpha_4+\alpha_1-\alpha_4\alpha_1/\beta_1)$

$p_9(t) = z_7 p_1(t)$; Where $z_7 = (\alpha_3+\alpha_2-\alpha_3\alpha_2/\beta_2)$
$p_{10}(t) = z_8 p_1(t)$; Where $z_8 = (\alpha_4+\alpha_2-\alpha_4\alpha_2/\beta_4)$
$p_{11}(t) = z_9 p_1(t)$; Where $z_9 = (\alpha_4+\alpha_3-\alpha_4\alpha_3/\beta_3)$

The Probability $p_i(t)$ is determined by using normalizing condition

$$\sum_{i=0}^{11} p_i = 1$$

$$p_1 + p_2 + p_3 + p_4 + p_5 + p_6 + p_7 + p_8 + p_9 + p_{10} + p_{11} = 1$$

$$p_1 + z_0 p_1(t) + z_1 p_1(t) + z_2 p_1(t) + z_3 p_1(t) + z_4 p_1(t) + z_5 p_1(t) + z_6 p_1(t) + z_7 p_1(t) + z_8 p_1(t) + z_9 p_1(t) = 1$$

$$(1 + z_0 + z_1 + z_2 + z_3 + z_4 + z_5 + z_6 + z_7 + z_8 + z_9)\, p_1(t) = 1$$

$$p_1(t) * V = 1$$

$$p_1(t) = 1/V$$

Where, $V = (1 + z_0 + z_1 + z_2 + z_3 + z_4 + z_5 + z_6 + z_7 + z_8 + z_9)$
$A_v = P_1$

Performance Analysis

The performances of AWP mostly affected by failure and repair rates of each subsystem.
The transition diagram includes all possible state (Working state and failed state) that is, failure rate is α_i and repair rate is β_i. The table 1 to 4 represents the availability decision for AWP.

Table 1: Availability for different values of failure and repair rate of GDC Machine

$\alpha_1 \rightarrow \beta_1 \downarrow$	0.002	0.004	0.006	0.008	0.010	Constant value
0.02	0.269	0.264	0.260	0.198	0.194	α_2=0.001 β_2=0.01 α_3=0.003 β_3=0.03 α_4=0.005 β_4=0.05
0.04	0.298	0.269	0.269	0.199	0.198	
0.06	0.299	0.272	0.274	0.205	0.203	
0.08	0.324	0.279	0.289	.212	0.213	
0.10	0.349	0.288	0.291	0.224	0.220	

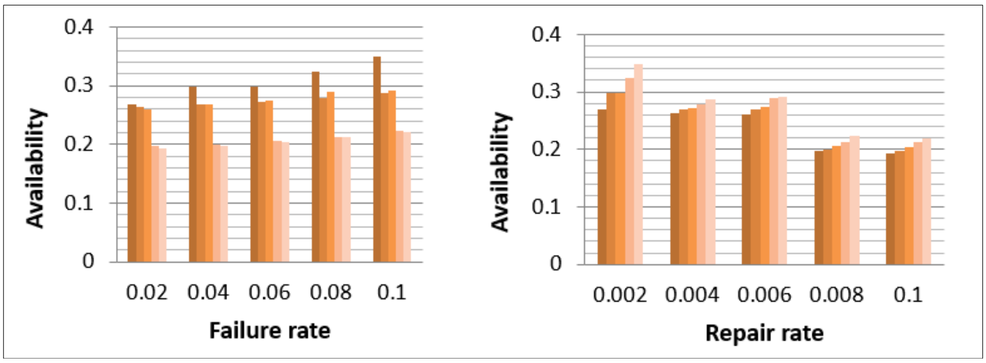

Fig 2: Variation of failure and repair rate of GDC Machine

Table 2: Availability for different value of failure and repair rate of CUTTING Machine

$\alpha_2 \rightarrow \beta_2 \downarrow$	0.001	0.002	0.003	0.004	0.005	Constant value
0.01	0.269	0.268	0.267	0.254	0.251	α_1=0.002 β_1=0.02 α_3=0.003 β_3=0.03 α_4=0.005 β_4=0.05
0.02	0.272	0.270	0.269	0.256	0.253	
0.03	0.281	0.276	0.274	0.265	0.256	
0.04	0.284	0.277	0.275	0.269	0.257	
0.05	0.289	0.278	0.278	0.270	0.268	

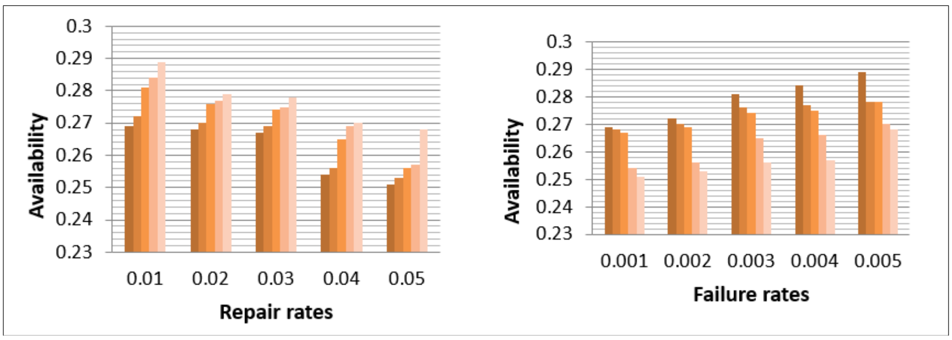

Fig 2: Variation of failure and repair rate of Cutting Machine

Table 3: Availability for different value of failure and repair rate of HEAT TREATMENT Machine

$\alpha_3 \rightarrow \beta_3 \downarrow$	0.003	0.005	0.007	0.009	0.011	Constant value
0.03	0.269	0.267	0.263	0.260	0.258	α_1=0.002 β_1=0.02 α_2=0.001 β_2=0.01 α_4=0.005 β_4=0.05
0.05	0.275	0.273	0.270	0.268	0.264	
0.07	0.279	0.278	0.275	0.273	0.270	
0.09	0.281	0.279	0.277	0.274	0.269	
0.11	0.284	0.283	0.280	0.277	0.275	

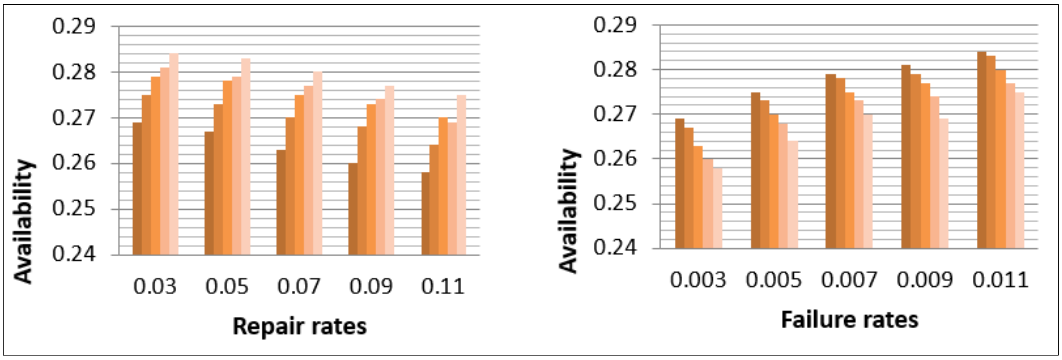

Fig 3: Variation of failure and repair rate of Heat Treatment Machine

Table 4: Value of availability for different value of failure and repair rate of SHOTBLASTING Machine

$\alpha_4 \rightarrow \beta_4 \downarrow$	0.005	0.007	0.009	0.011	0.013	Constant value
0.05	0.269	0.266	0.260	0.257	0.253	α_1=0.002
0.07	0.276	0.272	0.270	0.268	0.266	β_1=0.02
0.09	0.281	0.277	0.275	0.272	0.270	α_2=0.001
0.11	0.286	0.284	0.282	0.280	0.278	β_2=0.01
0.13	0.290	0.288	0.286	0.284	0.280	α_3=0.003 β_3=0.03

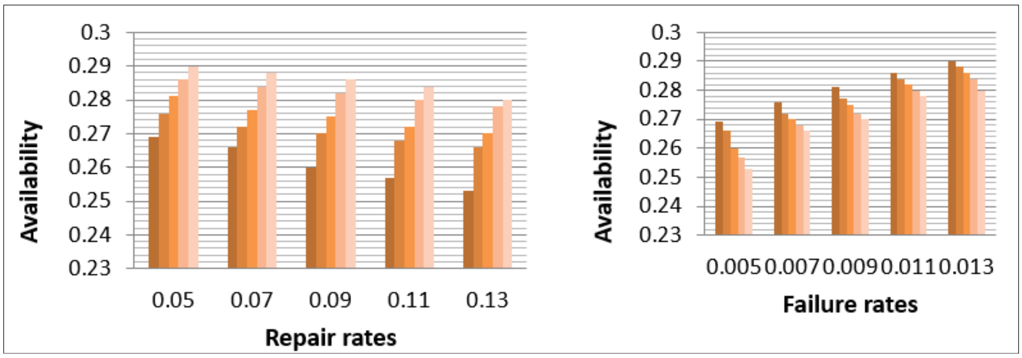

Fig 4: Variation of failure and repair rate of SHOT BLASTING Machine

Conclusion

The repair rate of GDC machine affect availability of system more effectively as comparison to another system. This modal is more realistic as compared to those models in which one failure is considered at one time. As in practical situations any number of failures can happen at one time. Further, we will extend our work to any random failures in random time without putting any assumption on number of failures. Subsystem name and their rank based on their "repair rates effect availability".

Subsystems Name	GDC Machine	Cutting Machine	Heat Treatment Machine	Shot blasting machine
Ranks	Ist	IInd	IIIrd	IVth

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