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Interquartile range control chart for six sigma under moderateness

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Abstract
Statistical process control (SPC) is the process of analyzing a random sample of a process's output and determining if the process is generating products with characteristics that fall within a predetermined range. SPC provides a response to the issue of whether or not the process is working well. The suggested six sigma-based control chart for mean with Inter-quartile range (IQR) under moderate distribution outperforms the Shewhart (1931) control chart for mean with moderate sample sizes.

Keywords: Inter-quartile range, robust control chart, Moderate distribution and six sigma

1. Introduction

When the underlying normality assumption is broken, one of the most often used statistical approaches is the robust method. These approaches give a helpful and practical alternative to classic statistical methods, providing more precise findings, frequently offering better statistical power and increased sensitivity while remaining efficient assuming the normal assumption is right (Moustafa Omar Ahmed Abu-Shawiesh, 2008) [3]. The standard deviation is a measure of the dispersion around the mean. As a result, calculating the standard deviation when utilizing the median as the measure of central tendency is impractical. When estimating the spread around the median, other statistics may be more relevant. Calculating the range is a common metric used to quantify the spread.

The range is calculated by subtracting the sample's smallest value, y_1, from its biggest value, y_n. The range has the same difficulty as the mean and the median in that it has the worst qualities of both. It is not resistive, like the mean. Either outlier in any direction will have a major impact on the range's value (Nuri Celik, 2015) [4]. It, like the median, disregards the numerical values of the majority of the data. That is not to imply that the range does not contain important information or that it is a simple statistic to construct.

To sidestep the challenge of dealing with outliers, we may construct an alternative measure of dispersion known as the Inter-quartile range (IQR). Subtracting the first quartile value (q1) from the third quartile value yields the Inter-quartile range (q3). We define q1 for a sample of observations as the order statistic below which 25% of the data falls. Similarly, q3 is defined as the order statistic below which 75% of the data falls.

The population IQR for a continuous distribution is defined as IQR=Q3-Q1, where Q3 and Q1 are determined by calculating the integrals 0.75 and $0.75 = \int_{-\infty}^{Q_3} f(x)dx$ and $0.25 = \int_{-\infty}^{Q_1} f(x)dx$. The function f(x) is continuous across the X support that fulfils the two conditions, (i) $f(x) \geq 0$ and $\int_{-\infty}^{\infty} f(x)dx = 1$.

2. Preliminaries & Methods of IQR (Concept of Inter-quartile Range)

The Inter-quartile range (IQR) is a measure of variability that is calculated by categorizing a data set into quartiles. A rank-ordered data collection is divided into four equal pieces by these quartiles. The values that split each section are known as the first, second, and third quartiles, and are symbolized by the letters Q1, Q2, and Q3, accordingly (Dewey and Whaley, 2005) [1].

- In the first half of the rank-ordered data set, Q1 is the "middle" value.
- Q2 is the set's median value.

- Q3 is the "middle" number in the rank-ordered data set's second half.
- Q3 minus Q1 is the Inter-quartile range.

2.1 Assumptions for the study

- If the data is discovered to be non-normal, robust six sigma-based control limits will be employed under moderateness.
- Businesses those are ready to incorporate the Inter-quartile Range (IQR) idea into their procedures.

2.2 Methods and materials

The six sigma-based control limits use Inter-quartile range (IQR) under moderate distribution in the course of "z-score," which corresponds to areas under the moderate curve of 0.25 and 0.75, respectively, to coin the quality control constant. Thus, Q3=0.8453+ and Q1=-0.8453+ imply that the use of the normal distribution is motivated by the central limit theorem.

$$f(x) = \frac{1}{\pi\delta} e^{-\frac{1}{\pi}\left(\frac{X-\mu}{\delta}\right)^2}, -\infty < X < \infty \text{ and } \delta > 0.$$

A six sigma-based control chart with Inter-quartile range (IQR) under moderate distribution for mean is one way. To determine the process standard deviation $\sigma_{6\sigma.MD:\bar{X}-IQR}$.

Apply the value of $\sigma_{6\sigma.MD:\bar{X}-IQR}$ in the control limits (Radhakrishnan and Balamurugan, 2012) [5].

$$\bar{\bar{X}} \pm \left(\frac{\delta_{6\sigma}}{\sqrt{n}}\right)\sigma_{6\sigma.MD:\bar{X}-IQR},$$

to get the six sigma based control limits for mean under moderate distribution.

where $\sigma_{6\sigma.MD:\bar{X}-IQR}$ is replaced instead of σ in the Shewhart 3-Sigma control chart and $\delta_{MD.6\sigma}$ is obtained using $p(z \le z_{6\sigma}) = 1-\alpha_1, \alpha_1 = 3.4 \text{ x } 10^{-6}$ and z is a standard moderate variate.

2.3 Example

The example offered by E.L. Grant in 1952 is taken into account here. The tensile strengths of specific aluminum-alloy castings are represented by the following statistics (psi) based on reading numbers.

2.3.1 Shewhart 3σ control chart for mean

The 3σ control limits suggested by Shewhart (1931) [6] are

$$\bar{\bar{X}} \pm \left(\frac{3\sigma}{\sqrt{n}}\right)$$

$$UCL = \bar{\bar{X}} + \left(\frac{3\sigma}{\sqrt{n}}\right) = 79 + \left(\frac{3\times 3.74}{\sqrt{5}}\right) = 84.02$$

$$CL = \bar{\bar{X}} = 79$$

$$LCL = \bar{\bar{X}} - \left(\frac{3\sigma}{\sqrt{n}}\right) = 79 - \left(\frac{3\times 3.74}{\sqrt{5}}\right) = 73.98$$

Table 1: Measurements of tensile strengths of certain aluminium-alloy

Reading Number	X_a	X_b	X_c	X_d	X_e	Mean	IQR_{z_S}
1-5	77	74	73	84	77	77	2.22
6-10	78	85	80	81	80	81	0.74
11-15	75	69	72	83	79	76	5.19
16-20	75	80	79	74	78	77	2.97
21-25	70	74	83	72	79	76	5.19
26-30	73	81	87	82	79	80	2.22
31-35	78	79	78	74	85	79	0.74
36-40	83	79	83	81	84	82	1.48
41-45	81	88	79	80	78	81	1.48
46-50	77	80	85	80	78	80	1.48
51-55	72	75	73	85	79	77	4.45
56-60	78	82	80	76	76	78	2.97
61-65	79	75	83	81	78	79	2.22
66-70	82	76	78	78	79	79	0.74
71-75	86	79	79	84	74	80	3.71
76-80	76	75	77	82	77	77	0.74
81-85	79	77	72	77	81	77	1.48
86-90	83	75	82	90	77	81	4.45
91-95	80	78	83	81	74	79	2.22
96-100	79	80	79	75	84	79	0.74
						$\bar{\bar{X}} = 79$	$\overline{IQR_{z_S}} = 2.37$

The resultant Figure 1 clearly shows that the process is under control, since all of the group numbers are inside the control limits and the control limit interval is 10.04 for n=5.

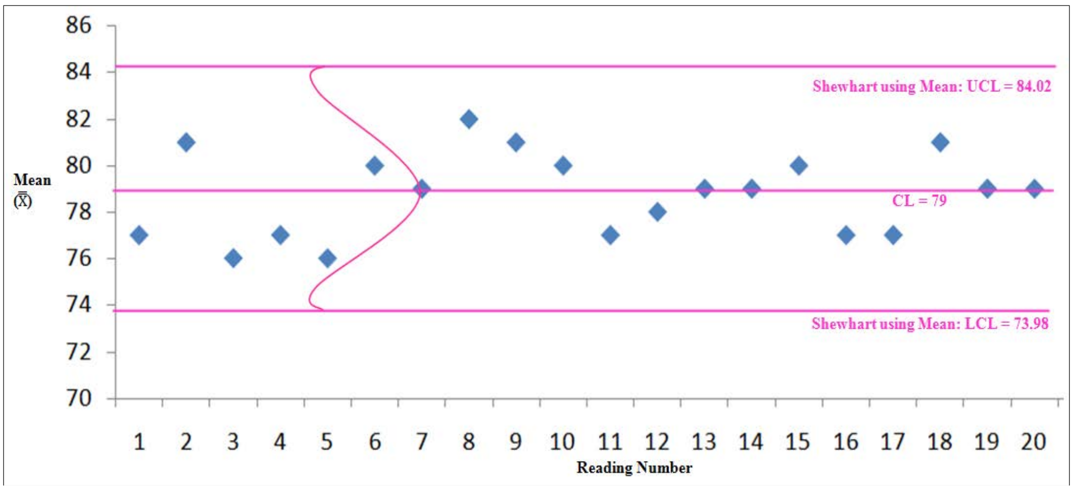

Fig 1: Shewhart control chart for mean

2.1.2 Inter quartile range (IQR) control chart for mean

The 3σ control limits based on IQR are $\bar{\bar{X}} \pm \left(\frac{3\overline{IQR}_{Z_s}}{\sqrt{n}}\right)$

$$UCL = \bar{\bar{X}} + \left(\frac{3\overline{IQR}_{Z_s}}{\sqrt{n}}\right) = 79 + \left(\frac{3\times 2.37}{\sqrt{5}}\right) = 82.18$$

$$CL = \bar{\bar{X}} = 82.18$$

$$LCL = \bar{\bar{X}} - \left(\frac{3\overline{IQR}_{Z_s}}{\sqrt{n}}\right) = 79 - \left(\frac{3\times 2.37}{\sqrt{5}}\right) = 75.82$$

Figure 2 shows that the process is under control, since the full group numbers are inside the control limits and the control limit interval is 6.36 for n=5.

3. Experimental Study

Under a modest distribution, the proposed Six Sigma-based Inter-quartile range (IQR) control chart for the mean difference between upper and lower specification limits is 4.45 (USL - LSL = 5.19 - 0.74).

It is known as the tolerance level (TL), and the process capacity (Cp) is set to 2.0. The value of $\sigma_{6\sigma.MD:\bar{X}-IQR}$ is 0.30. The six sigma based control limits for mean of inter quartile range (IQR) under moderate distribution for a specified tolerance level with the control limits $\bar{\bar{X}} \pm \left(\frac{\delta_{6\sigma}}{\sqrt{n}}\right)\sigma_{6\sigma.MD:\bar{X}-IQR}$,

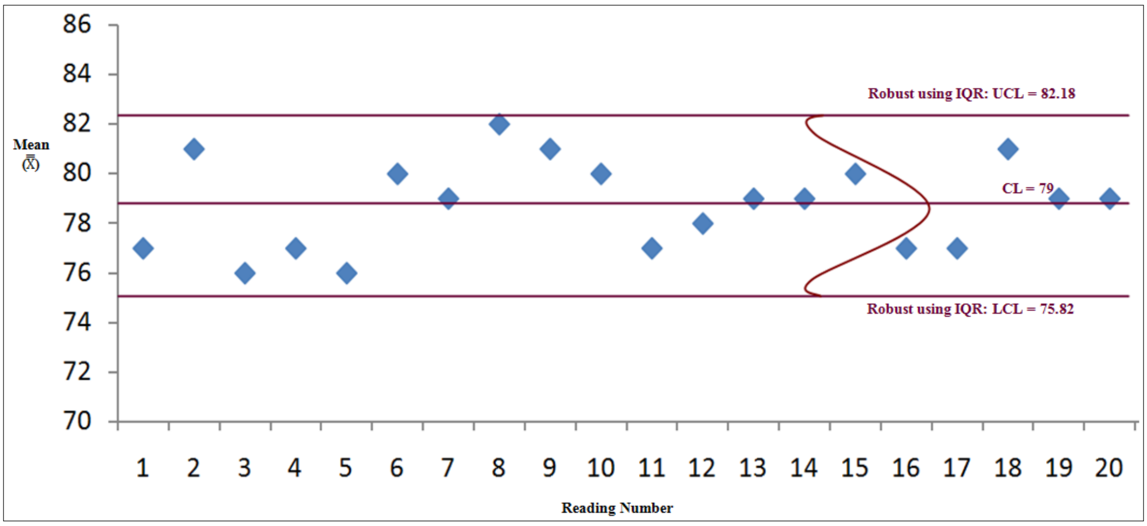

Fig 2: Inter quartile range (IQR) control chart for mean

$$UCL = \bar{\bar{X}} + \left(\frac{\delta_{6\sigma}}{\sqrt{n}}\right)\sigma_{6\sigma.MD:\bar{X}-IQR} = 79 + \left(\frac{3\times 0.30}{\sqrt{5}}\right) = 79.40$$

$$CL = \bar{\bar{X}} = 79$$

$$LCL = \bar{\bar{X}} - \left(\frac{\delta_{6\sigma}}{\sqrt{n}}\right)\sigma_{6\sigma.MD:\bar{X}-IQR} = 79 - \left(\frac{3\times 0.30}{\sqrt{5}}\right) = 78.60$$

The above data show that the process is out of control, since readings 2, 6, 8, 9, 10, 15, and 18 are above the higher control limit while readings 1, 3, 4, 5, 11, 12, 16, and 17 are below the lower control limit. For n=5, the control limit interval is 0.99.

Table 3: Assessment of Shewhart, IQR and Six sigma based IQR under moderate distribution

Control limits	Shewhart control chart	IQR	Six sigma based IQR under moderate distribution
LCL	73.98	75.82	78.60
CL	79.00	79.00	79.00
UCL	84.02	82.18	79.40

The results show that the process is under control when the control limits of Shewhart 3–Sigma and IQR are employed, but it is out of control when the six sigma-based control limits of IQR under moderate distribution are utilized. Under moderate distribution, the control limits interval of IQR based on six sigma is less than the control limits interval of Shewhart and IQR.

4. Conclusion

We provided a control chart based on six sigma with a solid IQR under a modest distribution for the mean. The numerical example results show that the proposed robust method performs better in the presence of moderate distribution, as many points fall outside the control limits than the existing control charts, and the six sigma based control limits interval of IQR under moderate distribution is smaller than the control limits intervals of Shewhart and IQR. It is obvious that the product/service is not of the required quality; thus, the process/system must be modified and improved.

Furthermore, in the situation of non-normality, it is advised to adopt the proposed six sigma based robust control charts under moderate distribution as an alternative to the Shewhart control chart for mean. The proposed six sigma-based control charts will not only help the company provide higher quality, but it will also boost consumer fulfillment and self-assurance.

5. References

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