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Kelvin-Helmholtz instability in flowing dusty and partially ionized plasmas

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Abstract
We investigate the effect of different flow velocities on Kelvin-Helmholtz instability arising in partially ionized dusty plasmas. The basic equations governing the motion of partially ionized dusty plasma have been linearized to obtain the dispersion relation by applying the boundary conditions. Dispersion relation is solved numerically to discuss the growth rate of unstable mode. It is found that unstable mode doesn't appear in the absence of relative motion but our results show a stabilizing effect for lower values of relative motion. It is also found that growth time scale of unstable modes increases as the relative motion increases and the flow in region without dust particles has a dominant effect on growth rate of unstable mode.

Keywords: Dusty plasma, partially ionized plasma, instabilities

1. Introduction

The physics of dusty plasmas is a subject of great interest because of its importance for various applications in laboratory and industrial plasmas and in several situations in astrophysical plasma environments, such as planetary ring system, cometary tails and Earth's lower ionospheric regions. Dusty plasma is a three-component plasma consisting of electrons, ions and charged dust particles. The dust grains are generally silicates, magnetite, graphite and amorphous carbon. The radius of grains varies from nanometer to micrometer. The grains have charge of several orders of electrons. The charged dust particles naturally occur in their environments and play a significant role in the collective response of plasmas.

Waves and instabilities in dusty plasmas have been studied by performing laboratory experiments by Merlino *et al*. (1997) [18] and Merlino *et al*. (1998) [19]. A large number of papers have been devoted to the study of waves and instabilities in many astrophysical environments, such as Earth's aurora (Hillinan and Davis (1970)) [13], protoplanetary disks (Michikoshi and Inutsuka (2006)) [20], the magnetopause (Guo *et al*. (2010)) [12], and planetary magnetospheres (Ogilvie and Fitzenreiter (1989)) [23]. The Kelvin-Helmholtz Instability (KHI) is a macroscopic magnetohydrodynamic instability which arises at the interface between two fluids in relative motion (Chandrasekhar (1961)) [4] Drazin and Reid (1981)) [7]. Kelvin-Helmholtz Instability (KHI) has been studied in many astrophysical plasmas such as the magnetopause (Hasegawa (1975), Miura (1984)) [15, 21], Earth's aurora (Farrugia *et al*. (1994)) [9] cometary tails (Ershkovich *et al*. (1986)) [8], protoplanetary disks (Gomez and Ostriker (2005)) [10], jets and outflows (Keppens and Toth (1999), Baty and Keppens (2002), Bodo *et al*. (1995), Gomez *et al*. (2014)) [16, 1, 3, 11]. The KHI is affected fully or partially by the presence of several special conditions or fluid properties such as gravity, magnetic fields and viscosity. Complete investigation of dynamical properties at a shear layer may be useful for the presence or absence of the KHI which can in principle be used to constrain the properties of the fluid.

To understand various physical phenomena occurring in the environment of space and interplanetary medium, the Kelvin-Helmholtz instability has been extensively investigated by many authors (Hardee and Stone (1997), Downes and Ray (1998), Rosen *et al*. (1999), Birk and Wiechen (2002) and Watson *et al*. (2007)) [14, 6, 25, 2, 29]. Multi-fluid simulations of KHI in partially ionized dusty plasma showing the stabilizing effect for massive dust grains and destabilizing effect for high charge numbers have been presented by Birk and Wiechen (2002) [2]. Shadmehri and Downes (2007) [27] studied the linear theory of KH instability in a layer of ions and neutrals with finite thickness. They found that perturbations with wavelength comparable to layer's thickness are significantly affected by the thickness of the layer. Soler *et al*. (2012) [28] studied the KH instability in partially ionized compressible plasmas and found that the domain of instability depends strongly on collision frequency and density contrast. Therefore, in the absence of certain stabilizing factors such as surface tension, partially ionized incompressible plasmas are always unstable in the presence of a velocity shear. The KH instability in dusty plasma with sheared magnetic field has also been studied by Kumar *et al*. (2013) [17]. Shadmehri and Downes (2008) [22] studied the K-H instability at the interface between a partially ionized dusty outflows and the ambient material. In the present study, we aim to extend the work of Shadmehri and Downes on KH instability by considering different flow velocities U_1 and U_2 along x – direction in plasma media on both sides of the interface. Following Pandey and Vladimirov (2007) [24], and Shadmeheri and Downes (2008) [22], we briefly discuss the basic set of equations and assumptions in section 2. We obtain dispersion relation by applying the boundary conditions in section 3. In section 4, we numerically solve the dispersion relation and discuss the results.

2. Basic and Linearized Equations

We consider the multi-component plasma consisting of ions, electrons, neutrals and charged dust particles on both sides of the interface and follow the analysis by Pandey and Vladimirov (2007) [24] and Shadmeheri and Downes (2008) [22] in order to discuss the basic equations governing the motion of partially ionized dusty plasmas. The continuity and the momentum equations for the bulk fluid are

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho v) = 0, \tag{1}$$

$$\rho\left(\frac{\partial v}{\partial t} + v.\nabla v\right) = -\nabla p + \frac{J\times B}{c}. \tag{2}$$

Here ρ is the bulk fluid density, **v** is the velocity of bulk fluid, p is the pressure, **J** is displacement current, **B** is the magnetic field and c is the speed of light.

The next assumption which simplifies magnetic induction equation is that electrons and ions are assumed well coupled to the magnetic fields. The induction equation can be written as (Pandey and Vladimirov (2007)) [24]

$$\frac{\partial B}{\partial t} = \nabla \times \left[(v\times B) - \frac{1+\Theta}{Zen_d} J\times B\right], \tag{3}$$

where $\Theta = \left(1+\frac{\nu_{nd}}{\nu_{ni}}\right)\beta_d^2$ and $\beta_d = \frac{\omega_{cd}}{\nu_{dn}}$. The symbols Z_e, n_d, ν_{nd} and ν_{ni} denote the charge on dust grain, number density of dust grain, collisional frequency of neutrals with dust and collisional frequency of neutrals with ions respectively.

Equations (1) - (3) together with equation

$$\nabla.B = 0, \tag{4}$$

are our required basic equations governing the motion of partially ionized medium to study the K-H instability.

We consider small perturbations from the equilibrium as

$$v = U + v', \; B = B_0 + B',$$

$$p = p_0 + p', \; \rho = \rho_0 + \rho', \tag{5}$$

where v', B', p' and ρ' are the perturbations in velocity, magnetic field, pressure and density respectively. The equilibrium velocity U and magnetic field B_0 are taken along x-direction as $U = U\hat{x}$ and $B_0 = B_0\hat{x}$.

Using equation (5), we linearize equations (1) - (4) as

$$\frac{\partial \rho'}{\partial t} + \nabla.(\rho_0 v') = 0, \tag{6}$$

$$\rho_0\left(\frac{\partial v'}{\partial t} + U.\nabla v'\right) = -\nabla p' + \frac{(\nabla\times B')\times B_0}{4\pi}, \tag{7}$$

$$\frac{\partial B'}{\partial t} = \nabla\times\left[(U\times B' + v'\times B_0) - \frac{1+\Theta}{Zen_d}\{(\nabla\times B')\times B_0\}\right], \tag{8}$$

$$\nabla.B' = 0. \tag{9}$$

We consider a simple model to discuss K-H instability by assuming incompressibility in order to simplify our analytical calculations and linearize the basic equations. We Fourier analyze the perturbations of all quantities as

$$f'(z,x,t) = \tilde{f}(z)\exp\left[i(\omega t + k_x x + k_y y)\right]. \tag{10}$$

Using equation (10) in linearized equations and dropping tilde on perturbed physical quantities in the following analysis, we obtain

$$ik_xv_x + ik_yv_y + \frac{dv_z}{dz} = 0, \quad (11)$$

$$\rho_0\Omega v_x = -k_xp, \quad (12)$$

$$\rho_0\Omega v_y = -k_yp + \frac{B_0}{4\pi}(k_xB_y - k_yB_x), \quad (13)$$

$$\rho_0\Omega v_z = i\frac{dp}{dz} - \frac{iB_0}{4\pi}\left(ik_xB_z - \frac{dB_x}{dz}\right), \quad (14)$$

$$\Omega\, B_x = k_xB_0v_x + \eta k_xB_0\left(\frac{dB_y}{dz} - ik_yB_z\right), \quad (15)$$

$$\Omega\, B_y = k_xB_0v_y - \eta k_xB_0\left(\frac{dB_x}{dz} - ik_xB_z\right), \quad (16)$$

$$\Omega\, B_z = k_xB_0v_z - \eta k_xB_0(k_xB_y - k_yB_x), \quad (17)$$

$$ik_xB_x + ik_yB_y + \frac{dB_z}{dz} = 0, \quad (18)$$

where,

$$\Omega = \omega + k_xU,\ \eta = \frac{c}{4\pi}\frac{1+\ominus}{Zen_d} = \frac{1}{4\pi}\frac{B_0}{\rho_0\,\omega_{mcd}},$$

$\omega_{mcd} = \left(\frac{\rho_{d_0}}{\rho_0}\right)\left(\frac{1}{1+\ominus}\right)\left(\frac{ZeB_0}{m_dc}\right)$ is the modified dust cyclotron frequency. Now if we multiply equation (16) by k_x and equation (17) by k_y and then add the resulting equations, we obtain the following equation from (15) as

$$i\Omega\frac{dv_z}{dz} = -\frac{k^2p}{\rho_0} + \frac{k_yB_0}{4\pi\rho_0}(k_xB_y - k_yB_x), \quad (19)$$

where, $k^2 = k_x^2 + k_y^2$. If we substitute p from equation (19) into equation (14), we have

$$\Omega(D - k^2)v_z = \frac{k_xB_0}{4\pi}(D - k^2)B_z, \quad (20)$$

where, $D \equiv \frac{d^2}{dz^2}$. From equations (12) and (13), we have

$$k_xv_y - k_yv_x = \frac{k_xB_0}{4\pi\rho_0\omega}(k_xB_y - k_yB_x). \quad (21)$$

Using equations (15) and (16), we obtain

$$k_xB_y - k_yB_x = \frac{k_xB_0}{4\pi\rho_0\omega}(k_xv_y - k_yv_x) - \frac{i\eta k_xB_0}{\Omega}(D - k^2)B_z. \quad (22)$$

From equations (21) and (22), we obtain

$$k_xB_y - k_yB_x = -i\frac{\left(\frac{\eta k_xB_0}{\Omega}\right)}{1-\left(\frac{k_x^2{B_0}^2}{4\pi\Omega^2}\right)}(D - k^2)B_z. \quad (23)$$

Substituting this equation into equation (21), we have

$$B_z = \frac{k_xB}{\Omega}v_z - \frac{\left(\frac{\eta^2k_x^2{B_0}^2}{\emptyset^2}\right)}{1-\left(\frac{k_x^2{B_0}^2}{4\pi\Omega^2}\right)}(D - k^2)B_z. \quad (24)$$

Equations (20) and (24) are the main equations and can be combined to form a single differential equation for v_z as

$$(D - q^2)(D - k^2)v_z = 0, \quad (25)$$

Where,

$$q^2 = k^2 - \frac{1}{4\pi\rho\eta^2}\left(\frac{V_Ak_x}{\Omega}\right)^2\left[\left(\frac{\Omega}{V_Ak_x}\right)^2 - 1\right]^2, \quad (26)$$

and $V_A = \frac{B_0}{\sqrt{4\pi\rho_0}}$ is the Alfven speed.

3. Dispersion Relation

We now consider two plasma media $z < 0$ and $z > 0$ across the interface z = 0 in a plasma and assume that the streaming of the plasma takes place in both regions $z < 0$ and $z > 0$ along x-direction as

$$U = \begin{cases} U_1 \text{ for } z < 0 \\ U_2 \text{ for } z > 0 \end{cases}$$

Where, U_1 and U_2 are constant. The magnetic field B_0 acting along x −direction is assumed as

$$B_0 = \begin{cases} B_{01} \text{ for } z < 0 \\ B_{02} \text{ for } z > 0 \end{cases}$$

Where, B_{01} and B_{02} are constant. All other equilibrium physical quantities are assumed constant in each medium.

The general solution of the equation (25) can be written as a linear combination of $exp(\pm kz)$ and $exp(\pm qz)$. Thus the solution to the equation (25), continuous at $z = 0$ and decaying as $|z| \to \infty$ in both media $z < 0$ and $z > 0$ is as follows:

$$v_z = \begin{cases} C_1 e^{kz} + C_2 e^{q_1 z}, \text{ for } z < 0 \\ C_3 e^{-kz} + C_4 e^{-q_2 z}, \text{ for } z > 0 \end{cases}$$

Where, C_1, C_2, C_3 and C_4 are the arbitrary constants. The parameters q_1 and q_2 given by equation (26) are assumed to have a positive real part so as to render the perturbations bounded at infinity. From equation (26), we have

$$q_1^2 = k^2 \left\{1 - \frac{\omega_{mcd}^2}{k^2 U_1^2} \left[\frac{M_1^2 (x_1+1)^2 - 1}{x_1+1}\right]^2\right\}, \tag{27}$$

$$q_2^2 = k^2 \left\{1 - \frac{\omega_{mcd}^2}{k^2 U_2^2} \left[\frac{M_2^2 (x_2+1)^2 - 1}{x_2+1}\right]^2\right\}, \tag{28}$$

Where, $M_1 = \frac{U_1}{V_{A_1}}, M_2 = \frac{U_2}{V_{A_2}}$ and $x_1 = \frac{\omega}{k_x U_1}$, $x_2 = \frac{\omega}{k_x U_2}$

There are four boundary conditions to be satisfied at the interface $z = 0$.These are the continuity of normal and tangential components of the magnetic field, the vertical displacement and the total pressure across the interface (Chandrasekhar (1961) [4], Roy Choudhury and Lovelace (1986) [26]; Chhajlani & Vyas (1991) [5]). Applying these boundary conditions at the interface $z = 0$, we obtain

$$U_2(x_2+1)(C_1 + C_2) = U_1(x_1+1)(C_3 + C_4), \tag{29}$$

$$U_2(x_2+1)[1 - M_1^2(x_1+1)^2]C_2 = U_1(x_1+1)[1 - M_2^2(x_2+1)^2]C_4, \tag{30}$$

$$U_1(x_1+1)(kC_1 + q_1 C_2) = -(kC_3 + q_2 C_4)U_2(x_2+1), \tag{31}$$

$$U_2(x_2+1)[kC_1 + q_1 M_1^2(x_1+1)^2 C_2] = U_1(x_1+1)[-kC_3 - q_2 M_2^2(x_2+1)^2 C_4]. \tag{32}$$

The system of equations (29) - (32) has a non-trivial solution when the determinant of coefficients is zero. So, after some algebra, we obtain the following dispersion relation

$$k(\alpha^2 - 1)(\beta - 1) = 2[\alpha^2\beta q_1 + q_2] - (1 + \alpha^2)[\beta q_1 M_1^2(x_1+1)^2 + q_2 M_2^2(x_2+1)^2], \tag{33}$$

Where, $\alpha = \frac{U_1(x_1+1)}{U_2(x_2+1)}$ and $\beta = \frac{1 - M_2^2(x_2+1)^2}{1 - M_1^2(x_1+1)^2}$.

Since it is extremely difficult to write the dispersion relation (33) in polynomial from so we aim to solve it by Maple 18 software to get all its roots in complex form. We consider only those roots of equation (33) which have negative imaginary part and provide us the positive values of the real parts of q_1 and q_2 for studying the Kelvin-Helmholtz instability in flowing dusty and partially ionized plasmas.

4. Results and Discussion

If we consider the initial flow U_1 and U_2 in regions $z < 0$ *and* $z > 0$ as $U_1 = U$, and $U_2 = -U$, and magnetic field $B_{01} = B_{02}$ then our dispersion relation (33) reduces to the dispersion relation (38) obtained by Shadmehri and Downes (2008) [22]. If U_1 and U_2 are different, $B_{01} = B_{02}$, and $\omega_{mcd} = 0$ in dusty plasma medium $z > 0$, then the dispersion relation (33) reduces to

$$\sqrt{1-\frac{1}{f}\left(\frac{x^2-1}{x}\right)^2}-\frac{(x^2-z^2)^2}{x^2(1-z^2)(2-x^2-z^2)}-\frac{z^2-x^2}{x^2(1-z^2)}=0. \quad (34)$$

If we simplify the dispersion relation (34) to remove the square root, it becomes

$$x^4(4f+12)+6x^8+x^4z^4(4f+34)+x^6(-4f-13)+x^6z^2(8f+34)+x^4z^2(-8f-34)+10x^8z^4+2x^4z^8+x^6z^4(-4f-30)+10x^6z^6-14x^4z^6-14x^8z^2-x^{10}-x^{10}z^4-x^6z^8-2x^8z^6+2x^{10}z^2-4x^2+x^2z^4(4f-13)+12x^2z^2+x^2z^8(4f-1)+x^2z^6(-8f+6)-4fz^8-4fz^4+8fz^6=0. \quad (35)$$

Where, $x=y+M_1$, $z=y+M_2$, $y=\frac{\omega}{k_x V_A}$ and $f=\left(\frac{\omega_{mcd}}{kV_A}\right)^{-2}$.

The dispersion relation (35) is solved numerically to compute growth rate in an astrophysical situation akin to the situation having molecular hydrogen density and ratio of ionized to natural mass density as 10^3 cm^3 and 10^{-6}. The bulk density of dust particles and ratio to neutral density are assumed to be $125g$ / cm^3 and 10^6.

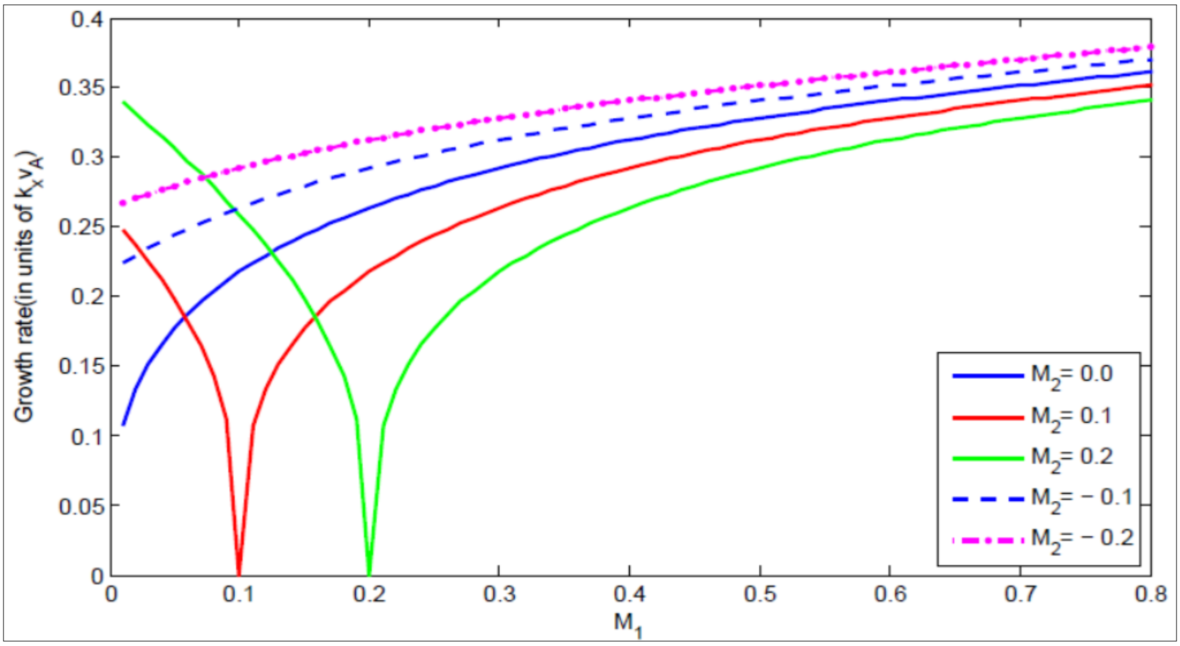

Fig 1: Variation of growth rate with Mach number M_1 in region $z < 0$ for different values of Mach number M_2 in region $z > 0$ with parameter f = 0.4.

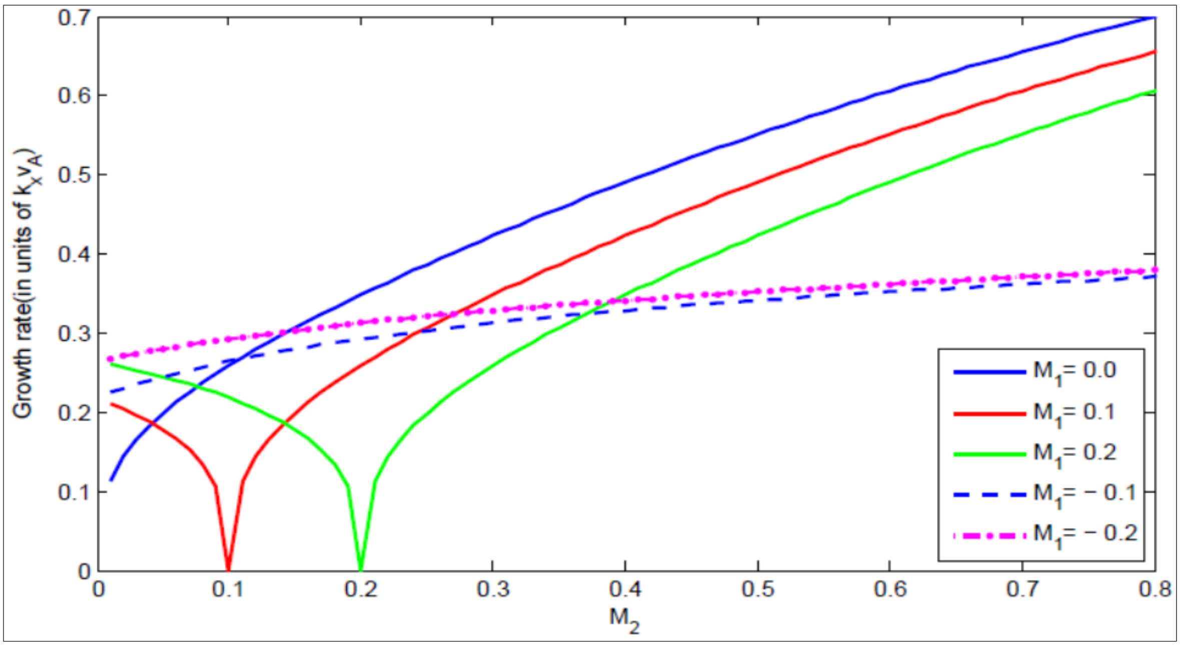

Fig 2: Variation of growth rate with Mach number M_2 in region $z > 0$ for different values of Mach number M_1 in region $z < 0$ with parameter f = 0.4.

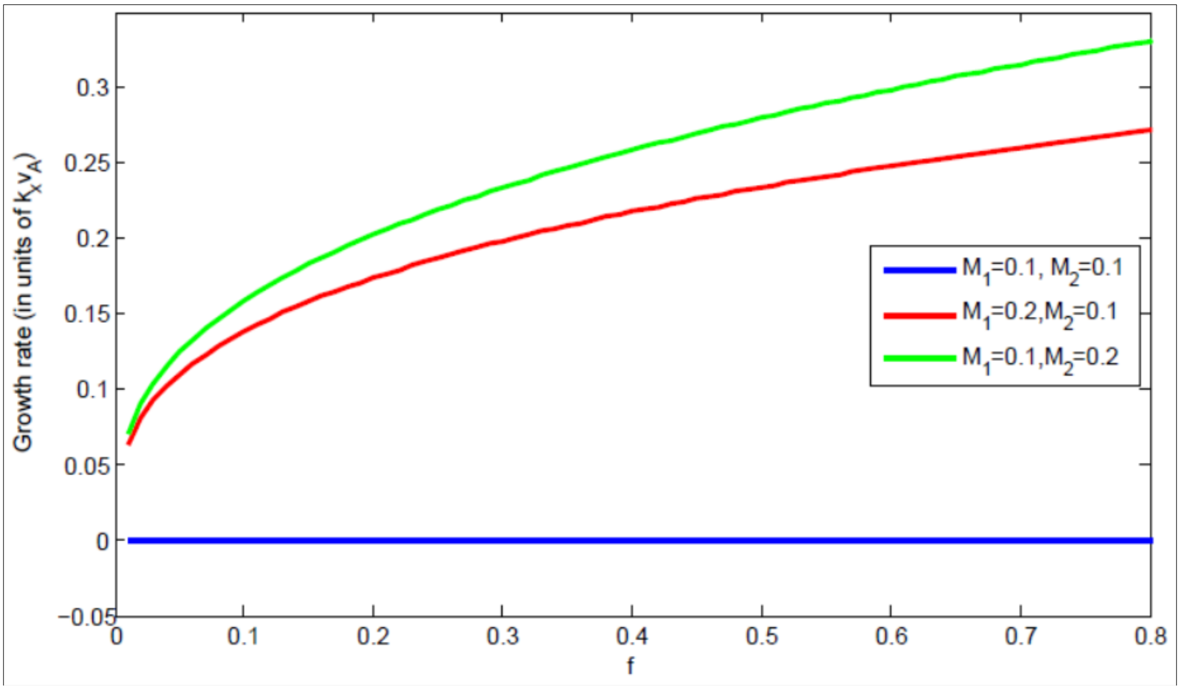

Fig 3: Variation of growth rate with parameter f for different values of Mach number M_1 and Mach number M_2.

Fig.1 shows the variation of growth rate (in units of k_xV_A) with Mach number M_1 in medium $z < 0$ for different values of Mach number M_2 in medium $z > 0$ keeping the parameter f fixed at 0.4. In the absence of flow in region $z > 0$, growth rate increases with the increase in Mach number M_1. In the presence of flow in region $z > 0$, growth rate of instability decreases till $M_2 < M_1$ and becomes zero for $M_2 = M_1$ then increases as the value of Mach number M_1 increases due to relative flow between the two media. If we consider negative values of Mach number M_2, it is observed that growth rate of unstable mode increases with the increase in Mach number M_1 as in the case of absence of flow in region $z > 0$.

Fig.2 depicts the variation of growth rate with Mach number M_2 for different values of Mach number M_1 and f = 0.4. The pattern of variation of growth rate is same as in fig.1 except instability grows slightly faster. This could be due to the absence of dust particle in the region $z > 0$. When Mach number M_1 is considered to be negative growth rate of unstable mode is smaller than that of in case of absence of flow in region $z < 0$.

Growth rate of the instability as a function of parameter f has been shown in fig.3. It can be seen that growth rate of instability is zero, when no relative motion is taking place. It can be observed that Mach number M_2 plays a leading role in the growth rate of the instability.

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