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## The Laplace transform decomposition method applied to the numerical solution of the food chain ecoepidemic model

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### Abstract

The aim of this study is to theoretically investigate the model parameters of the food chain ecoepidemic model using the Laplace transform decomposition method. The first-order autonomous differential equations governing the model were reduced via the Laplace transform and Adomian decomposition to an algebraic equation, where the inverse Laplace transform gives the pertinent parameters. The analytically obtained solution which is the partial sum of the infinite series is then treated with Pade approximation to improve its convergence. To validate the results obtained, we compare the numerical solution via LADM with other semi-analytical methods viz LADM-Pade and Daftardar-Jafari (DJM). The results obtained are presented in tables and figures. The comparison showed, there is excellent agreement between the methods. It is observed the method is valid, efficient, applicable, robust, and computationally convenient to handle both linear and nonlinear differential as well partial differential equations.

**Keywords:** Laplace decomposition method (LADM), ecoepidemic model, Pade approximation, food chain, Daftardar-Jafari method (DJM), numerical solution

### 1. Introduction

In the field of ecology and biological sciences, trophic levels containing food chains play a pivotal role. Though this area of research is relatively nascent, researchers have done extensive research on this area with a view to understanding the ever-changing behaviour of the food web, modelling and analysing the changes in the pertinent parameters of the model. Earlier studies in the food chain model were carried out by Gard *et al.* 1979<sup>[1]</sup>, Hallam and Gard 1980<sup>[2]</sup>, Shukla 1981 *et al.*<sup>[3]</sup> and Kirlinger 1988<sup>[4]</sup>. These studies were incisive and elaborate but couldn't incorporate all the intriguing factors that affects the food web and its attendant competition. The extinction of the food web was modelled by<sup>[5]</sup> using a single variable parameter. The study of these mathematical models has grown among academics the world over especially in recent times due to the importance of food in the ecosystem. This now encompasses studies of diseases affecting the predator-prey both at the top and bottom, disease affecting interacting species in each population, effect of disease in aquatic lives, disease in parasitic and symbiotic environment and roles of parasites in ecology and many more. The dynamics and nature of diseases causing organism as well as others factor which breeds competition has equally been giving devoted attention by scholars.

Most recently, Tripathi *et al.* 2004<sup>[6]</sup> examined the prey-predator model with an emphasis on a reserved area. The study reveals that, when the parameter bearing the reserved area reaches a certain threshold, Hopf bifurcation sets in. It was further shown that better stabilizing influence results when predator refuse is introduced using Beddington-De Angeles functional response. This, the study opined is useful to national parks where prey and predators co-exist together, and the prey wished to be protected<sup>[7]</sup>. The Lokta-Volterra system with pest was investigated by Venturino 1993<sup>[8]</sup>. In this study, the introduction of the pest was a check to keep predators at bay so the prey population wouldn't be utterly harmed by the predators<sup>[9-10]</sup>. independently analysed the prey-predator models including the SIS, SIR with infected prey. It was observed in this study that, when the prey population is vulnerable due to the disease

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introduced, the predators will persistently be present when indeed they are supposed to go extinction. Though most of these models are first order nonlinear autonomous differential equations, several approaches have been adopted to solve them for analytical and approximate solutions. These include variational iteration method (VIM) [12-13], Energy balance method [14-15], Homotopy Analysis method (HAM) [16-17], Homotopy perturbation method (HPM) [18], Adomian decomposition method (ADM) [19], Perturbation method [20] and others.

Similarly, the modified Laplace Adomian decomposition method (LADM) have proved to be potent and extensively applied to solve several linear and nonlinear problems in contemporary times since it accelerates the rate of convergence of the approximate solution to the exact solution [21]. investigated the SIR model that incorporate child disease with preventive vaccine strategy using LADM. The system of linear and nonlinear PDEs have been studied by [22]. The study showed when more terms are taken, the convergence of the approximate solution was improved compared to ADM. The Duffing equation, Falkner-Skan equation, systems of nonlinear differential equations, nonlinear coupled PDEs, HIV infection model of CD4+Tcells, Newell-Whitehead-Segel equation, Systems of ordinary differential equations, linear and nonlinear Volterra Integral equation, Nonlinear Volterra Integro-differential equation, nth order integro-differential equation, two-dimensional viscous fluid with shrinking sheet, convection diffusion-dissipation equation, crime deterrence model, dynamics of atmospheric carbon dioxide have all been investigated using LADM [23-33].

Equally, the Daftardar-Jafari method (DJM) presented by Daftardar-Gejji Varsha, and Hossein Jafari have also been used to solve linear and nonlinear problems including PDEs, Jeffery-Hamel flow problems, nonlinear functional equation, fractional physical differential equation and fractional gas dynamics equation with coupled Burger's equation [34-38].

In this present research, we theoretically investigate the model parameters in the Ecoepidemic model of food chain consisting of three trophic levels namely: top predators, intermediate predators and a bottom subpopulation comprising infected and susceptible preys. The study is organized as follows. In section 1, the detailed introduction of the study is given. The basics of the Laplace Adomian decomposition method (LADM) is presented in section 2. The fundamentals of the Daftardar-Jafari method (DJM) are discussed in section 3. Section 4 gives the application of the LADM to the model to ascertain the model parameters. In section 5, the results are presented in tables, figures, and their explanation. Finally, the conclusion of the study is drawn in section 6.

**2. Laplace Decomposition method (LADM)**

In this subsection, we outline the basics of the fundamentals of the fusion Laplace transformation and Adomian decomposition method (LADM)

Consider a functional differential equation of the form

$$L[u(x)] + R[u(x)] + N[u(x)] = g(x) \tag{1}$$

Subject to the initial condition

$$u(x, 0) = f(x), \frac{\partial u(x,0)}{\partial t} = h(x) \tag{2}$$

Rearranging the above, we obtain the following relation for

$$L[u(x)] = g(x) - R[u(x)] - N[u(x)] \tag{3}$$

Applying Laplace transform on both sides of Eq. (1), supposing the highest differential operator is of order two and using the differentiation property, we get

$$\begin{aligned} s^2 \mathcal{L}\{u(x)\} - sh(x) - f(x) &= \mathcal{L}\{g(x)\} - \mathcal{L}\{Ru(x)\} - \mathcal{L}\{Nu(x)\} \\ s^2 \mathcal{L}\{u(x)\} &= sh(x) + f(x) + \mathcal{L}\{g(x)\} - \mathcal{L}\{Ru(x)\} - \mathcal{L}\{Nu(x)\} \\ \mathcal{L}\{u(x)\} &= \frac{h(x)}{s} + \frac{f(x)}{s^2} + \frac{1}{s^2} \mathcal{L}\{g(x)\} - \frac{1}{s^2} \mathcal{L}\{Ru(x)\} - \frac{1}{s^2} \mathcal{L}\{Nu(x)\} \end{aligned} \tag{4}$$

Next, we apply the inverse transform on both sides of Eq. (4), we obtain

$$u(x) = \phi(x) - \mathcal{L}^{-1} \left[ \frac{1}{s^2} \mathcal{L}\{Ru(x)\} - \frac{1}{s^2} \mathcal{L}\{Nu(x)\} \right] \tag{5}$$

Where  $\phi(x)$  is the term arising from the first three terms on the right-hand side of Eq. (5)

Next, we assume the solution as decomposing series in the form

$$u(x) = \sum_{n=0}^{\infty} u_n(x) \tag{6}$$

Similarly, the nonlinear terms are written in terms of the Adomian polynomials

$$Nu(x) = \sum_{n=0}^{\infty} A_n \tag{7}$$

Where the  $A_n^s$  represents the Adomian polynomials defined in the form

$$A_n = \frac{1}{n!} \frac{d^n}{d\lambda^n} [N(\sum_{k=0}^{\infty} \lambda^k y_k)]_{\lambda=0}, n = 0, 1, 2, 3 \quad (8)$$

Plugging Eqs (6) and (7) into Eq. (5), we obtain

$$\sum_{n=0}^{\infty} u_n(x) = \phi(x) - \mathcal{L}^{-1} \left[ \frac{1}{s^2} \mathcal{L}\{R \sum_{n=0}^{\infty} u_n(x)\} - \frac{1}{s^2} \mathcal{L}\{N \sum_{n=0}^{\infty} A_n\} \right] \quad (9)$$

Matching both sides of Eq. (9), we obtain an iterative algorithm in the form

$$u_0(x) = \phi(x)$$

$$u_1(x) = -\mathcal{L}^{-1} \left[ \frac{1}{s^2} \mathcal{L}\{R \sum_{n=0}^{\infty} u_0(x)\} - \frac{1}{s^2} \mathcal{L}\{N \sum_{n=0}^{\infty} A_0\} \right]$$

$$u_2(x) = -\mathcal{L}^{-1} \left[ \frac{1}{s^2} \mathcal{L}\{R \sum_{n=0}^{\infty} u_1(x)\} - \frac{1}{s^2} \mathcal{L}\{N \sum_{n=0}^{\infty} A_1\} \right] \quad (10)$$

$$u_3(x) = -\mathcal{L}^{-1} \left[ \frac{1}{s^2} \mathcal{L} \left\{ R \sum_{n=0}^{\infty} u_2(x) \right\} - \frac{1}{s^2} \mathcal{L} \left\{ N \sum_{n=0}^{\infty} A_2 \right\} \right]$$

$$u_{n+1}(x) = -\mathcal{L}^{-1} \left[ \frac{1}{s^2} \mathcal{L}\{R \sum_{n=0}^{\infty} u_n(x)\} - \frac{1}{s^2} \mathcal{L}\{N \sum_{n=0}^{\infty} A_n\} \right]$$

Then the solution of the differential equation is obtained as the sum of decomposed series in the form

$$u(x) \approx u_0(x) + u_1(x) + u_2(x) + \dots \quad (11)$$

### 3. Padé Approximation

In Mathematics and other applied sciences, power series representation of a function is usually in truncated form. To approximate these functions to an appreciable degree, polynomials are used because their singularities are easily noticeable in each finite region. However, the radius of convergence may not be large enough to contain two boundaries, for this reason, power series is not always the best method to approximate a function. To overcome this inherent hurdle, a new approximation is applied to the solution obtained using power series as a quotient of two functions with varying degrees in a finite interval.

Padé approximation has been widely used to approximate several problems and has tremendous applications especially in computer calculation because it gives a better approximation without truncating its power series and still in problems where the series diverges. The different Padé approximants are obtained with the use of symbolic software Mathematica. [See 39-42]

A rational approximation to a function  $f(x)$  on  $[a, b]$  is the quotient of two polynomials,  $P_N(x)$  and  $Q_M(x)$  of degrees  $N$  and  $M$  respectively. It is denoted by

$$[N/M](x)$$

$$\text{That is, } [N/M](x) = \frac{P_N(x)}{Q_M(x)}, a \leq x \leq b \quad (12)$$

Now consider the formal power series

$$f(x) = \sum_{k=0}^{\infty} c_k x^k \quad (13)$$

$$f(x) = \frac{P_N(x)}{Q_M(x)} + O(x^{N+M+1})$$

Rearranging gives

$$f(x) - \frac{P_N(x)}{Q_M(x)} = O(x^{N+M+1}) \quad (14)$$

Multiply both sides of Eq. (14) by a constant keep it unchanged, hence we impose the normalization condition.

$$Q_M(0) = 1.0 \quad (15)$$

Next, we require that  $P_N(x)$  and  $Q_M(x)$  have non-common factors, so we write the coefficients of the  $P_N(x)$  and  $Q_M(x)$  as follows



$$\frac{dV}{dt} = -lV + eSV - hVW + qIV \frac{dI}{dt} = \beta IS - nIV - \gamma I - vI \tag{21}$$

$$\frac{dS}{dt} = aS \left(1 - \frac{S+I}{K}\right) - cVS - \beta SI + \gamma I$$

Subject to the initial condition

$$W(0) \geq 0, V(0) \geq 0, I(0) \geq 0, S(0) \geq 0 \tag{22}$$

Where the parameters  $W, V, I$  and  $S$  have their usual meanings as follows

$W$  = Top predator

$V$  =Intermediate population

$I$  =Infected subpopulation of the bottom prey population

$S$  =Susceptible subpopulation of the bottom prey

Taking the Laplace transform of the system in Eq. (21), we get

$$\mathcal{L}\left\{\frac{dW}{dt}\right\} = \mathcal{L}\{-mW\} + \mathcal{L}\{pVW\}$$

$$\mathcal{L}\left\{\frac{dV}{dt}\right\} = \mathcal{L}\{-lV\} + \mathcal{L}\{eSV\} - \mathcal{L}\{hVW\} + \mathcal{L}\{qIV\} \mathcal{L}\left\{\frac{dI}{dt}\right\} = \mathcal{L}\{\beta IS\} - \mathcal{L}\{nIV\} - \mathcal{L}\{\gamma I\} - \mathcal{L}\{vI\} \tag{23}$$

$$\mathcal{L}\left\{\frac{dS}{dt}\right\} = \mathcal{L}\left[ aS \left(1 - \frac{S+I}{K}\right) \right] - \mathcal{L}\{cVS\} - \mathcal{L}\{\beta SI\} + \mathcal{L}\{\gamma I\}$$

$$\mathcal{L}\left\{\frac{dW}{dt}\right\} = -m\mathcal{L}\{W\} + p\mathcal{L}\{VW\}$$

$$\mathcal{L}\left\{\frac{dV}{dt}\right\} = -l\mathcal{L}\{V\} + e\mathcal{L}\{SV\} - h\mathcal{L}\{VW\} + q\mathcal{L}\{IV\}$$

$$\mathcal{L}\left\{\frac{dI}{dt}\right\} = \beta\mathcal{L}\{IS\} - n\mathcal{L}\{IV\} - \gamma\mathcal{L}\{I\} - v\mathcal{L}\{I\} \tag{24}$$

$$\mathcal{L}\left\{\frac{dS}{dt}\right\} = \frac{a}{s}\mathcal{L}\{S\} - \frac{a}{Ks}\mathcal{L}\{S^2\} - \frac{a}{Ks}\mathcal{L}\{SI\} - c\mathcal{L}\{VS\} - \beta\mathcal{L}\{SI\} + \gamma\mathcal{L}\{I\}$$

Applying the Laplace transform formulae for the derivative, we obtain

$$s\mathcal{L}\{W\} - W(0) = -m\mathcal{L}\{W\} + p\mathcal{L}\{VW\}$$

$$s\mathcal{L}\{V\} - V(0) = -l\mathcal{L}\{V\} + e\mathcal{L}\{SV\} - h\mathcal{L}\{VW\} + q\mathcal{L}\{IV\}$$

$$s\mathcal{L}\{I\} - I(0) = \beta\mathcal{L}\{IS\} - n\mathcal{L}\{IV\} - \gamma\mathcal{L}\{I\} - v\mathcal{L}\{I\} \tag{25}$$

$$s\mathcal{L}\{S\} - S(0) = \frac{a}{s}\mathcal{L}\{S\} - \frac{a}{Ks}\mathcal{L}\{S^2\} - \frac{a}{Ks}\mathcal{L}\{SI\} - c\mathcal{L}\{VS\} - \beta\mathcal{L}\{SI\} + \gamma\mathcal{L}\{I\}$$

Using the initial condition to the above system and rearranging, we have the equivalent expression of the form

$$\mathcal{L}\{W\} = \frac{1}{s} - \frac{m}{s}\mathcal{L}\{W\} + \frac{p}{s}\mathcal{L}\{A\}$$

$$\mathcal{L}\{V\} = \frac{1}{s} - \frac{l}{s}\mathcal{L}\{V\} + \frac{e}{s}\mathcal{L}\{B\} - \frac{h}{s}\mathcal{L}\{A\} + \frac{q}{s}\mathcal{L}\{C\}$$

$$\mathcal{L}\{I\} = \frac{1}{s} + \frac{\beta}{s}\mathcal{L}\{D\} - \frac{n}{s}\mathcal{L}\{C\} - \left(\frac{\gamma}{s} + \frac{v}{s}\right)\mathcal{L}\{I\} \tag{26}$$

$$\mathcal{L}\{S\} = \frac{1}{s} + \frac{a}{s^2}\mathcal{L}\{S\} - \frac{a}{Ks^2}\mathcal{L}\{E\} - \left(\frac{a}{Ks^2} + \frac{\beta}{s}\right)\mathcal{L}\{D\} - \frac{c}{s}\mathcal{L}\{B\} + \frac{\gamma}{s}\mathcal{L}\{I\}$$

$$\text{Where } A = VW, B = SV, C = IV, D = IS, E = S^2 \tag{27}$$

By the LADM, we represent the linear terms as an infinite series of the form

$$W = \sum_{n=0}^{\infty} W_n, V = \sum_{n=0}^{\infty} V_n, I = \sum_{n=0}^{\infty} I_n, S = \sum_{n=0}^{\infty} S_n \tag{28}$$

Similarly, the nonlinear terms as Adomian polynomials of the form

$$A = \sum_{n=0}^{\infty} A_n, B = \sum_{n=0}^{\infty} B_n, C = \sum_{n=0}^{\infty} C_n, D = \sum_{n=0}^{\infty} D_n, E = \sum_{n=0}^{\infty} E_n \tag{29}$$

Where  $A_n, B_n, C_n, D_n$  and  $E_n$  are the so-called Adomian polynomials expressed equivalently as follows

$$\begin{aligned} A_0 &= V_0 W_0 \\ A_1 &= V_0 W_1 + V_1 W_0 \\ A_2 &= V_0 W_2 + V_1 W_1 + V_2 W_0 \\ A_3 &= V_0 W_3 + V_1 W_2 + V_2 W_1 + V_3 W_0 \\ A_4 &= V_0 W_4 + V_1 W_3 + V_2 W_2 + V_3 W_1 + V_4 W_0 \\ A_5 &= V_0 W_5 + V_1 W_4 + V_2 W_3 + V_3 W_2 + V_4 W_1 + V_5 W_0 \end{aligned} \tag{30}$$

$$\begin{aligned} B_0 &= S_0 V_0 \\ B_1 &= S_0 V_1 + S_1 V_0 \\ B_2 &= S_0 V_2 + S_1 V_1 + S_2 V_0 \\ B_3 &= S_0 V_3 + S_1 V_2 + S_2 V_1 + S_3 V_0 \\ B_4 &= S_0 V_4 + S_1 V_3 + S_2 V_2 + S_3 V_1 + S_4 V_0 \\ B_5 &= S_0 V_5 + S_1 V_4 + S_2 V_3 + S_3 V_2 + S_4 V_1 + S_5 V_0 \end{aligned} \tag{31}$$

$$\begin{aligned} C_0 &= I_0 V_0 \\ C_1 &= I_0 V_1 + I_1 V_0 \\ C_2 &= I_0 V_2 + I_1 V_1 + I_2 V_0 \\ C_3 &= I_0 V_3 + I_1 V_2 + I_2 V_1 + I_3 V_0 \\ C_4 &= I_0 V_4 + I_1 V_3 + I_2 V_2 + I_3 V_1 + I_4 V_0 \\ C_5 &= I_0 V_5 + I_1 V_4 + I_2 V_3 + I_3 V_2 + I_4 V_1 + I_5 V_0 \end{aligned} \tag{32}$$

$$\begin{aligned} D_0 &= I_0 S_0 \\ D_1 &= I_0 S_1 + I_1 S_0 \\ D_2 &= I_0 S_2 + I_1 S_1 + I_2 S_0 \\ D_3 &= I_0 S_3 + I_1 S_2 + I_2 S_1 + I_3 S_0 \\ D_4 &= I_0 S_4 + I_1 S_3 + I_2 S_2 + I_3 S_1 + I_4 S_0 \\ D_5 &= I_0 S_5 + I_1 S_4 + I_2 S_3 + I_3 S_2 + I_4 S_1 + I_5 S_0 \end{aligned} \tag{33}$$

$$\begin{aligned} E_0 &= S_0^2 \\ E_1 &= S_0^2 \\ E_2 &= S_1^2 \\ E_3 &= S_2^2 \\ E_4 &= S_3^2 \\ E_5 &= S_4^2 \end{aligned} \tag{34}$$

Plugging Eqs. (28) and (29) into Eq. (26), we obtain

$$\begin{aligned} \mathcal{L}\left\{\sum_{n=0}^{\infty} W_n\right\} &= \frac{1}{s} - \frac{m}{s} \mathcal{L}\left\{\sum_{n=0}^{\infty} W_n\right\} + \frac{p}{s} \mathcal{L}\left\{\sum_{n=0}^{\infty} A_n\right\} \\ \mathcal{L}\{\sum_{n=0}^{\infty} V_n\} &= \frac{1}{s} - \frac{l}{s} \mathcal{L}\{\sum_{n=0}^{\infty} V_n\} + \frac{e}{s} \mathcal{L}\{\sum_{n=0}^{\infty} B_n\} - \frac{h}{s} \mathcal{L}\{\sum_{n=0}^{\infty} A_n\} + \frac{q}{s} \mathcal{L}\{\sum_{n=0}^{\infty} C_n\} \\ \mathcal{L}\{\sum_{n=0}^{\infty} I_n\} &= \frac{1}{s} + \frac{\beta}{s} \mathcal{L}\{\sum_{n=0}^{\infty} D_n\} - \frac{n}{s} \mathcal{L}\{\sum_{n=0}^{\infty} C_n\} - \left(\frac{\gamma}{s} + \frac{v}{s}\right) \mathcal{L}\{\sum_{n=0}^{\infty} I_n\} \\ \mathcal{L}\{\sum_{n=0}^{\infty} S_n\} &= \frac{1}{s} + \frac{a}{s^2} \mathcal{L}\{\sum_{n=0}^{\infty} S_n\} - \frac{a}{Ks^2} \mathcal{L}\{\sum_{n=0}^{\infty} E_n\} - \left(\frac{a}{Ks^2} + \frac{\beta}{s}\right) \mathcal{L}\{\sum_{n=0}^{\infty} D_n\} - \frac{c}{s} \mathcal{L}\{\sum_{n=0}^{\infty} B_n\} + \frac{\gamma}{s} \mathcal{L}\{\sum_{n=0}^{\infty} I_n\} \end{aligned} \tag{35}$$

Matching both sides of Eq. (35) we have the following iterative algorithm for the control parameters of the model as follows

$$\begin{aligned} \mathcal{L}\{W_0\} &= \frac{1}{s} \\ \mathcal{L}\{W_1\} &= -\frac{m}{s} \mathcal{L}\{W_0\} + \frac{p}{s} \mathcal{L}\{A_0\} \\ \mathcal{L}\{W_2\} &= -\frac{m}{s} \mathcal{L}\{W_1\} + \frac{p}{s} \mathcal{L}\{A_1\} \end{aligned}$$

$$\mathcal{L}\{W_3\} = -\frac{m}{s}\mathcal{L}\{W_2\} + \frac{p}{s}\mathcal{L}\{A_2\} \tag{36}$$

$$\mathcal{L}\{W_{n+1}\} = -\frac{m}{s}\mathcal{L}\{W_n\} + \frac{p}{s}\mathcal{L}\{A_n\}$$

$$\mathcal{L}\{V_0\} = \frac{1}{s}$$

$$\mathcal{L}\{V_1\} = -\frac{l}{s}\mathcal{L}\{V_0\} + \frac{e}{s}\mathcal{L}\{B_0\} - \frac{h}{s}\mathcal{L}\{A_0\} + \frac{q}{s}\mathcal{L}\{C_0\}$$

$$\mathcal{L}\{V_2\} = -\frac{l}{s}\mathcal{L}\{V_1\} + \frac{e}{s}\mathcal{L}\{B_1\} - \frac{h}{s}\mathcal{L}\{A_1\} + \frac{q}{s}\mathcal{L}\{C_1\}$$

$$\mathcal{L}\{V_3\} = -\frac{l}{s}\mathcal{L}\{V_2\} + \frac{e}{s}\mathcal{L}\{B_2\} - \frac{h}{s}\mathcal{L}\{A_2\} + \frac{q}{s}\mathcal{L}\{C_2\} \tag{37}$$

$$\mathcal{L}\{V_{n+1}\} = -\frac{l}{s}\mathcal{L}\{V_n\} + \frac{e}{s}\mathcal{L}\{B_n\} - \frac{h}{s}\mathcal{L}\{A_n\} + \frac{q}{s}\mathcal{L}\{C_n\}$$

$$\mathcal{L}\{I_0\} = \frac{1}{s}$$

$$\mathcal{L}\{I_1\} = \frac{\beta}{s}\mathcal{L}\{D_0\} - \frac{n}{s}\mathcal{L}\{C_0\} - \left(\frac{\gamma}{s} + \frac{v}{s}\right)\mathcal{L}\{I_0\}$$

$$\mathcal{L}\{I_2\} = \frac{\beta}{s}\mathcal{L}\{D_1\} - \frac{n}{s}\mathcal{L}\{C_1\} - \left(\frac{\gamma}{s} + \frac{v}{s}\right)\mathcal{L}\{I_1\}$$

$$\mathcal{L}\{I_3\} = \frac{\beta}{s}\mathcal{L}\{D_2\} - \frac{n}{s}\mathcal{L}\{C_2\} - \left(\frac{\gamma}{s} + \frac{v}{s}\right)\mathcal{L}\{I_2\} \tag{38}$$

$$\mathcal{L}\{I_{n+1}\} = \frac{\beta}{s}\mathcal{L}\{D_n\} - \frac{n}{s}\mathcal{L}\{C_n\} - \left(\frac{\gamma}{s} + \frac{v}{s}\right)\mathcal{L}\{I_n\}$$

$$\mathcal{L}\{S_0\} = \frac{1}{s}$$

$$\mathcal{L}\{S_1\} = \frac{a}{s^2}\mathcal{L}\{S_0\} - \frac{a}{Ks^2}\mathcal{L}\{E_0\} - \left(\frac{a}{Ks^2} + \frac{\beta}{s}\right)\mathcal{L}\{D_0\} - \frac{c}{s}\mathcal{L}\{B_0\} + \frac{\gamma}{s}\mathcal{L}\{I_0\}$$

$$\mathcal{L}\{S_2\} = \frac{a}{s^2}\mathcal{L}\{S_1\} - \frac{a}{Ks^2}\mathcal{L}\{E_1\} - \left(\frac{a}{Ks^2} + \frac{\beta}{s}\right)\mathcal{L}\{D_1\} - \frac{c}{s}\mathcal{L}\{B_1\} + \frac{\gamma}{s}\mathcal{L}\{I_1\}$$

$$\mathcal{L}\{S_3\} = \frac{a}{s^2}\mathcal{L}\{S_2\} - \frac{a}{Ks^2}\mathcal{L}\{E_2\} - \left(\frac{a}{Ks^2} + \frac{\beta}{s}\right)\mathcal{L}\{D_2\} - \frac{c}{s}\mathcal{L}\{B_2\} + \frac{\gamma}{s}\mathcal{L}\{I_2\} \tag{39}$$

$$\mathcal{L}\{S_{n+1}\} = \frac{a}{s^2}\mathcal{L}\{S_n\} - \frac{a}{Ks^2}\mathcal{L}\{E_n\} - \left(\frac{a}{Ks^2} + \frac{\beta}{s}\right)\mathcal{L}\{D_n\} - \frac{c}{s}\mathcal{L}\{B_n\} + \frac{\gamma}{s}\mathcal{L}\{I_n\}$$

Applying the inverse Laplace transform of both sides of Eq. (38)-(39), we obtain the following iteratives for the parameters as follows

$$\mathcal{L}\{W_0\} = \frac{1}{s}$$

$$\mathcal{L}\{V_0\} = \frac{1}{s}$$

$$\mathcal{L}\{I_0\} = \frac{1}{s} \tag{40}$$

$$\mathcal{L}\{S_0\} = \frac{1}{s}$$

The next iterate of the problem become

$$\begin{aligned} \mathcal{L}\{W_1\} &= -\frac{m}{s^2} + \frac{p}{s^3} \\ \mathcal{L}\{V_1\} &= -\frac{l}{s^2} + \frac{1}{s^3}(e - h + q) \\ \mathcal{L}\{I_0\} &= \frac{1}{s^3}(\beta - n) - \frac{1}{s^2}(\gamma + v) \\ \mathcal{L}\{S_0\} &= \frac{\gamma}{s^2} + \frac{1}{s^3}(a - c) - \frac{1}{s^4}\left(\frac{a}{K} + a + K\beta s\right) \end{aligned} \tag{41}$$

Evaluating the Laplace transform of Eqs. (40) and (41) and taking the inverse Laplace transform, we obtain the first and approximate solutions of the model

$$W_0(t), V_0(t), I_0(t), S_0(t) \text{ and } W_1(t), V_1(t), I_1(t), S_1(t)$$

Similarly, the succeeding iterative solution of the parameters of interest,

$$W_2(t), W_3(t), \dots, W_n(t), V_2(t), V_3(t), \dots, V_n(t), I_2(t), I_3(t), \dots, I_n(t) \text{ and}$$

$$S_2(t), S_3(t), \dots, S_n(t) \text{ are obtained recursively in a similar manner.}$$

### 5. Numerical Simulation

Now to obtain the solutions of the control parameters of the model in explicit form, we apply the LADM and take the following values via simulation.  $W(0) = 1, V(0) = 1, I(0) = 1, S(0) = 1$  for the four parameters of interest. Next, we take

$$a = 3, K = 5, m = 1, p = 0.2, l = 3, e = 1, h = 0.5, q = 1, \beta = 2, n = 0.5, c = 1, \gamma = 10 \text{ and } v = 0$$

Using the above, we obtain the first few approximations to the parameters given as

$$\begin{aligned} \mathcal{L}\{W_0\} &= \frac{1}{s} \\ \mathcal{L}\{V_0\} &= \frac{1}{s} \\ \mathcal{L}\{I_0\} &= \frac{1}{s} \\ \mathcal{L}\{S_0\} &= \frac{1}{s} \\ \mathcal{L}\{W_1\} &= -\frac{1}{s^2} + \frac{0.2}{s^3} \\ \mathcal{L}\{V_1\} &= -\frac{3}{s^2} + \frac{1.5}{s^3} \\ \mathcal{L}\{I_1\} &= \frac{1.5}{s^3} - \frac{10}{s^2} \\ \mathcal{L}\{S_1\} &= \frac{10}{s^2} + \frac{2}{s^3} - \frac{10s + 3.6}{s^4} \end{aligned} \tag{42}$$

Taking the inverse Laplace transform of both sides of Eqs. (42) and (43), we obtain the partial sum of the parameters of interest as follows.

$$\begin{aligned} W(t) &= 1 - t + 0.1t^2 \\ V(t) &= 1 - 3t + 0.75t^2 \quad I(t) = 1 - 10t + 0.75t^2 \\ S(t) &= 1 + 10t - 4t^2 - 0.6t^3 \end{aligned} \tag{44}$$

Next, to match the obtained solution as far as possible, we calculate the [4/4] Pade approximant as an infinite series solution of the model parameters in the form.

$$W_{Pade}(t) = \frac{1 - 0.592234493t - 0.1919t^2 - 0.07501815263499813t^3 + 0.011579479t^4}{1 + 0.4077655t + 0.1157947t^2 - 6.2852429 \times 10^{-17}t^3 + 5.08749 \times 10^{-17}t^4}$$

$$V_{Pade}(t) = \frac{1 - 2.465125694725606t - 0.676625t^2 - 0.132835t^3 + 0.1334914134t^4}{1 + 0.53487t + 0.17799t^2 - 1.00099 \times 10^{-16}t^3 + 7.27842796422 \times 10^{-17}t^4}$$

$$I_{Pade}(t) = \frac{1 - 9.825540t - 0.972136t^2 - 0.0937050t^3 + 0.01684120098788147t^4}{1 + 0.174459t + 0.02245493t^2 + 9.5768 \times 10^{-17}t^3 + 8.641975565 \times 10^{-17}t^4} \tag{45}$$

$$S_{Pade}(t) = \frac{1 + 10.2331861108t - 1.668131125t^2 - 1.5327475498t^3 - 0.1399121324835628t^4}{1 + 0.2331868t - 1.2310707 \times 10^{-14}t^2 - 6.264653 \times 10^{-16}t^3 - 4.025035122201 \times 10^{-15}t^4}$$

**6. Results and Discussion**

In this section, we present the results in Tables and Graphs and their interpretation. Comparison is made between the result obtained using ADM, LADM-Pade and Daftardar-Jafari method (DJM).

**Table 1:** Numerical Computation for Top predators W(t)

t	LADM	LADM-PADE	Daftardar-Jafari (DJM)
0	1.00000	1.00000	1.00000
0.2	0.80400	0.80400	0.80400
0.4	0.61600	0.61920	0.61600
0.6	0.43600	0.44320	0.43600
0.8	0.26400	0.27680	0.26400
1.0	0.10000	0.12000	0.10000
1.2	-0.00560	-0.02720	-0.05600

**Table 2:** Numerical Computation for Intermediate predator V(t)

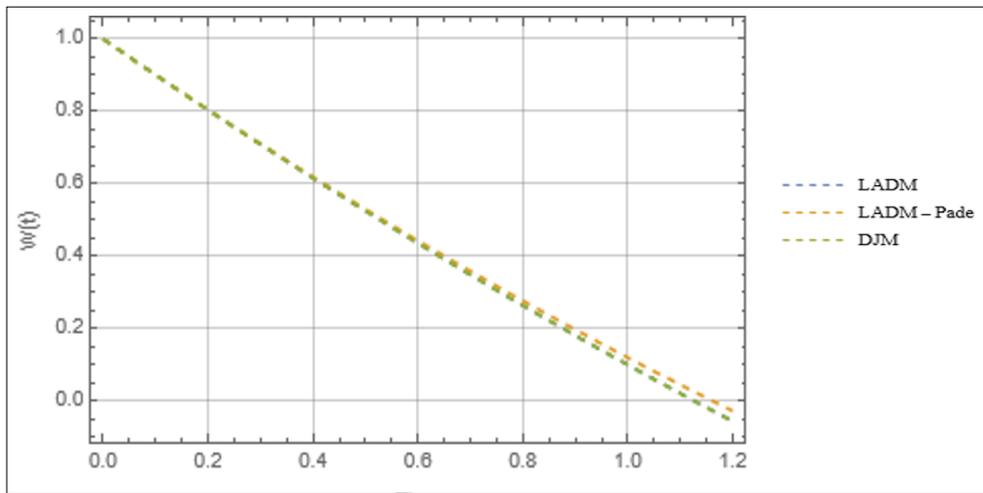
t	LADM	LADM-PADE	Daftardar-Jafari (DJM)
0	1.00000	1.00000	1.00000
0.2	1.55500	1.55600	1.55500
0.4	2.02400	2.02400	2.02000
0.6	2.40400	2.40400	2.39500
0.8	2.69600	2.69600	2.68000
1.0	2.90000	2.90000	2.87500
1.2	3.01600	3.01600	2.98000

**Table 3:** Numerical Computation for Infected prey in subpopulation I(t)

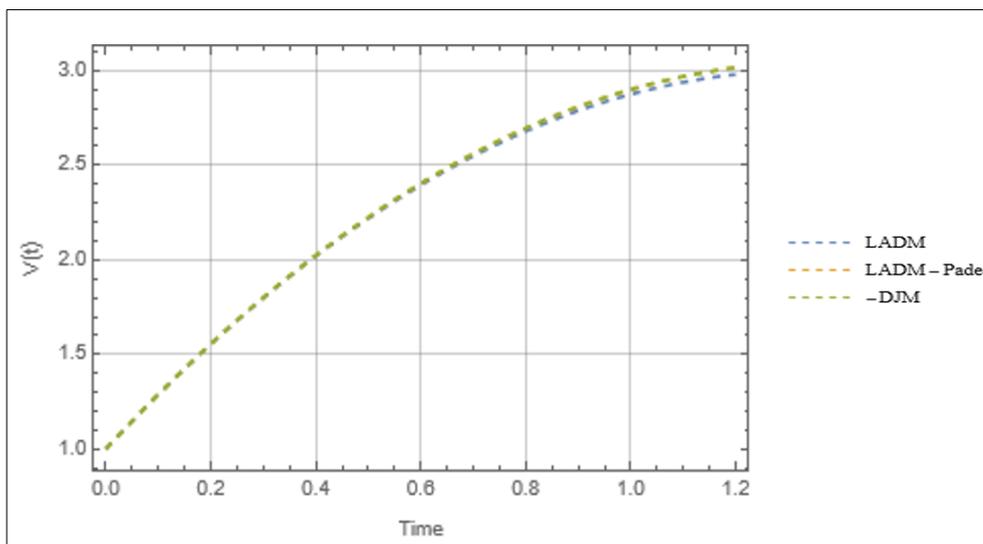
t	LADM	LADM-PADE	Daftardar-Jafari (DJM)
0	1.00000	1.00000	1.00000
0.2	2.10000	2.00000	2.10000
0.4	1.60000	1.56000	1.60000
0.6	-0.50000	-0.51000	-0.54000
0.8	-4.20000	-4.16000	-4.13200
1.0	-9.50000	-9.51000	-9.51100
1.2	-16.40000	-16.35000	-16.32000

**Table 4:** Numerical Computation for Susceptible prey in subpopulation S(t)

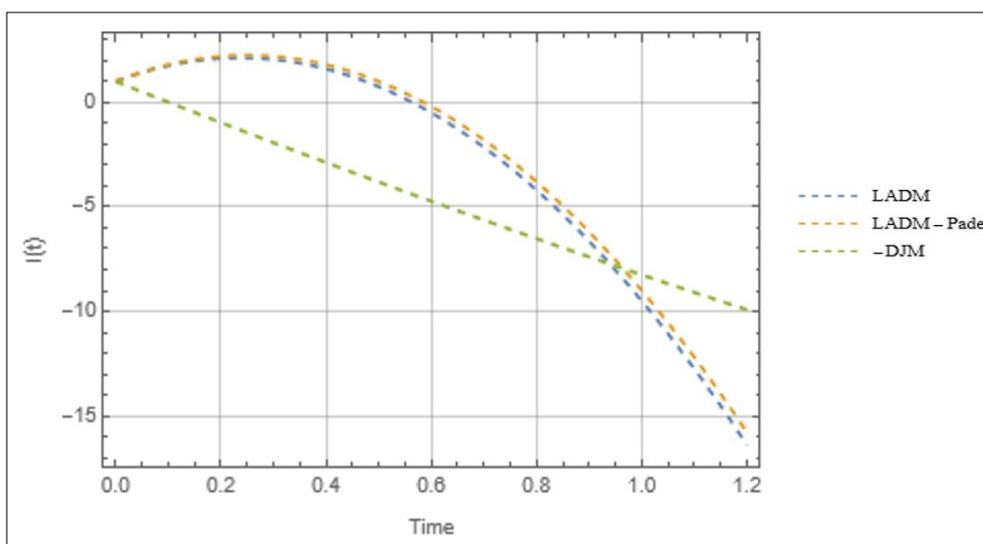
t	LADM	LADM-PADE	Daftardar-Jafari (DJM)
0	0.00000	0.00000	0.00000
0.2	-1.84480	-1.83500	-1.84480
0.4	-3.47840	-3.47790	-3.47830
0.6	-4.92960	-4.92950	-4.92950
0.8	-6.22720	-6.22710	-6.22730
1.0	-7.40000	-7.41000	-7.41200
1.2	-8.47680	-8.47670	-8.47680



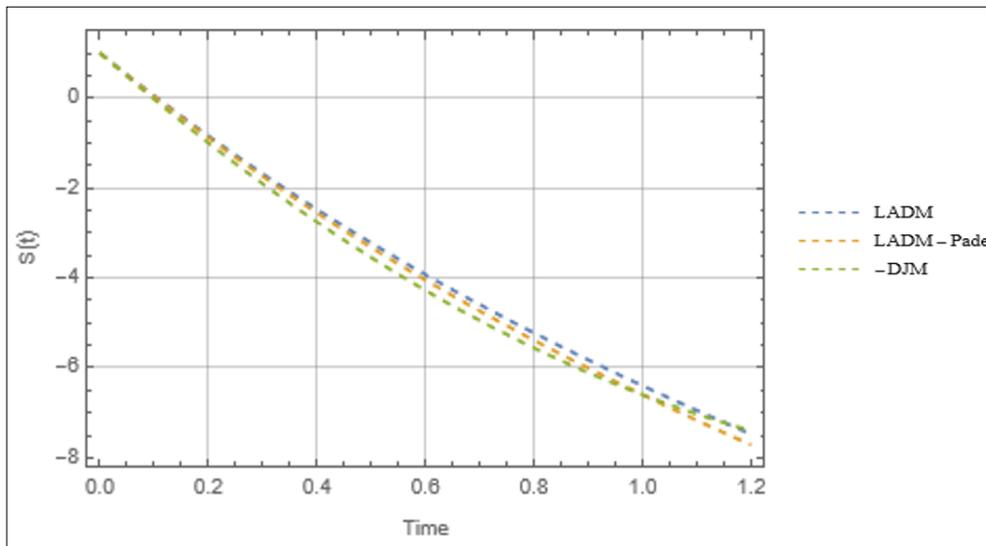
**Fig 1:** Comparison of LADM, LADM-Pade and R-K for Top predator at fixed Time



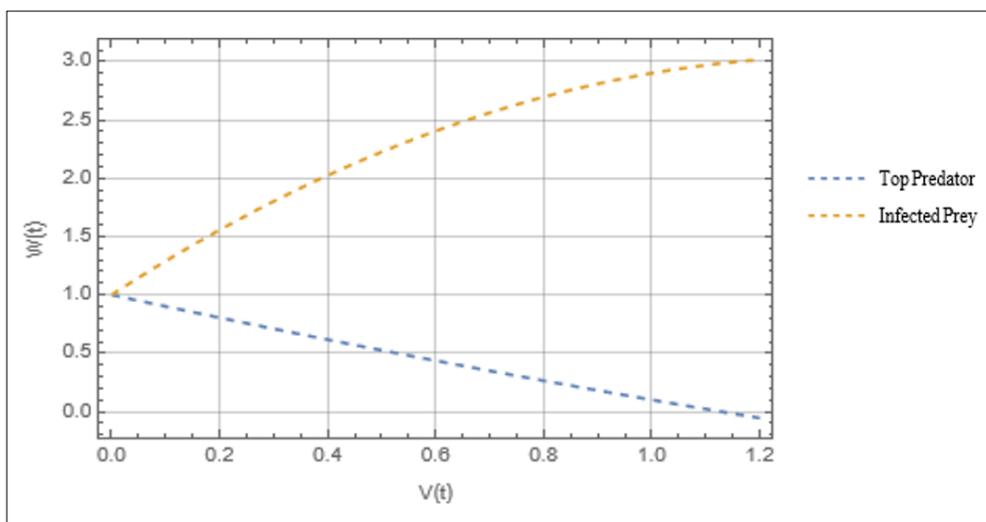
**Fig 2:** Comparison of LADM, LADM-Pade and R-K for Intermediate Prey



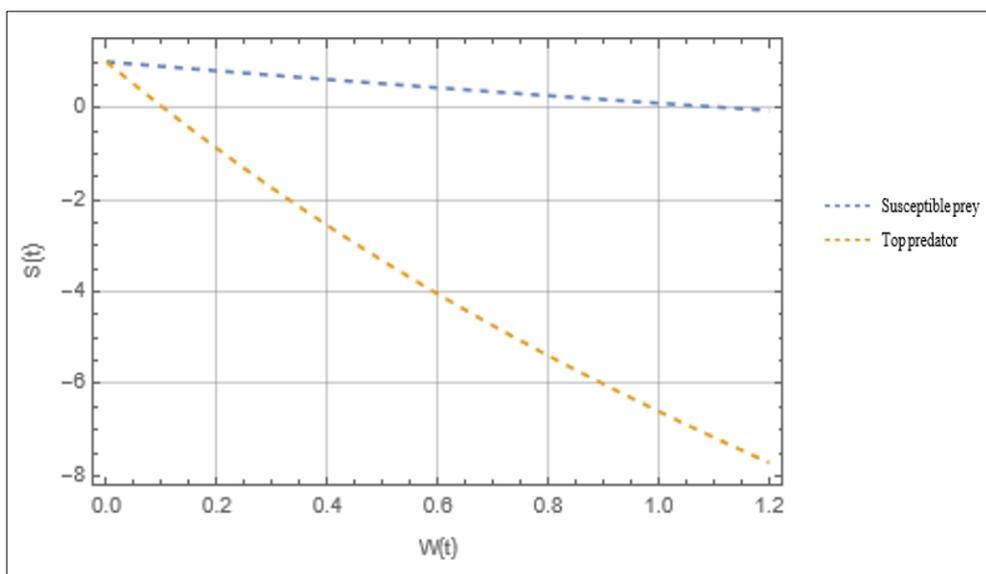
**Fig 3:** Comparison of LADM, LADM-Pade and R-K for Infected Prey Population



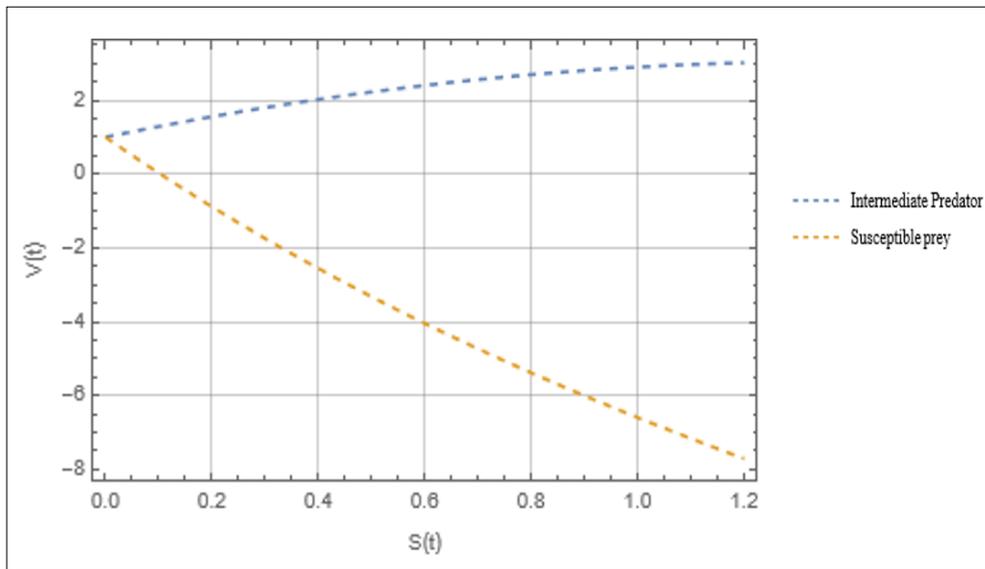
**Fig 4:** Comparison of LADM, LADM-Pade and DJM for Susceptible Prey Population



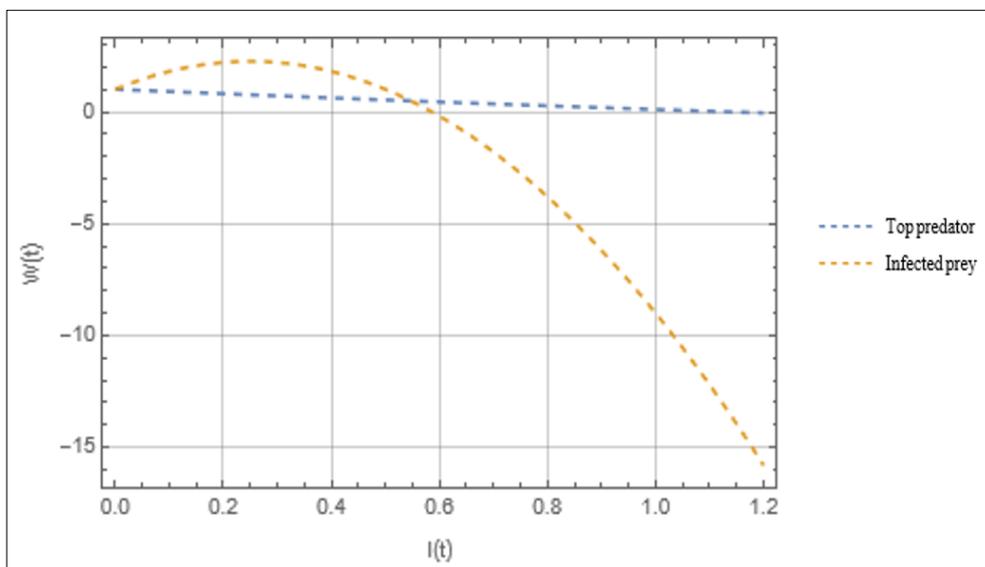
**Fig 5:** Variation of Top Predator and Intermediate Predator at fixed Time



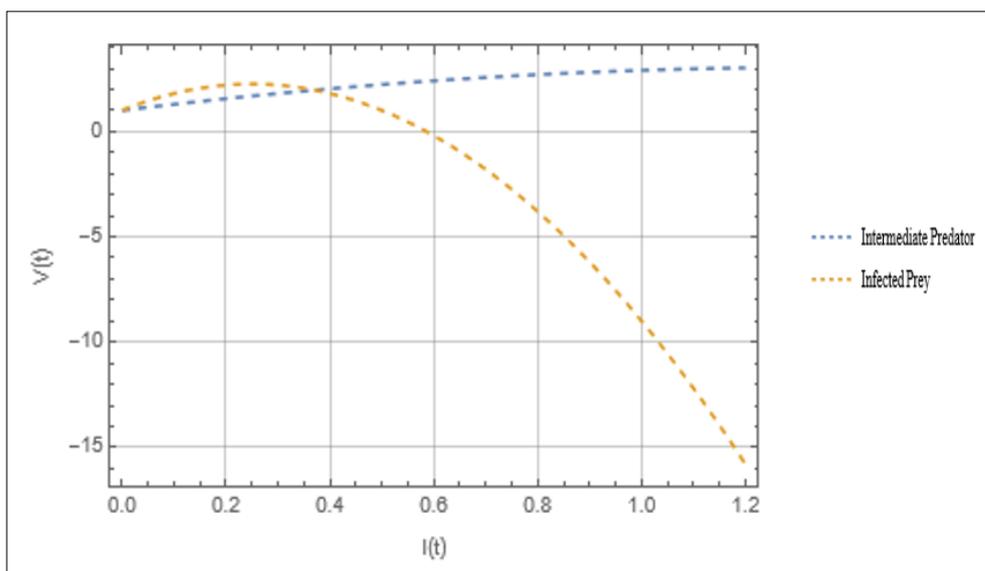
**Fig 6:** Comparison of Susceptible Prey and Top Predator for constant Time



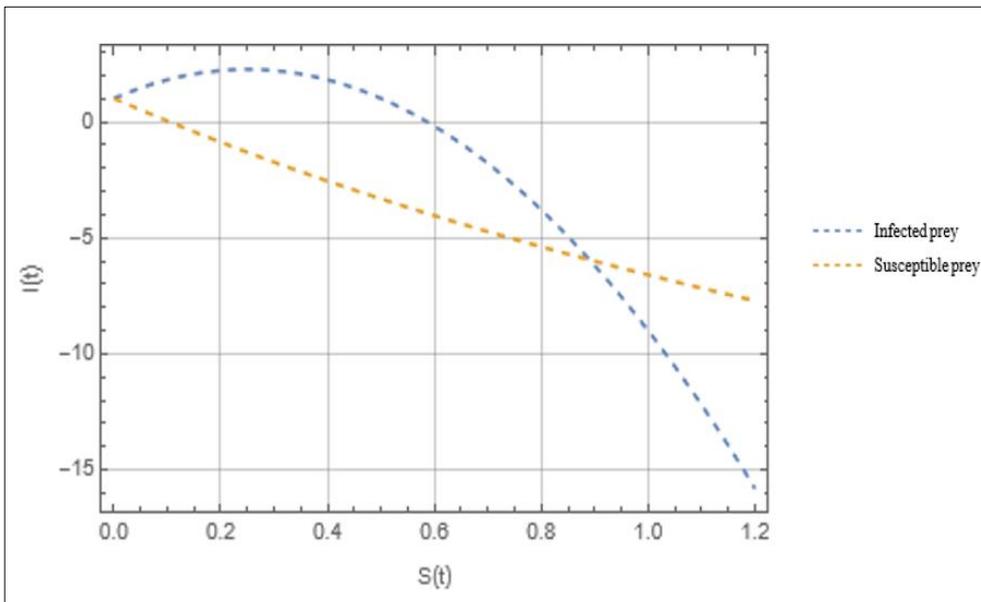
**Fig 7:** Comparison of Intermediate Predator and Susceptible Prey at fixed Time



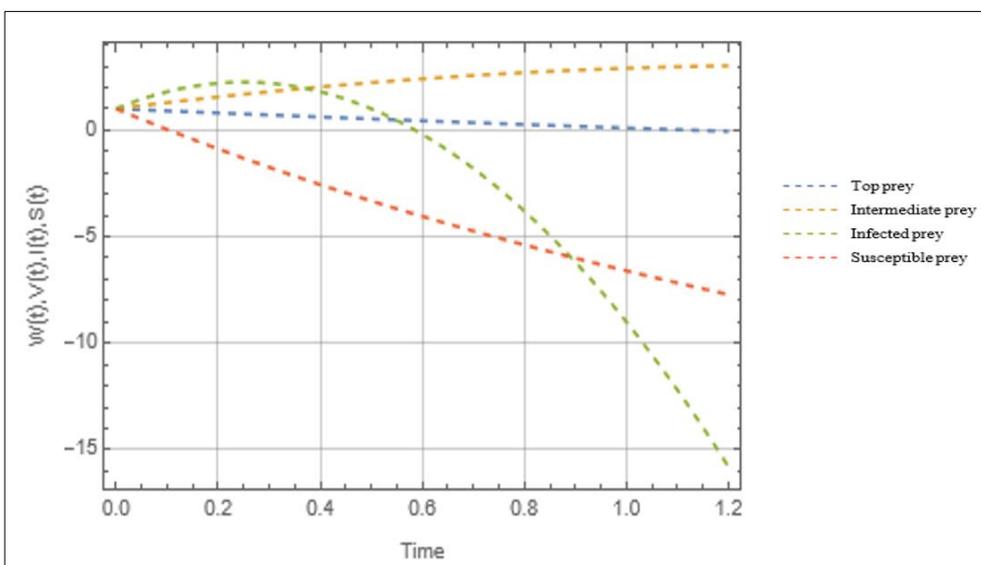
**Fig 8:** Comparison of Top predator and Infected Prey at fixed Time



**Fig 9:** Comparison of Intermediate predator and Infected prey for Fixed Time



**Fig 10:** Comparison of Infected Prey and Susceptible Prey at fixed Time



**Fig 11:** Comparison of Top predator, Intermediate predator, Infected prey and Susceptible prey at fixed Time

**7. Concluding Remarks**

In this present work, we investigate the model parameters of the food chain ecoepidemic model with three trophic levels consisting of top predators, intermediate predators and bottom predators comprising susceptible and infected subpopulation using the Laplace decomposition method. The solutions were obtained and matched with an improved LADM-Pade approximation to increase its convergence. In the final analysis, we compare the results with fourth order Runge-Kutta method. The result shows, the method is robust, applicable, correct, valid, and efficient with less computational work.

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