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Moderate distribution with six sigma control chart for range

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Abstract
Fuzzy set theory is a powerful mathematical tool for evaluating the vagueness-related uncertainty that can be expressed linguistically in certain situations. Control charts are the most effective and unobtrusive way of statistical process control. Many times, data is gathered in quantitative form; nevertheless, there are many qualitative aspects that cannot be stated numerically, such as appearance, smoothness, and colour, among others. In this research, we develop a six sigma-based fuzzy control limit for a range with a moderate distribution.

Keywords: Fuzzy control chart, moderate distribution, process capability and six sigma

1. Introduction
Control charts are commonly used for monitoring and analyzing a manufacturing process. The ability of control charts to detect process alterations and identify abnormal conditions in the process is what gives them their power (Amirzadeh, 2008) [1]. If 'w' is a sample statistic that measures some quality characteristic of interest, and the mean and standard deviation of w are w, then the control limits are defined as where A is the "distance" of the control limits from the centre line, expressed in standard deviation units (Shewhart, 1924) [5].
Control limitations might not be so exact in many instances. Uncertainty is caused by the measurement system, which includes operators and gauges, as well as ambient circumstances.
Fuzzy set theory is a good technique for dealing with uncertainty in this scenario. Using membership functions, numerical control limits can be converted to fuzzy control limits (Sevil Senturk and Nihal Erginel, 2009) [4]. They compared the fraction faulty chart to the conventional chart.
The Fuzzy Multinomial Chart with Variable Sample Size outperforms the conventional chart. Fuzzy multinomial charts (FM-chart) are an extension of normal control charts that deal with linguistic categories and variable sampling size (VSS), and they compared FM-chart to the ordinary p–chart and EWMA Control Chart (Kawa Jamal Rashid and Suzan Haydar, 2014) [2].
It is observed that the FM chart with VSS outperforms the conventional charts; this method is more sensitive, accurate, and cost-effective in allowing decision makers to regulate the operating system at an early stage, particularly when sample sizes fluctuate. We present a six sigma-based fuzzy control chart for range under moderate distribution of numerical samples in this research study.
In section 2 preliminaries and methods of Control charts. Review some fundamental experimental results in section 3 and we draw the conclusion in Section 4.

2. Preliminaries and Methods
A main goal of statistical process control is to increase product productivity and quality (SPC). The control chart, invented by Walter Shewhart in 1924 [5], is one of the most important tools in statistical process control. The Shewhart control chart, which plots the data produced by a process on a chart restricted by upper and lower specification limits, provides a clear image of the state of a process [2001].
The trapezoidal fuzzy numbers' average range is represented $\left(\bar{R}_a, \bar{R}_b, \bar{R}_c, \bar{R}_d\right)$.

$$\left(\bar{R}_a,\bar{R}_b,\bar{R}_c,\bar{R}_d\right)=\left(\frac{\sum_{j=1}^{m}R_{aj}}{m},\frac{\sum_{j=1}^{m}R_{bj}}{m},\frac{\sum_{j=1}^{m}R_{cj}}{m},\frac{\sum_{j=1}^{m}R_{dj}}{m}\right),\ j=1,2,...m$$

Where

$$\bar{R}_r=\frac{\sum_{j=1}^{m}R_{rj}}{m},r=a,b,c,d\ \ and\ \ j=1,2,...m$$

$$R_{rj}=\left(X_{\max.aj}-X_{\min.dj},X_{\max.bj}-X_{\min.cj},X_{\max.cj}-X_{\min.bj},X_{\max.dj}-X_{\min.aj}\right),\ j=1,2,...m$$

$\left(X_{\max.aj},X_{\max.bj},X_{\max.cj},X_{\max.dj}\right)$, are the maximum trapezoidal fuzzy numbers for each sample and $\left(X_{\min.aj},X_{\min.bj},X_{\min.cj},X_{\min.dj}\right)$ are the minimum trapezoidal fuzzy numbers for each sample.

The Shewhart (1924) [5] control limits for range (R) are given below:

$$UCL_{\bar{R}}=\bar{R}+3d_3\left(\frac{\bar{R}}{d_2}\right)$$

$$CL_{\bar{R}}=\bar{R}$$

$$LCL_{\bar{R}}=\bar{R}-3d_3\left(\frac{\bar{R}}{d_2}\right)$$

where d_2 andd_3 are a control chart co-efficient.

The fuzzy control limits for range are as follows:

$$\begin{aligned}U\tilde{C}L_{\bar{R}}&=\left(\bar{R}_a,\bar{R}_b,\bar{R}_c,\bar{R}_d\right)+\left(\frac{3d_3}{d_2}\right)\left(\bar{R}_a,\bar{R}_b,\bar{R}_c,\bar{R}_d\right)\\&=\left(\bar{R}_a+\frac{3d_3}{d_2}\bar{R}_a,\bar{R}_b+\frac{3d_3}{d_2}\bar{R}_b,\bar{R}_c+\frac{3d_3}{d_2}\bar{R}_c,\bar{R}_d+\frac{3d_3}{d_2}\bar{R}_d\right)\\&=\left(U\tilde{C}L_{a.\bar{R}},U\tilde{C}L_{b.\bar{R}},U\tilde{C}L_{c.\bar{R}},U\tilde{C}L_{d.\bar{R}}\right)\\C\tilde{L}_{\bar{R}}&=\left(\bar{R}_a,\bar{R}_b,\bar{R}_c,\bar{R}_d\right)\end{aligned}$$

$$\begin{aligned}L\tilde{C}L_{\bar{R}}&=\left(\bar{R}_a,\bar{R}_b,\bar{R}_c,\bar{R}_d\right)-\left(\frac{3d_3}{d_2}\right)\left(\bar{R}_a,\bar{R}_b,\bar{R}_c,\bar{R}_d\right)\\&=\left(\bar{R}_a-\frac{3d_3}{d_2}\bar{R}_a,\bar{R}_b-\frac{3d_3}{d_2}\bar{R}_b,\bar{R}_c-\frac{3d_3}{d_2}\bar{R}_c,\bar{R}_d-\frac{3d_3}{d_2}\bar{R}_d\right)\\&=\left(L\tilde{C}L_{a.\bar{R}},L\tilde{C}L_{b.\bar{R}},L\tilde{C}L_{c.\bar{R}},L\tilde{C}L_{d.\bar{R}}\right)\end{aligned}$$

The proposed standard deviation $\left(\sigma_{r.FMD:6\sigma},\ r=a,b,c,d\right)$ for a six sigma-based fuzzy control chart with a moderate distribution using process capability is,

$$C_p=\frac{USL_{r.RFC_p}-LSL_{r.RFC_p}}{6\sigma},r=a,b,c,d\ .$$

Under moderate distribution, a JAVA script (Radhakrishnan and Balamurugan, 2012) [3] is used to calculate by the specified tolerance level from the relation,

$$\frac{\left(\dfrac{\sum_{j=1}^{m} R_{rj}}{m}\right) d_3}{d_2}, \; r = a,b,c,d \;\; and \;\; j = 1,2,...m.$$

To obtain the six sigma based fuzzy control limits for mean utilizing range under Moderate distribution, enter the value in the control limits. The value of $A_{FMD:6\sigma}$ is obtained using $p(z \le z_{6\sigma}) = 1-\alpha_1, \alpha_1 = 3.4 \text{ x } 10^{-6}$ and z is a standard moderate variate.

As a result, the following is the result of the suggested six sigma-based fuzzy control limits for range under moderate distribution:

$$U\tilde{C}L_{\bar{R}:C_p} = \left(\bar{R}_a + A_{FMD:6\sigma}\tilde{\sigma}_{aR.FC_p}, \bar{R}_b + A_{FMD:6\sigma}\tilde{\sigma}_{bR.FC_p}, \bar{R}_c + A_{FMD:6\sigma}\tilde{\sigma}_{cR.FC_p}, \bar{R}_d + A_{FMD:6\sigma}\tilde{\sigma}_{dR.FC_p}\right)$$

$$= \left(U\tilde{C}L_{a.\bar{R}:C_p}, U\tilde{C}L_{b.\bar{R}:C_p}, U\tilde{C}L_{c.\bar{R}:C_p}, U\tilde{C}L_{d.\bar{R}:C_p}\right)$$

$$C\tilde{L}_{\bar{R}:C_p} = \left(\bar{R}_a, \bar{R}_b, \bar{R}_c, \bar{R}_d\right)$$

$$L\tilde{C}L_{\bar{R}:C_p} = \left(\bar{R}_a - A_{FMD:6\sigma}\tilde{\sigma}_{aR.FC_p}, \bar{R}_b - A_{FMD:6\sigma}\tilde{\sigma}_{bR.FC_p}, \bar{R}_c - A_{FMD:6\sigma}\tilde{\sigma}_{cR.FC_p}, \bar{R}_d - A_{FMD:6\sigma}\tilde{\sigma}_{dR.FC_p}\right)$$

$$= \left(L\tilde{C}L_{a.\bar{R}:C_p}, L\tilde{C}L_{b.\bar{R}:C_p}, L\tilde{C}L_{c.\bar{R}:C_p}, L\tilde{C}L_{d.\bar{R}:C_p}\right)$$

3. Experimental Study

Consider the procedure by which a company in the Salem District manufactures coils. The primary data gathered and shown in Table-1 were obtained by randomly selecting samples of size 5 from the process and measuring the 'between' measurement resistance values (in ohms) of the coils for the application. Table-2 shows the results of converting these measurements into trapezoidal fuzzy numbers (TFN) using a computer software.

Table 1: Resistance values (in ohms) of coils

Sample No.	X_1		X_2		X_3		X_4		X_5	
1	0.52	0.55	0.49	0.51	0.56	0.57	0.49	0.50	0.52	0.53
2	0.52	0.53	0.51	0.52	0.52	0.54	0.51	0.53	0.45	0.47
3	0.51	0.52	0.53	0.54	0.54	0.55	0.49	0.55	0.50	0.51
4	0.42	0.45	0.42	0.44	0.46	0.56	0.51	0.54	0.53	0.54
5	0.48	0.50	0.47	0.51	0.50	0.50	0.57	0.58	0.52	0.53
6	0.55	0.56	0.49	0.50	0.50	0.52	0.48	0.49	0.50	0.51
7	0.49	0.53	0.52	0.54	0.49	0.55	0.46	0.48	0.49	0.50
8	0.43	0.46	0.49	0.52	0.49	0.51	0.50	0.53	0.50	0.51
9	0.54	0.55	0.48	0.53	0.51	0.53	0.47	0.49	0.48	0.50
10	0.50	0.52	0.49	0.51	0.48	0.49	0.48	0.52	0.46	0.47
11	0.47	0.50	0.55	0.57	0.50	0.52	0.49	0.50	0.48	0.50
12	0.50	0.54	0.56	0.57	0.47	0.51	0.48	0.51	0.48	0.50
13	0.46	0.48	0.52	0.54	0.50	0.51	0.53	0.54	0.50	0.51
14	0.50	0.51	0.48	0.51	0.48	0.52	0.51	0.53	0.44	0.45
15	0.49	0.51	0.50	0.51	0.54	0.55	0.48	0.52	0.49	0.50

Table 2: Trapezoidal Fuzzy measurement resistance values of coils

Sample No.	X_1				X_2				X_3			
1	0.45	0.52	0.55	0.58	0.48	0.49	0.51	0.53	0.55	0.56	0.57	0.58
2	0.50	0.52	0.53	0.55	0.45	0.51	0.52	0.55	0.50	0.52	0.54	0.56
3	0.47	0.51	0.52	0.56	0.52	0.53	0.54	0.58	0.53	0.54	0.55	0.56
4	0.40	0.42	0.45	0.48	0.41	0.42	0.44	0.49	0.41	0.46	0.56	0.57
5	0.45	0.48	0.50	0.53	0.45	0.47	0.51	0.54	0.46	0.50	0.50	0.52
6	0.52	0.55	0.56	0.57	0.47	0.49	0.50	0.53	0.47	0.50	0.52	0.53
7	0.46	0.49	0.53	0.56	0.51	0.52	0.54	0.55	0.45	0.49	0.55	0.56
8	0.40	0.43	0.46	0.47	0.44	0.49	0.52	0.55	0.46	0.49	0.51	0.54
9	0.53	0.54	0.55	0.57	0.41	0.48	0.53	0.56	0.46	0.51	0.53	0.55
10	0.49	0.50	0.52	0.54	0.44	0.49	0.51	0.54	0.45	0.48	0.49	0.52
11	0.41	0.47	0.50	0.53	0.54	0.55	0.57	0.58	0.46	0.50	0.52	0.55
12	0.48	0.50	0.54	0.56	0.54	0.56	0.57	0.59	0.42	0.47	0.51	0.53

13	0.44	0.46	0.48	0.50	0.51	0.52	0.54	0.55	0.48	0.50	0.51	0.56
14	0.47	0.50	0.51	0.53	0.45	0.48	0.51	0.57	0.46	0.48	0.52	0.58
15	0.46	0.49	0.51	0.55	0.48	0.50	0.51	0.56	0.53	0.54	0.55	0.56

Sample No.	**X_4**				**X_5**			
1	0.47	0.49	0.50	0.52	0.49	0.52	0.53	0.54
2	0.49	0.51	0.53	0.55	0.42	0.45	0.47	0.49
3	0.48	0.49	0.55	0.58	0.48	0.50	0.51	0.52
4	0.46	0.51	0.54	0.57	0.50	0.53	0.54	0.55
5	0.56	0.57	0.58	0.59	0.49	0.52	0.53	0.54
6	0.46	0.48	0.49	0.50	0.48	0.50	0.51	0.52
7	0.44	0.46	0.48	0.51	0.47	0.49	0.50	0.55
8	0.48	0.50	0.53	0.55	0.49	0.50	0.51	0.53
9	0.46	0.47	0.49	0.54	0.45	0.48	0.50	0.52
10	0.45	0.48	0.52	0.56	0.42	0.46	0.47	0.48
11	0.41	0.49	0.50	0.54	0.45	0.48	0.50	0.52
12	0.45	0.48	0.51	0.55	0.47	0.48	0.50	0.52
13	0.48	0.53	0.54	0.55	0.48	0.50	0.51	0.53
14	0.50	0.51	0.53	0.54	0.42	0.44	0.45	0.48
15	0.46	0.48	0.52	0.55	0.41	0.49	0.50	0.51

The average of range for the trapezoidal fuzzy numbers are represented as $\left(\bar{R}_a, \bar{R}_b, \bar{R}_c, \bar{R}_d\right)$.

$$R_{a1} = \left(X_{\max.aj} - X_{\min.dj}, X_{\max.bj} - X_{\min.cj}, X_{\max.cj} - X_{\min.bj}, X_{\max.dj} - X_{\min.aj}\right)$$
$$= (0.55-0.52, 0.56-0.50, 0.57-0.49, 0.58-0.45)$$
$$R_{a2} = (0.50-0.49, 0.52-0.47, 0.54-0.45, 0.56-0.42)$$

and so on....

$$\bar{R}_a = \frac{R_{a1}+R_{a1}+...+R_{a15}}{15} = \frac{0.03+0.01+...+0.02}{15} = 0.0173$$
$$\bar{R}_b = \frac{R_{b1}+R_{b1}+...+R_{b15}}{15} = \frac{0.06+0.05+...+0.04}{15} = 0.0520$$
$$\bar{R}_c = \frac{R_{c1}+R_{c1}+...+R_{c15}}{15} = \frac{0.08+0.09+...+0.07}{15} = 0.0887$$
$$\bar{R}_d = \frac{R_{d1}+R_{d1}+...+R_{d15}}{15} = \frac{0.13+0.14+...+0.15}{15} = 0.1427$$

$$\left(\bar{R}_a, \bar{R}_b, \bar{R}_c, \bar{R}_d\right) = (0.0173, 0.520, 0.0887, 0.1427)$$

The fuzzy control limits for range are as follows:

$$U\tilde{C}L_{\bar{R}} = (0.0173, 0.0520, 0.0887, 0.1427) + \left(\frac{3\times 0.864}{2.326}\right)(0.0173, 0.0520, 0.0887, 0.1427)$$
$$= \begin{pmatrix} 0.0173 + \frac{3\times 0.864}{2.326}0.0173, 0.0520 + \frac{3\times 0.864}{2.326}0.0520, \\ 0.0887 + \frac{3\times 0.864}{2.326}0.0887, 0.1427 + \frac{3\times 0.864}{2.326}0.01427 \end{pmatrix}$$
$$= (0.0366, 0.1099, 0.1875, 0.3016)$$
$$C\tilde{L}_{\bar{R}} = (0.0173, 0.0520, 0.0887, 0.1427)$$
$$L\tilde{C}L_{\bar{R}} = (0.0173, 0.0520, 0.0887, 0.1427) - \left(\frac{3\times 0.864}{2.326}\right)(0.0173, 0.0520, 0.0887, 0.1427)$$

$$= \begin{pmatrix} 0.0173 - \dfrac{3\times 0.864}{2.326} 0.0173, 0.0520 - \dfrac{3\times 0.864}{2.326} 0.0520, \\ 0.0887 - \dfrac{3\times 0.864}{2.326} 0.0887, 0.1427 - \dfrac{3\times 0.864}{2.326} 0.01427 \end{pmatrix}$$

$$= \left(-0.0020 \approx 0, -0.0059 \approx 0, -0.0101 \approx 0, -0.0163 \approx 0\right)$$

The proposed standard deviation

$$USL_{aR.FC_p} - LSL_{aR.FC_p} = 0.01486 - 0.00000 \Rightarrow \tilde{\sigma}_{aR.FC_p} = 0.00124$$
$$USL_{bR.FC_p} - LSL_{bR.FC_p} = 0.03343 - 0.01114 \Rightarrow \tilde{\sigma}_{bR.FC_p} = 0.00186$$
$$USL_{cR.FC_p} - LSL_{cR.FC_p} = 0.05200 - 0.02229 \Rightarrow \tilde{\sigma}_{cR.FC_p} = 0.00248$$
$$USL_{dR.FC_p} - LSL_{dR.FC_p} = 0.06315 - 0.04086 \Rightarrow \tilde{\sigma}_{dR.FC_p} = 0.00186$$

Therefore the resultant of proposed fuzzy control limits for range using process capability is given below:

$$U\tilde{C}L_{\bar{R}:C_p} = \left(\bar{R}_a + A_{FMD:6\sigma}\tilde{\sigma}_{aR.FC_p}, \bar{R}_b + A_{FMD:6\sigma}\tilde{\sigma}_{bR.FC_p}, \bar{R}_c + A_{FMD:6\sigma}\tilde{\sigma}_{cR.FC_p}, \bar{R}_d + A_{FMD:6\sigma}\tilde{\sigma}_{dR.FC_p}\right)$$
$$= \begin{pmatrix} [0.0173 + 4.5\times 0.00124], [0.0520 + 4.5\times 0.00186], \\ [0.0887 + 4.5\times 0.00248], [0.1427 + 4.5\times 0.00186] \end{pmatrix}$$
$$= (0.0229, 0.0604, 0.0998, 0.1510)$$
$$C\tilde{L}_{\bar{R}:C_p} = \left(\bar{R}_a, \bar{R}_b, \bar{R}_c, \bar{R}_d\right) = (0.0173, 0.0520, 0.0887, 0.1427)$$

$$L\tilde{C}L_{\bar{R}:C_p} = \left(\bar{R}_a - A_{FMD:6\sigma}\tilde{\sigma}_{aR.FC_p}, \bar{R}_b - A_{FMD:6\sigma}\tilde{\sigma}_{bR.FC_p}, \bar{R}_c - A_{FMD:6\sigma}\tilde{\sigma}_{cR.FC_p}, \bar{R}_d - A_{FMD:6\sigma}\tilde{\sigma}_{dR.FC_p}\right)$$
$$= \begin{pmatrix} [0.0173 - 4.5\times 0.00124], [0.0520 - 4.5\times 0.00186], \\ [0.0887 - 4.5\times 0.00248], [0.1427 - 4.5\times 0.00186] \end{pmatrix}$$
$$= (0.0118, 0.0436, 0.0775, 0.1343)$$

It demonstrates that the estimated number of samples required to identify a shift of'multiple of' under the six sigma-based control chart for a fuzzy range with a moderate distribution is more agile than the existing fuzzy range control limits.

4. Conclusion

The developed six sigma-based fuzzy control chart for range under moderate distribution, methodologies selected and explained in the study article by using process capability (Cp) as the only base. This article discusses the use of a fuzzy control chart with a moderate distribution, which eliminates the flaws of existing control charts. It specifically displays one of the six sigma-based fuzzy control charts and demonstrates the ease of use in practice using real-world data.

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