International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
Maths 2022; 7(1): 112-117

www.mathsjournal.com
Received: 19-09-2021
Accepted: 09-11-2021

Dr. S Geetha
Assistant Professor, Department of Statistics, PSG College of Arts and Science, Coimbatore, Tamil Nadu, India

S Saranya
Research Scholar, Department of Statistics, PSG College of Arts and Science, Coimbatore, Tamil Nadu, India

Corresponding Author:
Dr. S Geetha
Assistant Professor, Department of Statistics, PSG College of Arts and Science, Coimbatore, Tamil Nadu, India

Construction of multiple dependent state sampling plan for variables based on logistic distribution

Dr. S Geetha and S Saranya

DOI: https://doi.org/10.22271/maths.2022.v7.i1b.780

Abstract
Acceptance sampling plans are specific rules used in statistical quality control to reach a decision on acceptance or rejection of lots of manufactured products submitted for inspection subject to the quality requirements of the sampled units. In order to reduce the inspection time and cost, the variable sampling plans are used as it needs comparatively small sample size. The multiple dependent state sampling plan is one of the conditional sampling plans which acquires the sample information from present lot as well as the prior lots for disposition of a lot. The MDS plan is applicable in the situations where lots are submitted for inspection in sequence of production from a process having a constant proportion non-conforming. MDS plan under the assumption of normality are found in literature. In this article, a multiple dependent state sampling plan is developed for sentencing lots of products whose quality characteristics follows a Logistic distribution. The proposed plan is designed by considering the two points on the OC curve, by formulating it as a non-linear optimization problem, when standard deviation is known and unknown. The tables are constructed for the selection of the proposed plan parameters. The proposed plan is compared with the single variable sampling plan based on Logistic distribution.

Keywords: Multiple dependent sampling plan, logistic distribution, operating characteristic curve, acceptable quality level, limiting quality level

Introduction

The concept of multiple dependent (or deferred) state sampling plan for attribute, developed by Wortham and Baker (1976) [19], is one of the conditional sampling procedure. In this plan, the decision on accepting or rejecting of a lot is based on the sample information from the current lot together with consecutive preceding lots (in the case of dependent state sampling) or upcoming lots (in the case of deferred state sampling). Vaerst (1982) [18] further modified the operating procedure and characteristic function of multiple dependent (or deferred) state sampling plan for attributes by Wortham and Baker to make it on a par with chain sampling plan (ChSP-1). The Multiple Dependent State Sampling plan is applicable in the case of Type B situations (i.e., sampling from a continuous process) where lots are submitted for inspection serially in the order of production. Govindaraju and Subramani (1990) [8] have developed a table and procedure for finding the multiple deferred state sampling plan of the type MDS-1 involving a minimum sum of risks for specified acceptable quality level and limiting quality level. Soundararajan and Vijayaraghavan (1990) [16] have designed multiple deferred state MDS-1 (0, 2) sampling plan indexed by acceptable quality level and limiting quality level. Balamurali and Jun (2007) [2] developed MDS Sampling plan for variables, when the quality characteristics is normally distributed. The decision upon the acceptance of the lot is based on the states of the preceding lots or on the states of the forthcoming lots. Aijun Yan, Sanyang Liu and Xiaojuan Dong (2016) [1] developed a multiple dependent state (MDS) sampling plan based on the coefficient of variation when the quality characteristic follows a normal distribution with unknown mean and variance. Muhammad Aslam, Balamurali and Chi-Hyuck Jun (2019) [12] proposed a new multiple dependent state sampling plan which comprises the features of existing multiple dependent state sampling plan and repetitive group sampling plan based on the process capability index where the quality characteristic under investigation follows the normal distribution with unknown mean and unknown variance.

Generally, sampling plans are designed by considering two points on the OC curve, namely, $(p_1, 1-\alpha)$ and (p_2, β), where p_1 and p_2 are termed as acceptable quality level (AQL) and limiting quality level (LQL), associated with producer's risk α and consumer's risk β, respectively. A well designed sampling plan must provide at least $(1-\alpha)$ probability of acceptance of a lot when the process fraction non-conforming is at AQL and not more than β probability of acceptance if the process fraction non-conforming is at the LQL. Thus the proposed variables MDS sampling plan will also be designed so as to pass through two points on the OC curve namely, $(AQL, 1-\alpha)$ and (LQL, β). In this paper, multiple dependent state sampling plan for variables is proposed when quality characteristics follow logistic distribution.

2. Logistic Distribution

Let X be a random variable distributed according to a Logistic distribution with two parameters a (location) and b (scale). The probability density function is given by

$$\mathrm{f(x{:}\,a,b)} = \frac{\mathrm{e}^{-(\mathrm{x}-\mathrm{a})/\mathrm{b}}}{b(1+\mathrm{e}^{-(\mathrm{x}-\mathrm{a})/\mathrm{b}})^2} \tag{1}$$

Where a is the mean and b is a scale parameter proportional to the standard deviation. Thus, the mean and variance are respectively given by a and $b^2\pi^2/3$. Taking transformation as $a = 0$ and $b = 1$, logistic distribution becomes standard logistic distribution with pdf and cdf respectively given by

$$\mathrm{f(x)} = \frac{\mathrm{e}^{-\mathrm{x}}}{(1+\mathrm{e}^{-\mathrm{x}})^2} \tag{2}$$

$$\text{and } \mathrm{F(x)} = \frac{1}{1+\mathrm{e}^{-\mathrm{x}}} \tag{3}$$

3. Variable MDS Plan for Unknown Sigma

In multiple dependent state sampling plan, the past lots will be considered for acceptance of the current lot, consequently the accepting or rejecting decision is effectively postponed. Although the MDS plan is reported to be well-organized in terms of the sample size required. The following assumptions should be considered for the application of MDS plan.

1. Lots are submitted for inspection sequentially in the order of production from a process having a constant proportion non-conforming.
2. The consumer has assurance in the supplier and there should be no reason to believe that a particular lot is poorer than the preceding lots.
3. The quality characteristic follows a Logistic distribution.

3.1 Operating Procedure

Suppose the quality characteristic follows Logistic Distribution with mean a and standard deviation $\sqrt{\pi^2/3}$. Let the upper specification limit be U. The proposed plan operates as follows:

Step1: From each submitted lot take a random sample of size n, $(x_1, x_2, \dots, x_n)$ and compute
$v = \frac{U-\bar{x}}{S}$, where $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ and $S = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2}$, S^2 is an unbiased estimate of σ^2.
Step2: Accept the lot if $v \geq k_a$, and reject the lot if $v < k_r$. If $k_r \leq v < k_a$, then accept the current lot provided that the foregoing m lots were accepted on the condition that $v \geq k_a$.
Thus the proposed unknown sigma variables MDS plan is characterized by four parameters, namely n – sample size, m – preceding lots, k_a – acceptance constant and k_r – acceptance constant.
It is known that $\bar{X} \pm k_a S$ for large sample size approximately follows (Duncan, 1986) [6]:

$$\bar{X} \pm k_a s \sim N\left(\mu \pm k_a\sigma, \frac{\sigma^2}{n} + k_a^2\frac{\sigma^2}{2n}\right)$$

The lot quality is defined by

$$p = P(X > U|\mu) = 1 - L\left(\frac{U-\mu}{S}\right) \tag{4}$$

Where L(.) is the cumulative distribution function of the standard Logistic Variate. Now, the OC function of the variable MDS sampling plan, which gives the proportion of lots that are expected to be accepted for a lot quality p, is defined by

$$P_a(p) = P(v \geq k_a) + P(k_r \leq v < k_a)P(v \geq k_a)^m \tag{5}$$

Where the first term in the right hand side represents the probability of accepting a lot based on a current sample and the second term is the probability of accepting a lot based on the states of the preceding lots.
The acceptable quality level and limiting quality level respectively defined by $p_1 = 1 - F(x_{p_1})$ and $p_2 = 1 - F(x_{p_2})$ where x_{p_1} and x_{p_2} are the values corresponding to standardized logistic distribution at p_1 and p_2.
Analogously, the lot acceptance probability for sigma unknown plan is rewritten as

$$P_a(p) = \Phi(\omega_2) + [\Phi(\omega_1) - \Phi(\omega_2)][\Phi(\omega_2)]^m \tag{6}$$

Where

$\omega_1 = (x_{p_1} - k_r)\sqrt{\frac{n}{1+k_r^2/2}}$ and $\omega_2 = (x_{p_2} - k_a)\sqrt{\frac{n}{1+k_a^2/2}}$

The proposed sampling plan parameters are obtained by considering two points on the OC curve namely, $(p_1, 1-\alpha)$ and (p_2, β), where α and β are respectively producer's risk and consumer's risk associated with p_1 and p_2.
For specified $(p_1, 1-\alpha)$ and (p_2, β) the parameters of the proposed plan should satisfy the following conditions

$$P_a(p_1) = \Phi(\omega_{21}) + [\Phi(\omega_{11}) - \Phi(\omega_{21})][\Phi(\omega_{21}]^m \geq 1-\alpha \quad (7)$$

$$P_a(p_2) = \Phi(\omega_{22}) + [\Phi(\omega_{12}) - \Phi(\omega_{22})][\Phi(\omega_{22}]^m \leq \beta \quad (8)$$

where ω_{11} be the value of ω_1 at $p = p1$, ω_{21} be the value of ω_2 at $p = p1$, ω_{12} be the value of ω_1 at $p = p_2$ and ω_{22} be the value of ω_2 at $p = p_2$.
In order to obtain the optimum plan parameters, the problem is formulated as an optimization problem as follows
Minimize n Subject to

$P_a(p_1) \geq 1-\alpha$

$P_a(p_2) \leq \beta$

$n \geq 1, k_a > k_r > 0$

To solve the non-linear optimization problem, the routine "fmincon" in the Matlab software was used which is based on the sequential quadratic programming algorithm proposed by Nocedal and Wright (1999) [13].

4. Variable MDS Plan for Known Sigma
Suppose the quality characteristic follows a Logistic distribution, with upper specification limit U, specified. The operating procedure for a proposed variable MDS sampling plan is as follows:

Step 1: Draw a sample of n' units from a lot and observe the measurements, $x_1, x_2, \dots, x_{n'}$ of quality characteristic, X and compute $v = \frac{U-\bar{x}}{\sigma}$.

Step 2: Accept the lot if $v \geq k_a{'}$ and reject the lot if $v < k_r{'}$ If $k_r' \leq v < k_a'$ then accept the current lot provided that the preceding m' lots were accepted on the condition that $v \geq k_a{'}$. Reject the lot otherwise. The proposed variable MDS plan is designated by four parameters namely n', m', k_a', k_r'. The fraction non-conforming in a lot will be expressed as

$$p = P\{X > U|\mu\} = 1 - L\left(\frac{U-\mu}{\sigma}\right) \quad (9)$$

where L(.) is the cumulative distribution function of the standard Logistic Variate. Thus, the known mean is related to the fraction non-conforming p through (9). Thus, the probability of acceptance in the lot is defined by

$$P_a(p) = \Phi(\omega_2) + [\Phi(\omega_1) - \Phi(\omega_2)][\Phi(\omega_2)]^{m\prime} \quad (10)$$

Where

$\omega_1 = (k_{p_1} - k_r{'})\sqrt{n'}$ and $\omega_2 = (k_{p_2} - k_{a\sigma}{'})\sqrt{n'}$

The sampling plan parameters are obtained by considering two points on the OC curve, namely, $(p_1, 1-\alpha)$ and (p_2, β)
Values of the plan parameters for m $(or\ m')$ $= 1, 2$ & 3 for several combinations of (p_1 and p_2) under $\alpha = 0.05$ and $\beta = 0.1$ are tabulated.
It is observed that the proposed plan becomes single variable sampling plan with parameter n, k $(or\ n', k_a')$ when $m (or\ m')$ goes to infinity. Also when $k_a = k_r (\ k_a' = k_r{'})$ the proposed becomes variable single sampling plan.

5. Numerical Example
Suppose that it is preferred to set up a MDS sampling plan by giving protection to the consumer and the producer in terms of quality levels $p_1 = 0.01$ and $p_2 = 0.05$ associated with $\alpha = 0.05$ and $\beta = 0.10$, when the quality characteristics is distributed according to Logistic distribution. The plan parameters are $n = 21$, $k_a = 2.1436$, $k_r = 1.5482$, when $m = 2$. For each submitted lot, take a random sample of size 21, and compute $\bar{X}$, S and evaluate $v = \frac{U-\bar{X}}{S}$. Accept the lot if $v \geq 2.1436$ and reject the lot if $v \leq 1.5482$. If $1.5482 \leq v < 2.1436$, then accept the current lot provided that the preceding two lots were accepted on the condition that $v \geq 2.1439$, otherwise reject the lot.

6. Comparison of Sampling Plans

When sigma is unknown, the optimum plan parameters for $p_1 = 1\%, \alpha = 5\%, p_2 = 5\%$ and $\beta = 10\%$ are

i. $n = 21, k_a = 2.2025$ and $k_r = 1.7206$, when $m = 1$.
ii. $n = 21, k_a = 2.1436$ and $k_r = 1.5482$, when $m = 2$.
iii. $n = 22, k_a = 2.1164$ and $k_r = 1.4162$, when $m = 3$.

For the same protection, the optimum parameters of single variable sampling plan (S. Geetha and R.Vijayaraghavan, 2011) [17] are $n = 32, k = 2.022$

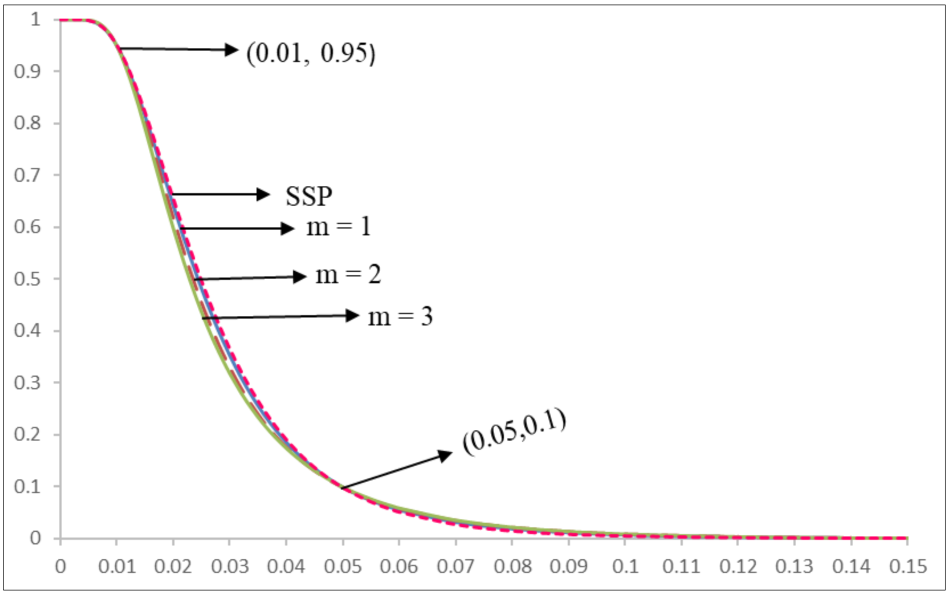

Fig 1: OC Curve for Variable MDS Plan for $m = 1, 2$ & 3 and Variable single sampling Plan

It can be observed from the OC curve that the proposed plan provides almost same protection for consumer and producer as the variable single sampling plan, but with smaller sample size. The proposed MDS plan is compared with single sampling plan for variables under Logistic distribution with respect to sample size. Table 1 is constructed for comparing the MDS and SSP, and can be inferred that the sample size required for proposed plan is comparably less.

Table 1: Comparison of MDS plan with SSP for Variables

p_1	p_2	Unknown Sigma		Known Sigma	
		MDS	SSP	MDS	SSP
0.001	0.002	276	435	39	59
0.0025	0.005	213	335	21	59
0.005	0.020	38	59	9	15
0.010	0.050	21	32	6	11
0.020	0.100	6	23	3	10

7. Conclusion

The intention of this article is to construct a multiple dependent state (MDS) sampling inspection plans for variables, in which the quality characteristics of interest follows logistic distribution. The sample size of the proposed MDS sampling plan for variables is comparably lesser than the conventional variable single sampling plan for logistic distribution. The proposed plan assures nearly the same preservation for producer and consumer as the single sampling plan for logistic distribution, but with a smaller sample size. Hence the plan can be implemented when the testing is destructive and costly. Tables are designed at various values of AQL and LQL for the selection of the proposed plan.

Table 2: Variable MDS Sampling plans for m $(or\ m') = 1$ indexed by AQL and LQL for $\alpha = 5\%$ and $\beta = 10\%$

p_1	p_2	Unknown Sigma			Known Sigma		
		n	k_a	k_r	n'	k_a'	k_r'
0.001	0.002	276	3.6717	3.4301	39	3.6863	1.8432
	0.004	63	3.5324	3.0905	9	3.5386	3.0053
	0.005	46	3.4869	2.9877	7	3.4951	2.8758
	0.010	20	3.3434	2.6852	3	3.3598	2.4724
	0.020	11	3.1965	2.4064	2	3.2235	2.0661
	0.040	6	3.0449	2.1370	1	3.0851	1.6537
	0.050	5	2.9948	2.0537	1	3.0398	1.5187
0.0025	0.005	213	3.1653	2.9256	21	3.2470	2.8941
	0.006	130	3.1287	2.8338	14	3.2115	2.7882

	0.010	48	3.0249	2.5897	7	3.1117	2.4907
	0.050	8	2.6816	1.9087	2	2.7917	1.5371
0.005	0.010	171	2.7811	2.5426	36	2.7830	2.5150
	0.020	38	2.6393	2.2085	9	2.6467	2.1087
	0.030	21	2.5539	2.0268	5	2.5661	1.8685
	0.040	15	2.4920	1.9026	4	2.5083	1.6963
	0.050	12	2.4432	1.8083	3	2.4630	1.5613
0.010	0.020	134	2.3950	2.1570	35	2.3971	2.1271
	0.040	29	2.2506	1.8224	9	2.2587	1.7147
	0.050	21	2.2025	1.7206	6	2.2135	1.5798
	0.060	16	2.1626	1.6390	5	2.1761	1.4683
	0.080	11	2.0981	1.5124	4	2.1161	1.2897
	0.100	9	2.0464	1.4152	3	2.0686	1.1481
0.020	0.040	101	2.0050	1.7659	34	2.0073	1.7333
	0.060	37	1.9189	1.5635	13	1.9246	1.4869
	0.080	22	1.8556	1.4253	8	1.8647	1.3083
	0.100	15	1.8050	1.3200	6	1.8172	1.1667
	0.200	6	1.6341	0.9932	3	1.6600	0.6982
0.050	0.100	64	1.4758	1.2286	31	1.4785	1.1917
	0.200	13	1.3103	0.8567	7	1.3213	0.7232
	0.300	6	1.1972	0.6282	4	1.2168	0.4118
	0.400	4	1.1022	0.4475	3	1.1312	0.1565
	0.500	3	1.0125	0.2846	2	1.0401	0

Table 3: Variable MDS Sampling plans for $m\ (or\ m') = 2$ indexed by AQL and LQL for $\alpha = 5\%$ and $\beta = 10\%$

p_1	p_2	Unknown Sigma			Known Sigma		
		n	k_a	k_r	n'	$k_a{}'$	$k_r{}'$
0.001	0.005	46	3.4274	2.8023	7	3.4292	2.5770
	0.010	20	3.2617	2.4593	3	3.2653	2.0443
	0.020	11	3.0943	2.1484	2	3.1002	1.5078
	0.050	6	2.8682	1.7703	1	2.87789	0.7850
0.005	0.010	170	2.7541	2.4436	36	2.7545	2.3857
	0.020	38	2.5878	2.0485	9	2.5894	1.8492
	0.040	15	2.4183	1.7010	4	2.4218	1.3046
	0.050	12	2.3628	1.5957	3	2.3670	1.1264
0.010	0.020	134	2.3693	1.1847	35	2.3683	1.9969
	0.050	21	2.1436	1.5482	6	2.1459	1.2741
	0.100	9	1.9659	1.2027	3	1.9705	0.7041
0.020	0.040	100	1.9776	1.6685	34	1.9781	1.6011
	0.100	15	1.7453	1.1458	6	1.7479	0.8529
	0.200	6	1.5523	0.7686	3	1.5575	0.2342
0.040	0.100	39	1.5210	1.1235	18	1.5219	1.0039
	0.200	10	1.3287	0.6989	5	1.3316	0.3852
	0.300	5	1.2005	0.4308	2	1.2047	0.0000
0.050	0.100	63	1.4474	1.1281	31	1.4480	1.0533
	0.200	13	1.2554	0.6836	7	1.2576	0.4346
	0.500	3	0.9228	0.0000	2	0.9081	0.0000

Table 4: Variable MDS Sampling plans for $m\ (or\ m') = 3$ indexed by AQL and LQL for $\alpha = 5\%$ and $\beta = 10\%$

p_1	p_2	Unknown Sigma			Known Sigma		
		n	k_a	k_r	n'	$k_a{}'$	$k_r{}'$
0.001	0.010	21	3.2238	2.2908	3	3.2242	1.6603
	0.020	11	3.0459	1.9625	2	3.0466	1.0069
	0.050	6	2.8062	1.5667	1	2.8074	0.1271
0.005	0.010	181	2.7421	1.3710	38	2.7421	1.3710
	0.020	40	2.5643	1.9247	9	2.5645	1.6163
	0.050	12	2.3248	1.4370	3	2.3253	0.7362
0.010	0.040	31	2.1754	1.5431	9	2.1756	1.2169
	0.050	22	2.1164	1.4162	7	2.1166	0.9999
	0.100	9	1.9274	1.0415	3	1.9280	0.3057
0.020	0.050	58	1.9064	1.4388	21	1.9065	1.2655
	0.100	16	1.7175	1.0107	6	1.7178	0.5713
	0.200	6	1.5124	0.5893	3	1.5129	0.0000
0.040	0.080	76	1.5663	1.1834	34	1.5664	1.0686
	0.200	11	1.2994	0.5462	6	1.2997	0.0875
	0.400	4	1.0511	0.0000	2	1.0468	0.0000
0.050	0.100	67	1.4346	1.0464	33	1.4347	0.9292
	0.200	13	1.2297	0.5423	8	1.2299	0.1758
	0.500	3	0.8750	0.0000	2	0.8656	0.0000

References

1. Aijun Yan, Sanyang Liu, Xiaojuan Dong. Designing a multiple dependent state sampling plan based on the coefficient of variation, Yan *et al*. Springer Plus. 2016;5(1):1447.
2. Balamurali S, Chi-Hyuck. Multiple Dependent State Sampling plan for lot acceptance based on measurement data, European Journal of Operation Research. 2007 Aug 1;180(3):1221-1230.
3. Balamurali S, Park H, Jun CH, Kim KJ, Lee J. Designing of variables repetitive group sampling plan involving minimum average sample number, Communication in Statistics-Simulation and Computation. 2005;34(3):799-809.
4. Chien-Wei Wu, Zih-Huei Wang. Developing a variables multiple dependent state sampling plan with simultaneous consideration of process yield and quality loss, International Journal of Production Research. 2017;55(8):2351-2364
5. Collani EV. A note on acceptance sampling for variables, Metrika. 1991 Dec;38(1):19-36.
6. Dunkan AJ. Quality control and Industrial Statistics, fifth ed. Richard D. Irwin, Homewood, Illinois; c1986.
7. Govindaraju K, Soundararajan V. Selection of single sampling plans for variables matching the MIL-STD-105 scheme. J Qual Technol. 1986 Oct 1;18(4):234-8.
8. Govindaraju K, Subramani K. Selection of Multiple deferred state MDS-1 sampling plan for given acceptable quality level and limiting quality level involving minimum risks. Journal of Applied Statistics. 1990;17(3):427-434.
9. Govindaraju K, Subramani K. Selection of multiple deferred (dependent) state sampling plans for given acceptable quality level and limiting quality level. Journal of Applied Statistics. 1993;20(3):423-428.
10. Govindaraju K, Balamurali S. Chain Sampling Plan for Variables Inspection. Journal of Applied Statistics. 1998;25(1):103-109.
11. Kuraimani K, Govindaraju K. Selection of multiple deferred (dependent) state sampling plans. Communications in Statistics-Theory and Methods. 1992;21(5):1339-1366.
12. Muhammad Aslam, Saminathan Balamurali, Chi-Hyuck. A new multiple dependent state sampling plan based on the process capability index, Communications in Statistics - Simulation and Computation, 2021 Jul 5;50(6):1711-27.
13. Nocedal J, Stephen J Wright, Numerical Optimization, Second Edition, Springer, USA; c2000.
14. Ritwik Bhattacharya, Muhammad Aslam. Generalized Multiple Dependent State Sampling Plans in Presence of Measurement Data, IEEE Access. 2020 Sep 3;8:162775-162784.
15. Schilling EG. Acceptance Sampling in Quality Control, Marcel Dekker, New York; c1982.
16. Soundararajan V, Vijayaraghavan R. Construction and Selection of multiple dependent (deferred) state sampling plan, Journal of Applied Statistics. 1990 Jan 1;17(3):397-409.
17. Geetha S, Vijayaraghavan R. Selection of Single Sampling Plans by Variables Based on Logistic Distribution, UGC sponsored National Conference on Recent Trends in Statistical and Computer Applications, Manonmaniam Sundaranar University. Tirunelveli, India, 2011, Mar 23
18. Vaerst R. A procedure to construct multiple deferred state sampling plan, Methods of Operation Research. 1982;37:477-485.
19. Wortham AW, Baker RC. Multiple deferred state sampling inspection, International Journal of Production Research. 1976 Oct 1;14(6):719-731.